

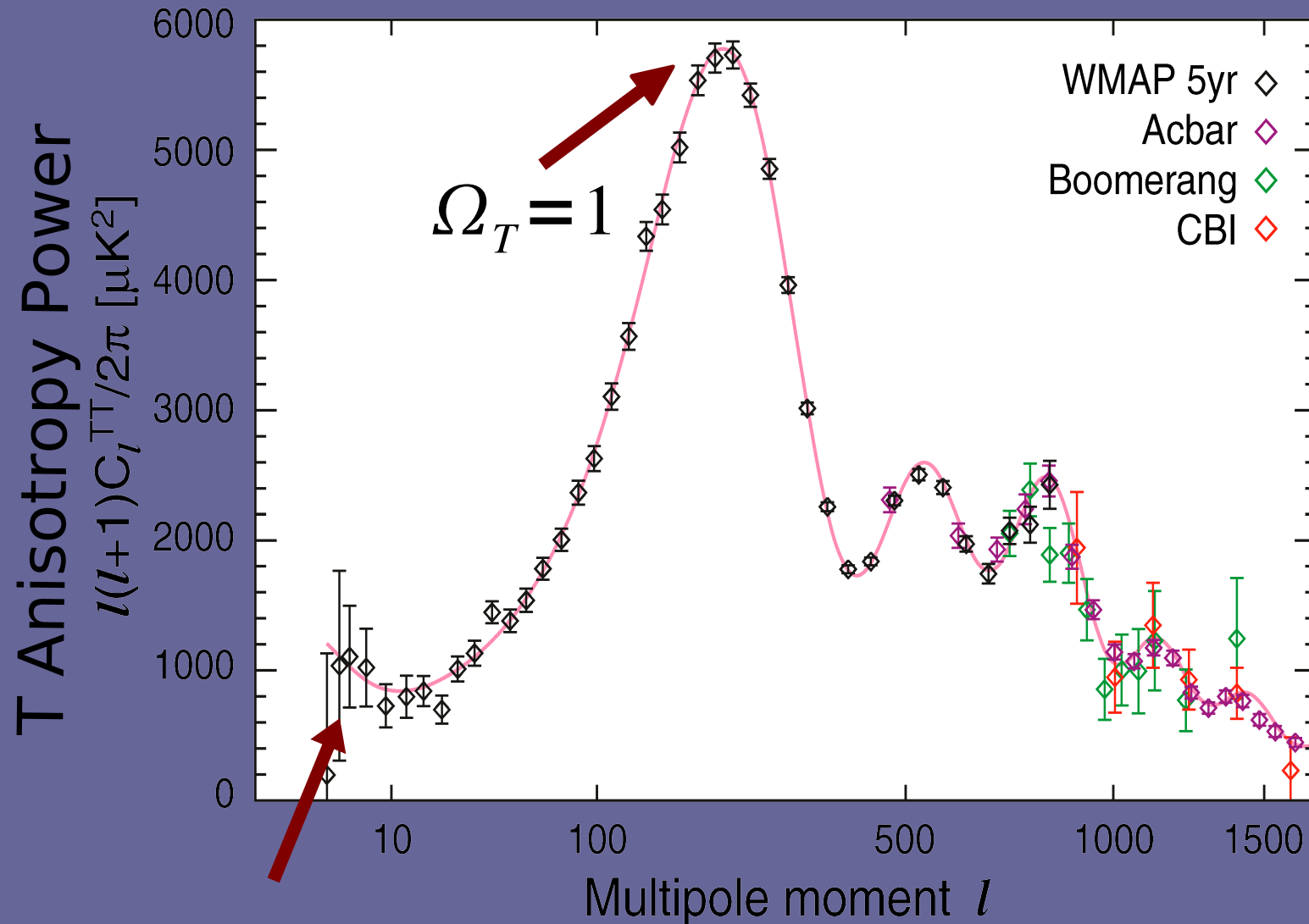
# Inflation & CMB

- Cosmological Observations:
  - Why inflation
  - “Slow-roll” inflation
  - Primordial spectrum
- Model building:
  - Susy (sugra) models of inflation:
    - “eta” problem
    - moduli problem
    - Inflation & Shift/Heisenberg sym.
- Summary

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# Cosmic Microwave Background Radiation

## Wilkinson Microwave Anisotropy Probe



astro-ph/0803.0593

COBE (1992)  
Boomerang  
Maxima  
DASI  
CBI  
VSA

PLANCK  
(14/05/09)

Largest observable scale  
 $O(3000 \text{ Mpc})$

Galaxy scale  
 $O(\text{Mpc})$

# Cosmological Parameters (LCDM)

TABLE 1

| Parameter                   | WMAP5-year                    | WMAP5+BAO+SN Mean             |
|-----------------------------|-------------------------------|-------------------------------|
| $H_0$                       | $71.9^{+2.6}_{-2.7}$ km/s/Mpc | $70.5 \pm 1.3$ km/s/Mpc       |
| $t_0^a$                     | $13.69 \pm 0.13$ Gyr          | $13.72 \pm 0.12$ Gyr          |
| $\Omega_b$ (barions)        | $0.0441 \pm 0.0030$           | $0.0456 \pm 0.0015$           |
| $\Omega_c$ (CDM)            | $0.214 \pm 0.027$             | $0.228 \pm 0.013$             |
| $\Omega_\Lambda$ (DE)       | $0.742 \pm 0.030$             | $0.726 \pm 0.015$             |
| Curvature ( $w \neq -1$ )   | $-0.063 < \Omega_k < 0.017$   | $-0.0179 < \Omega_k < 0.0081$ |
| Dark Energy (Constant $w$ ) | $-1.37 < 1 + w < 0.32$        | $-0.14 < 1 + w < 0.12$        |

**Homogeneous background**

**Flat Geometry**

**Primordial spectrum: ~adiabatic, scale-invariant, gaussian**

**Tensors?**

**Gaussian?**

|                          |                                  |                                    |
|--------------------------|----------------------------------|------------------------------------|
| $\Delta_R^2(k_0^b)$      | $(2.41 \pm 0.11) \times 10^{-9}$ | $(2.445 \pm 0.096) \times 10^{-9}$ |
| $n_s$                    | $0.963^{+0.014}_{-0.015}$        | $0.960 \pm 0.013$                  |
| Gravitational Wave       | $r < 0.43$                       | $r < 0.22$                         |
| Running Index ( $r=0$ )  | $-0.090 < dn_s/d \ln k < 0.019$  | $-0.068 < dn_s/d \ln k < 0.012$    |
| Gaussianity: Local       | $-9 < f_{NL}^{local} < 111$      |                                    |
| Gaussianity: Equilateral | $-151 < f_{NL}^{equil} < 253$    |                                    |

<sup>a</sup>The present-day age of the universe  
<sup>b</sup> $k_0 = 0.002 \text{ Mpc}^{-1}$ .

# Expanding Universe

## Flatness problem

$$\Omega_T = 1 \Rightarrow \Omega_T(t_{\text{nuc}}) - 1 \approx 10^{-16}$$

## Horizon problem

The observable Universe was larger than the **particle horizon** at LSS

## Inflation

Early period of accelerated expansion

$$\ddot{a} > 0 : P < -\rho/3$$

## Super-horizon perturbations?

Too small sub-horizon  
(**causal**) perturbations

Unwanted relics  
**monopoles**, moduli, gravitinos, ..

# Slow Roll Inflation

Scalar field rolling down its (flat) potential

**Energy & pressure**

$$\rho = \dot{\phi}^2/2 + V(\phi) \approx V(\phi) \approx \text{Constant}$$

$$P = \dot{\phi}^2/2 - V(\phi) \approx -V(\phi) \quad \text{negative pressure}$$

**“Flat” potential**

The curvature and the slope smaller than the (Hubble) expansion rate

**EOM**

$$\underline{\ddot{\phi} \ll 3H\dot{\phi}, \dot{\phi}^2 \ll V} \quad \Rightarrow \quad 3H\dot{\phi} \simeq -V_{\phi}$$

**Slow-roll parameters**

$$|\eta_{\phi}| = m_P^2 \left| \frac{V'''}{V} \right| < 1$$

$$\epsilon_{\phi} = \frac{m_P^2}{2} \left( \frac{V'}{V} \right)^2 < 1 \quad \leftarrow$$

$$\eta_H \simeq \eta_{\phi} - \epsilon_{\phi}$$

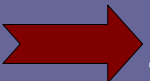
$$|\eta_H| = 2 m_P^2 \left| \frac{H'''}{H} \right| < 1$$

$$\epsilon_H = 2 m_P^2 \left( \frac{H'}{H} \right)^2 < 1$$

# Slow Roll Inflation

- The observable Universe should be within the horizon at the beginning of inflation

**No. of e-folds:**  $N_e \simeq 50 - 60$

- Quantum fluctuations  Primordial Spectrum

$$P_{\delta\phi} \simeq (H/2\pi)_{(k=aH)}^2 \quad P_R \simeq (H/\dot{\phi})^2 P_{\delta\phi} \simeq (5 \times 10^{-5})^2$$

**(  $H \sim \text{constant}$  ,  $P_R \sim \text{scale invariant}$  )**

- Gravitational waves:  $ds^2 = dt^2 - a(t)^2 (\delta_{ij} (1 + 2\psi) + h_{ij}) dx^i dx^j$

$$P_T \simeq 8 (H/(2\pi m_p))^2$$

$$r = P_T/P_R \simeq 16\epsilon_\phi$$

**Tensor-to-scalar ratio  $r < 1$**

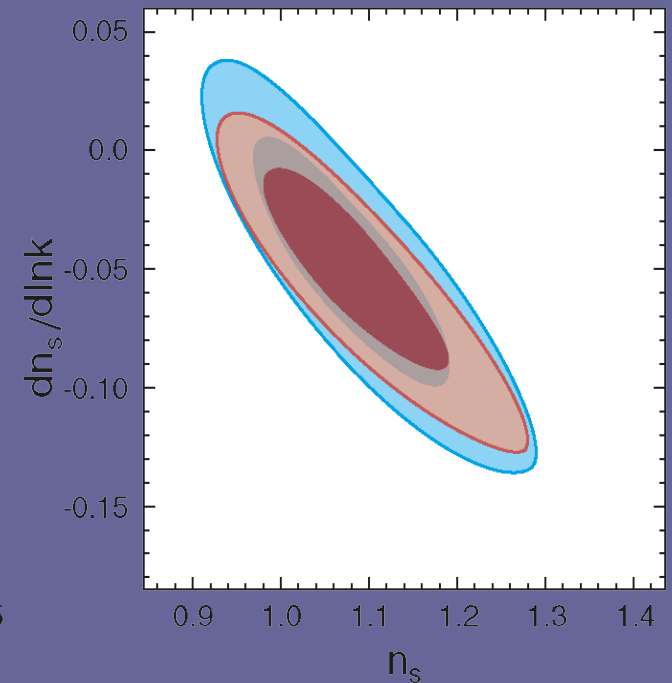
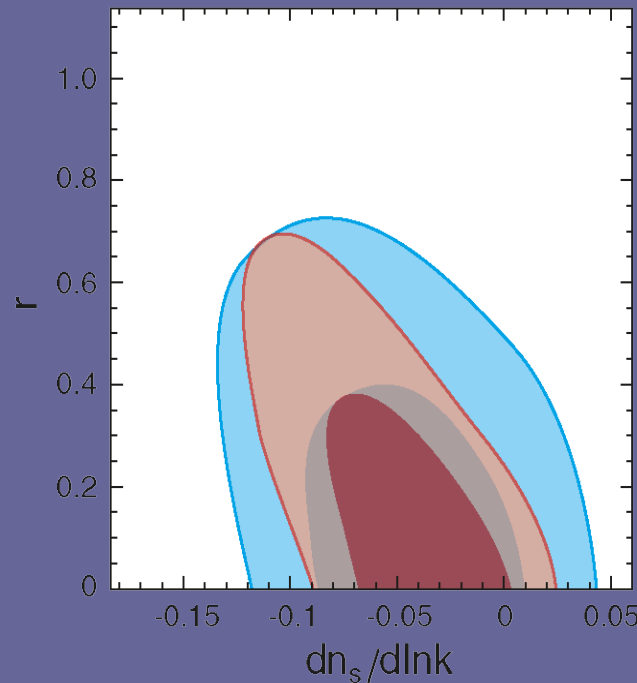
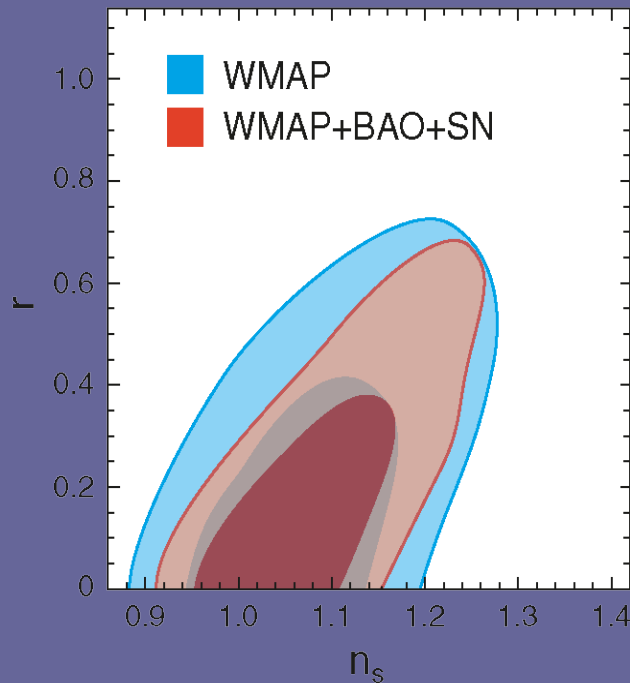
# Primordial Spectrum

$$P_R(k) = P_R(k_0) (k/k_0)^{n_s(k_0) - 1 + n'_s/2}$$

$$P_T(k) = P_T(k_0) (k/k_0)^{n_T}$$

Running:  $n'_s = dn_s/d\ln k$

( $k_0 = 0.002 \text{ Mpc}^{-1}$ )



Slow-roll inflation:

$$n_s \simeq 1 - 6\epsilon_\phi + 2\eta_\phi, \quad n_T \simeq -2\epsilon_\phi \simeq -r/8$$

$$n'_s \simeq 2\epsilon_\phi (8\eta_\phi - 6\epsilon_\phi + \xi_\phi)$$

$$(\xi_\phi = 2m_P^2 (V_{\phi\phi\phi}/V_\phi))$$

E. Komatsu et al. (WMAP):  
astro-ph/0803.0547

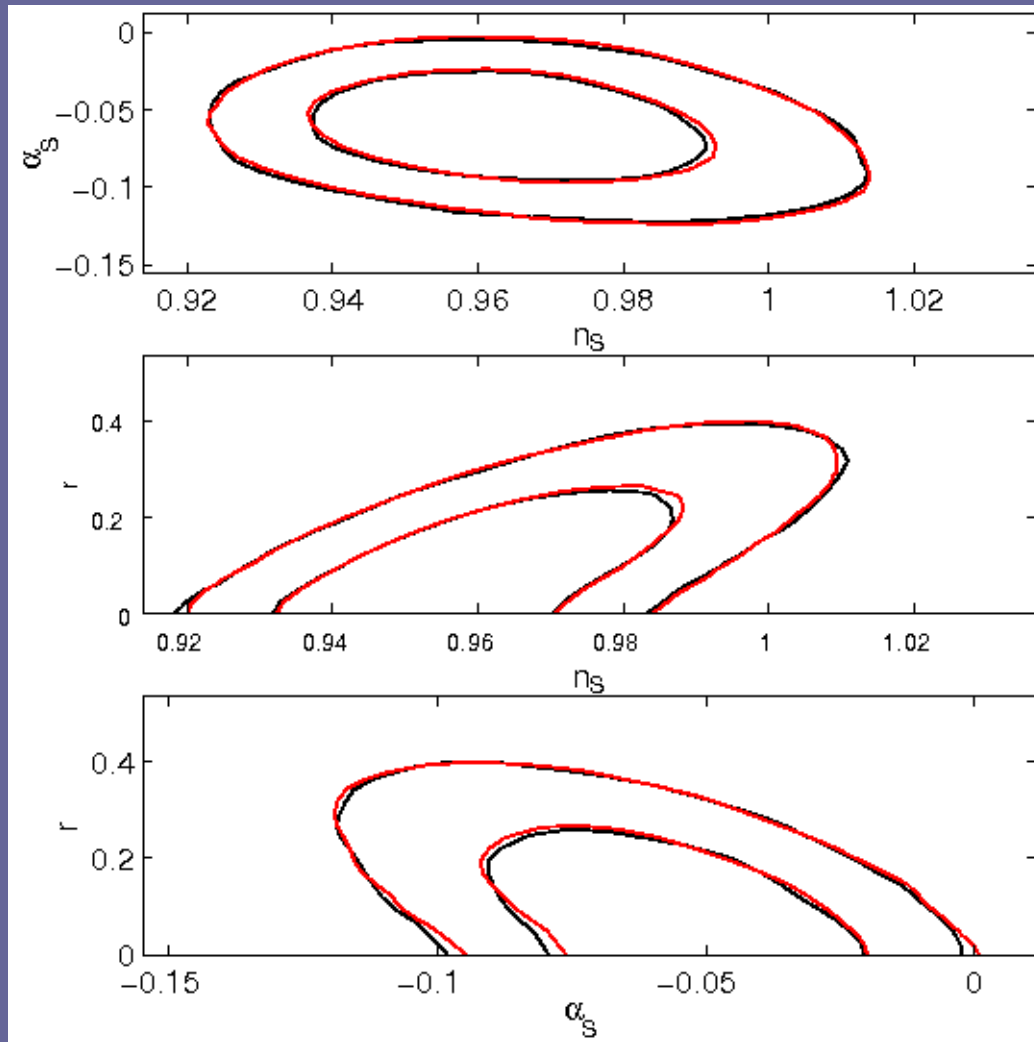
$n_s \simeq 0.96 \pm 0.015$   
(No tensor, no running)

# Primordial Spectrum

$$P_R(k) = P_R(k_0) (k/k_0)^{n_s(k_0) - 1 + n'_s/2}$$

$$P_T(k) = P_T(k_0) (k/k_0)^{n_T + n'_T/2}$$

( $k_0 = 0.002 \text{ Mpc}^{-1}$ )



Slow-roll:  
**Consistency relation**

**Red Gravity Waves**

$$n_T = -r/8 - (r/8)(1 - n_s - r/8)$$

$$n'_T = r/8(r/8 + n_s - 1)$$

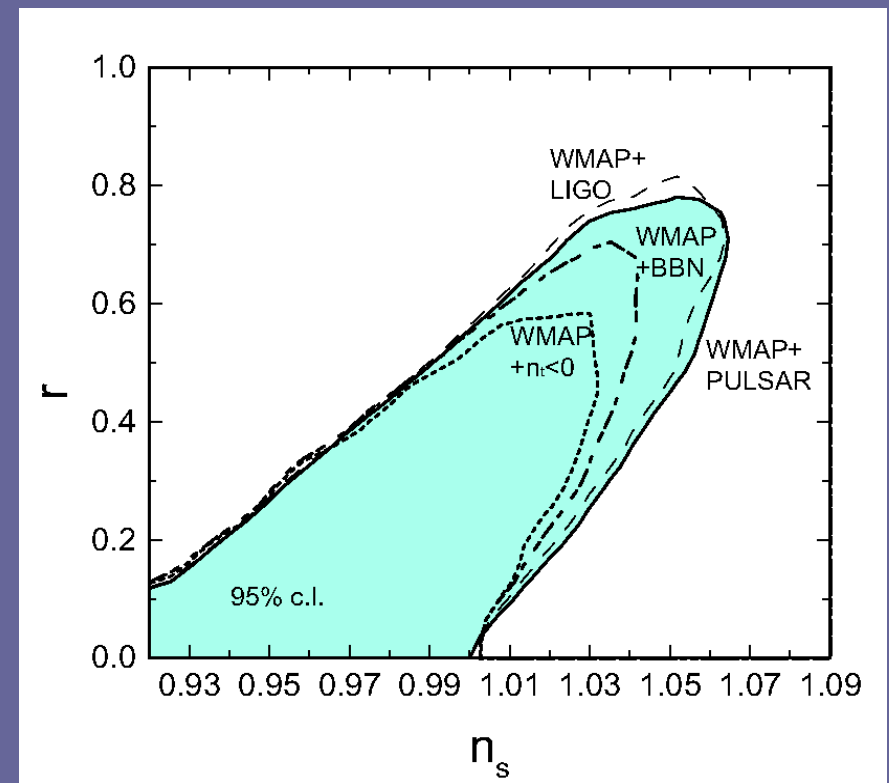
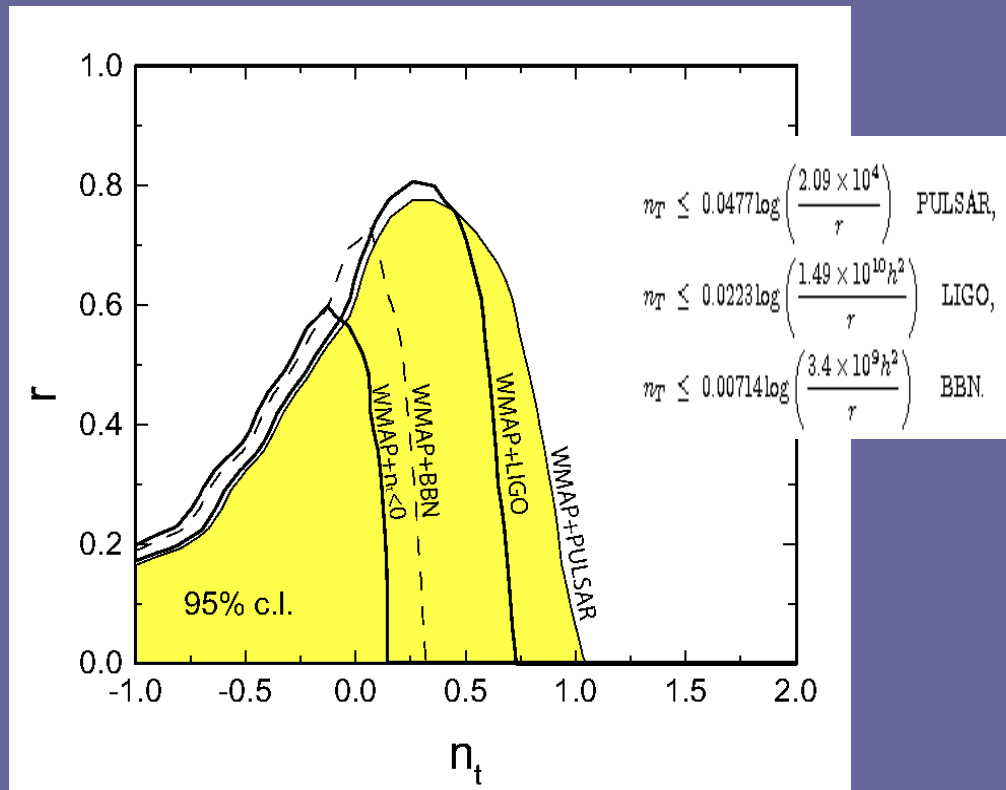
$$n_s = 0.964^{+0.039}_{-0.030}$$

$$r < 0.33$$

$$\alpha_s = n'_s = -0.063^{+0.061}_{-0.049}$$

# Primordial Spectrum: Blue Gravity Waves?

$$P_R(k) = P_R(k_0) (k/k_0)^{n_s(k_0)-1} \quad P_T(k) = P_T(k_0) (k/k_0)^{n_T} \quad (k_0 = 0.002 \text{ Mpc}^{-1})$$



No priors:  **$-1.51 < n_T < 2.62$**  ( $r > 0.05$ )

R. Camerini et al., 0802.1442: WMAP3 + ACBAR

# Primordial Spectrum: Quantum corrections

- Correlation functions are **IR (UV) divergent**

$$P_\phi(k) \simeq (H(t_k)/(2\pi))^2 \longrightarrow \langle \delta\phi^2 \rangle = \int \frac{dk}{k} P_\phi(k) \sim P_\phi \ln k$$

- Regularization scheme:

any effect on the primordial spectrum?

(a) Adiabatic subtraction (momentum space)

$$\langle \delta\phi^2 \rangle_{\text{ren}} = \int \frac{dk}{k} (P_\phi(k) - P_\phi(k)^{\text{ad}}) \sim \text{finite}$$

Parker, 0702216  
Agulló et al., 0901.0439

(b) Infrared mode matching

Janssen & Prokopec  
0906.0666

$$\delta\phi(t=t_0) = \delta\phi^{(0)}(t=t_0), \quad \langle \delta\phi^{(0)2} \rangle \sim \text{finite when } \epsilon = -\dot{H}/H^2 > 3/2$$

(c) The universe in a (comoving) box  $\sim L$  (IR cutoff)

$$\langle \delta\phi^2 \rangle = \int_{L^{-1}} \frac{dk}{k} P_\phi(k) \sim P_\phi \ln k L$$

Lyth, 0707.0361

“Minimal Box”  $\ln(H_0 L) \sim O(1)$

# Primordial Spectrum: Quantum corrections

(a) Adiabatic subtraction (momentum space):  $\langle T_{\mu\nu} \rangle_{\text{ren}} \sim \text{finite}$

$$\langle \delta \phi^2 \rangle_{\text{ren}} = \int dk k^2 (\phi_k^2 - \underbrace{1/(2W_k^{\text{ad}})})$$

Parker, hep-th/0702216  
Agulló et al., 0901.0439

Adiabatic invariant (vacuum contrib.)

$$W_k^{\text{ad}} \simeq (\omega_k^2 - 9H^2/4 - 3\dot{H}/2 + 3(\dot{\omega}_k/\omega_k)^2 - \ddot{\omega}_k/(2\omega_k))^{1/2} \simeq \omega_k (1 + \delta(z))^{1/2}$$

( $m \ll H$ ,  $\omega = H z$ ) ( $z = k/aH$ )

- Subtraction at  $k=aH$  ( $z=1$ ):

$$P_T(k) \simeq 8(H/(2\pi m_P))^2 \underline{\epsilon_\phi(t_k)} \quad P_R(k) \simeq (H/(2\pi m_P))^2 (\alpha \epsilon_\phi + \beta \eta_\phi)/(2\epsilon_\phi)$$

$$\underline{n_T} \simeq 2(\epsilon_\phi - \eta_\phi)$$

- But when  $z \ll 1 \Rightarrow \delta(z) > 1$  (IR regime):

$$k^3/W_k^{\text{ad}} \simeq H^2 z^4/(z^2 + \epsilon_\phi - 2) \ll k^3/\omega_k$$

Durrer, Marozzi, Rinaldi,  
0906.4772

**No effect on the primordial spectrum...**

# Primordial Spectrum: Quantum corrections

(b) Mode matching in the IR:  $\langle T_{\mu\nu} \rangle_{\text{ren}} \sim \text{finite}$  Janssen & Prokopec 0906.0666

$\langle \delta \phi^2 \rangle$  Is IR finite when  $\epsilon = -\dot{H}/H^2 > 3/2$  ( $q = \epsilon - 1 > 1/2$ )  
(deceleration parm.)

time  $\xrightarrow{\epsilon > 3/2 \text{ (RD = 2):}} \quad \epsilon < 3/2 \text{ (Inflation } \ll 1)$

$\sim$  No particle production  $\xrightarrow{t_0} \sim$  particle production (long wavelength modes)

$a \delta \phi_k(t < t_0) = u_k^{(0)}(t) \rightarrow \text{Bunch-Davies vacuum}$

$a \delta \phi_k(t > t_0) = \alpha_k u_k(t) + \beta_k \bar{u}_k(t) \rightarrow |\beta_k|^2 \neq 0$

$\rightarrow$  Quantum backreaction negligible at one loop order  
When  $q < -1$ , equivalent to placing the Universe in a (comoving) box at late times

# Non-gaussianity

(beyond linear order)

## ■ Spectrum:

$$\langle R(k_1)R(k_2) \rangle = (2\pi)^3 \delta(k_1 + k_2) P_R(k)$$

(Power spectrum)

$$\langle R(k_1)R(k_2)R(k_3) \rangle = (2\pi)^3 \delta(k_1 + k_2 + k_3) B_R(k_1, k_2, k_3)$$

(Bispectrum)

$$\vdots$$

$$B_R(k_1, k_2, k_3) = -\frac{6}{5} f_{\text{NL}} (P_R(k_1)P_R(k_2) + \text{permt.})$$

Non-linearity parameter:  $-9 < f_{\text{NL}}^{\text{local}} < 111$  ( $k_3 \ll k_1 \sim k_2$ )  
 (Planck:  $f_{\text{NL}} \sim \mathcal{O}(10)$ )  $-151 < f_{\text{NL}}^{\text{equil}} < 253$  ( $k_3 \sim k_1 \sim k_2$ )

- Single field inflation
- Canonical kinetic terms
- Slow-roll
- Initial vacuum state



$$f_{\text{NL}} \sim \mathcal{O}(\varepsilon, \eta)$$

Maldacena JHEP '03  
 Acquaviva et al. NPB '03

# Non-gaussianity

- **Large (detectable)  $f_{\text{NL}}$ :**

- **Multi-field inflation**

Byrnes, Choi, Hall 0812.0807

- **Non-canonical kinetic terms**

Seery, Lidsey JCAP '05

Chen et al. JCAP '07

- **End of inflation**

Bernardeu et al. PRD '04

Lyth JCAP '05

- **Curvaton**

Lyth et al. PRD '03

Enqvist, Nurmi JCAP '05

- **Warm inflation**

Moss, Xiong JCAP '07



- **Slow-roll + loops corrections**

Cogollo et al. 0806.1546

- ...

# Non-gaussianity

## Loops corrections:

Sloth NPB '06, NPB '07

Byrnes et al., 0705.4096

Seery 0707.3377, 0707.3378

$$P_{\phi}(k) \simeq P_{\phi}^{\text{tree}}(k) \left( 1 + P_{\phi}^{\text{tree}} \left( \frac{35}{6} \ln kL + \beta \right) \right)$$

$$P_R(k) \simeq P_{\phi}^{\text{tree}}(k) \left( (\partial_{\phi} N_e)^2 + \Pi^{\text{loop}} \right)$$

Several fields

$$\Rightarrow \Pi^{\text{loop}} \sim P_{\phi}^{\text{tree}}(k) (\partial_{\phi\phi}^2 N_e) \ln kL \quad \Rightarrow (\partial_{\alpha\beta}^2 N_e) \sim e^{N_e \eta_{\beta}}$$

Cogollo et al. 0806.1546

## 2-fields example:

Rodriguez, Valenzuela, 0811.4092

$\Phi \sim$  inflaton,  $\sigma \sim 0$ , light field

$$P_R^{\text{tree}} \simeq (H/2\pi)^2 / (\eta_{\phi} \phi)^2$$

$$B_R^{\text{tree}} \simeq -(P_R^{\text{tree}})^2 f(k_i) / \eta_{\phi}$$

$\Phi$  contribution

$$P_R^{\text{loop}} \simeq (\eta_{\sigma} e^{2N_e(\eta_{\phi} - \eta_{\sigma})} P_R^{\text{tree}})^2 \ln kL$$

$$B_R^{\text{loop}} \simeq (\eta_{\sigma} e^{2N_e(\eta_{\phi} - \eta_{\sigma})} P_R^{\text{tree}})^3 f(k_i) \ln kL$$

$\Phi, \sigma$  contribution

$P_R \sim$  tree-level,  $B_R \sim$  1-loop  $\Rightarrow f_{\text{NL}}$  sizeable

# Which (particle) model of inflation?

- Inflationary predictions consistent with observations
- Particle physics model building issues

# Large field models (chaotic inflation)

(Linde, PLB'83)

$$V(\phi) = V_0 (\phi/M)^p$$

“Slow roll” for large field values  
(larger than Planck scale)

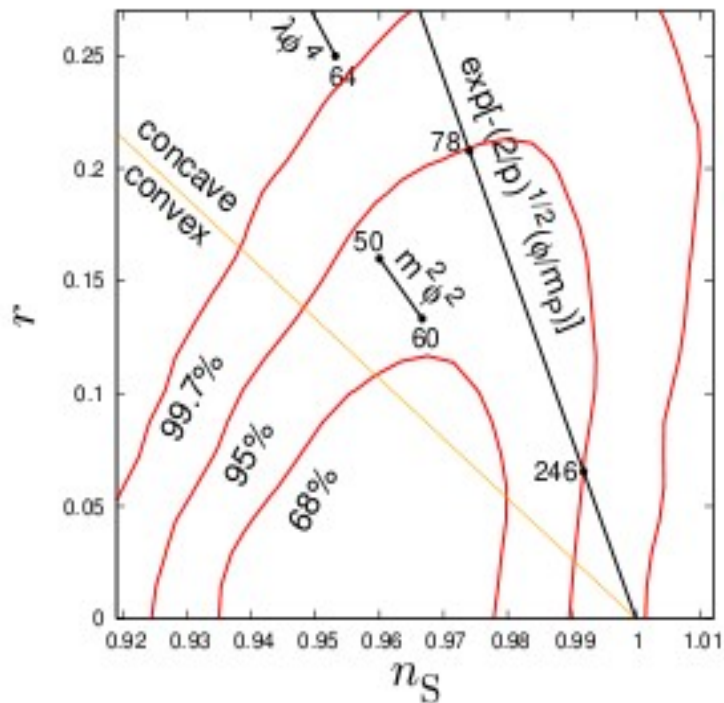
$$\eta_\phi = p(p-1) (m_P/\phi)^2 < 1$$

Tensor-to-scalar:  $r \simeq 4p/N_e$

$$p = 2 \quad r \simeq 0.13 \quad p = 4 \quad r \simeq 0.3$$

**$p > 2$  ruled out:  
too large tensor amplitude**

( $r < 0.33$  F. Finelli et al. 0912.0522)



-  **$p=4$  + 1-loop effective potential not excluded**

Senoguz & Shafi PLB '08

-  **$p=4$  + “just enough” inflation**

Ramirez & Schwarz 0912.4348

-  **$p=4$  + warm inflation**

MBG & Berera 0902.0521

# Small field models (false vacuum dominated)

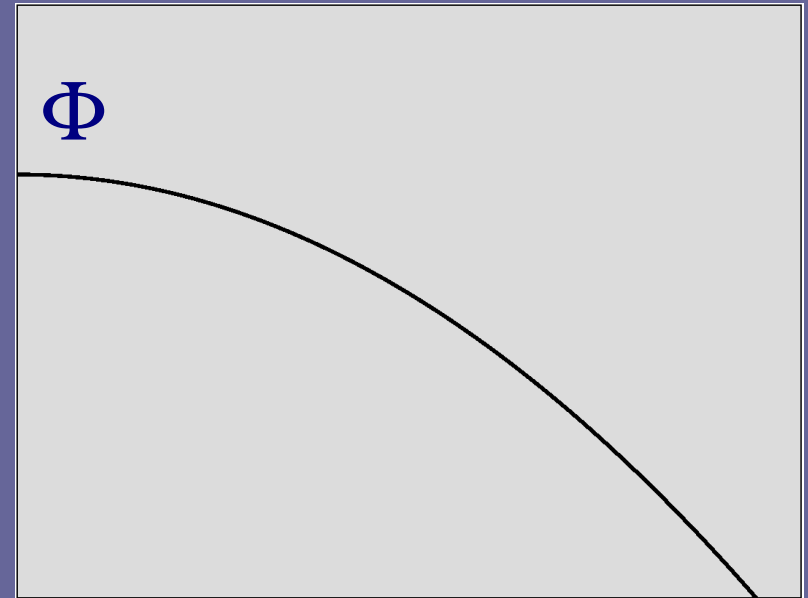
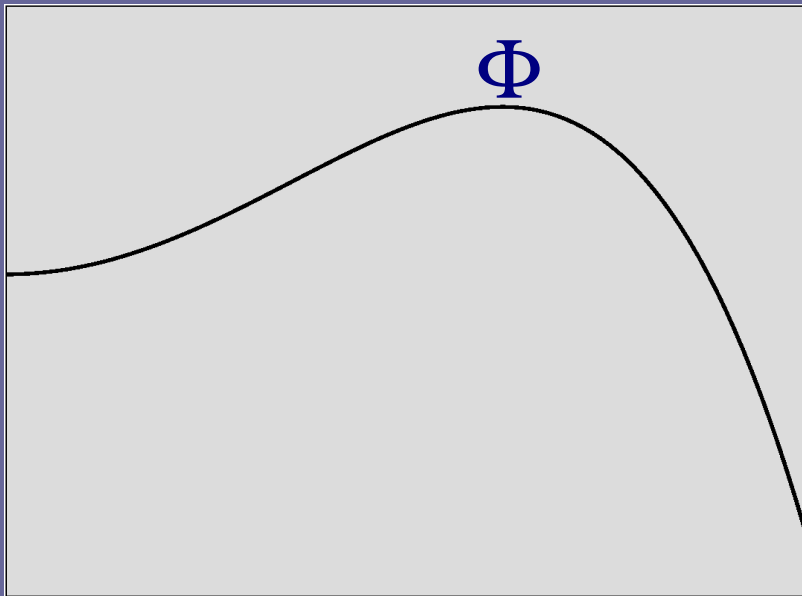
$$V(\phi) = V_0 \left( 1 \pm \left( \frac{\phi}{M} \right)^p \right) + \dots$$

Linde PLB '82

Albrecht & Steinhardt PRL '82

“Slow roll” for field values below Planck

$$\epsilon_\phi \ll \eta_\phi \longrightarrow n_s \simeq 1 + 2\eta_\phi \longrightarrow \text{Observations: } \eta_\phi < 0 \quad (V'' < 0)$$



Cosmological scales leave the horizon near the “top of a hill”

Tensor-to-scalar:  $r \ll 1$

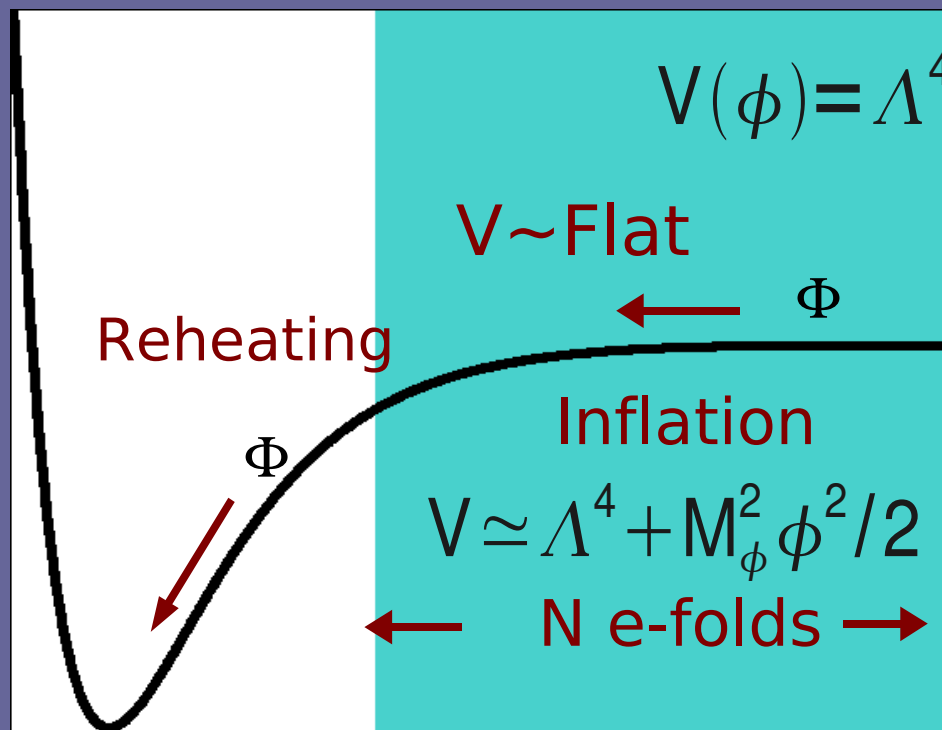
Alabidi & Lyth JCAP '06  
Kohri, Lin, Lyth JCAP '07

# Hybrid inflation (2 fields models)

Inflation  $\Rightarrow$  phase transition  $\Rightarrow$  reheating

False vacuum

Global minimum



$$V(\phi) = \Lambda^4 + \frac{M_\phi^2}{2} \phi^2 + \frac{\lambda}{4} \chi^4 + \frac{1}{2} (g^2 \phi^2 - \sqrt{\lambda} \Lambda^2) \chi^2$$

Inflation: above the critical value

The second field vanishes  
Potential  $\sim$  vacuum energy

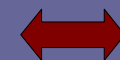
Global minimum: below critical

The second field gets a vev  
The potential vanishes

(a) Small (below Planck) field values

**(No tensor:  $r \ll 1$ )**

(b) Susy models



Flat directions

Linde, PLB'90

Copeland et al, PRD'94

# Supersymmetry

- **Fermion/boson** symmetry

- **Hierarchy** problem:  $m_W/m_P \sim O(10^{-17})$



SM scalar (Higgs) masses are not protected against large (quadratic) radiative corrections

**Susy:** no rad. Corrections

exact cancelation between fermions/scalars

**Soft Susy:** no quadratic rad. corrections

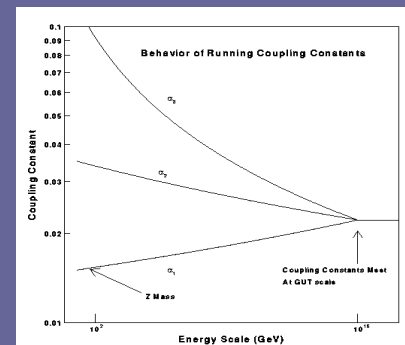
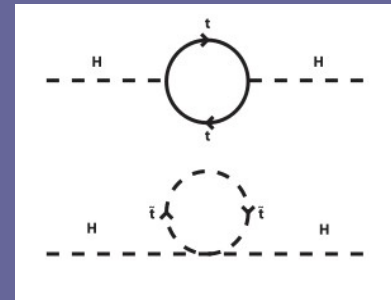


- **Grand Unification:**

- **Dark Matter candidate:**

Lightest Supersymmetric Particle

R-parity: susy particles produced in pairs, stable LSP i



# Supersymmetric Hybrid inflation:

**Susy superpotential:**  $W = \kappa \Phi H \bar{H} + \dots$

$\Phi$  = inflaton (gauge singlet)

$H = \bar{H}$  = GUT fields

$$V = V_F + V_D(H, \bar{H}) = \sum |W_i|^2 + V_D(H, \bar{H})$$

- **F-term inflation:**  $W = \kappa \Phi (H \bar{H} - \underline{M^2})$

Dvali, Shafi,  
Shraefer PRL '94

$$H = \bar{H} = 0 \quad \longrightarrow \quad V_0 = V_F = \kappa^2 M^4$$

- **D-term inflation:**  $V_D = \frac{g^2}{2} (\underline{\xi_{FI}} + \bar{H}^2 - H^2)^2$

Binetruy & Dvali PLB '96

Halyo PLB '96

$$H = \bar{H} = 0 \quad \longrightarrow \quad V_0 = V_D = \frac{g^2}{2} \xi_{FI}^4$$

- **Susy breaking:  $V$  lifted by 1-loop rad. correct.**

$$m_H^2(\phi), m_{\bar{H}}^2(\phi) \quad \longrightarrow \quad V = V_0 + \Delta V^{(1)}(\phi)$$

# Sugra inflation: “eta” problem:

- Global susy models should be embedded into a local susy (sugra) model

**Sugra Potential:** superpotential  $W$ + Kähler potential ( $K$ )

$$V(\phi) = e^K (|W_i + K_i W| K_{i\bar{j}}^{-1} |\bar{W}_{\bar{j}} + K_{\bar{j}} \bar{W}| - 3|W|^2) = |F_i|^2 - 3m_{3/2}^2 m_P^2$$

- Sugra corrections problematic for (slow-roll) inflation: they typically induce masses of order  $H$  for the scalars

→ **“eta” problem?** (F-term inflation)

$$V(\phi) = e^{K(\phi)/m_P^2} V_0 \sim V_0 \left( 1 + \frac{\phi^2}{m_P^2} + \dots \right) \sim V_0 + 3H^2 \phi^2 + \dots$$

→  $\eta_\phi \sim 1$

Copeland et al. PRD '94  
Dine, Randall, Thomas PRD '95

- Chaotic models :  $V(\phi) = e^{K(\phi)/m_P^2} V_0(\phi)$

too steep for  $\phi > m_P$

# Sugra F-term Hybrid Inflation

Susy superpotential:  $W_H = \kappa \Phi (H\bar{H} - M^2)$

Waterfall (GUT) fields:  $H \quad \bar{H}$  Singlet:  $\phi$

Minimal Kahler potential:  $K = |\Phi|^2 + |H|^2 + |\bar{H}|^2$

→  $W_H$  + Minimal Kähler potential: no quadratic term

$$V(\phi) \simeq \kappa^2 M^4 \left( 1 + \frac{\phi^4}{8m_p^4} + \dots \right) + \Delta V^{(1)}$$

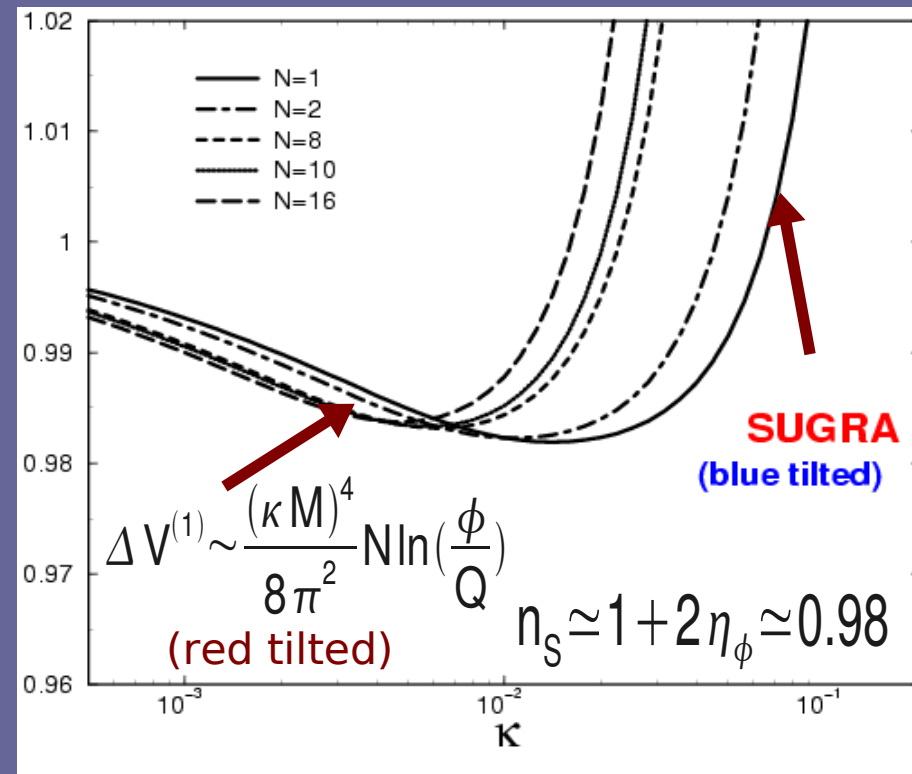
Sugra correction

Copeland et al. PRD '94

Linde & Riotto PRD '97

$n_s$  consistent with observations  
at 95% CL, but....

WMAP:  $n_s \simeq 0.96 \pm 0.015$   
(no tensor)



# “eta” problem: shift symmetry

- Symmetry of the Kahler potential

$\phi \rightarrow \phi + c \longrightarrow K = K(\phi - \bar{\phi})$  does not depend on  $\text{Re}(\phi)$

**no quadratic correction from Kahler**

- Chaotic inflation:

sym broken by small parameter in  $W$

$W = m \Phi^2 \longrightarrow$  unbounded from below for  $\text{Im}(\phi)$

$W = m \Phi X$  **OK!**

Kawasaki, Yamaguchi, Yanagida PRD'00

Hsu, Kallosh JHEP'04

Brax, Martin PRD'05

Antusch et al. PLB'09

- Hybrid inflation:

$$K = -(\phi - \bar{\phi})/2 + |H|^2 + |\bar{H}|^2$$

$\longrightarrow$  tachyonic  $\text{Im}(\phi)$ :

Extra contributions to Kahler required

# Sugra inflation: moduli problem(s)

Brunstein, Steinhardt PLB'92; M. Dine PLB'00

- Sugra as the low energy effective theory from strings:

Moduli fields: scalar d. of f. from extra dimensions

Dilaton  $S$ ,  $T$  modulus,.. :  $K \sim -\text{Ln}(S + S^*) - 3 \text{Ln}(T + T^*)$

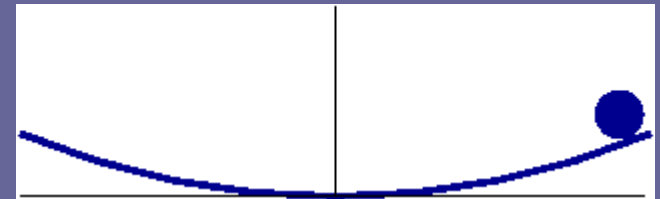
No  $W$ , flat  $V$  at tree-level, lifted by NP effects

- Long lived light d. of f.: ( $m_{3/2} \sim 1\text{TeV}$ )

must decay before nucleosynthesis

- Potential lifted by susy breaking effects:

Energy due to large amplitude ( $\sim m_p$ )  
oscillations may overclose the Universe



- Runaway potential during inflation:  $V \sim V_{\text{inf}} / (T + \bar{T})^3$

Kinetic energy dominates  $\longrightarrow$  **they must be stabilised**

Ellis et al. JCAP'06; Brax et al. JCAP'06; Davis, Postma JCAP'08

# Sugra inflation + moduli

- $K = K(\phi) - 3 \ln(T + T^*)$  ,  $W = W(\phi)$  , shift symmetry  $K(\phi - \phi^*)$

$$V = |F_\phi|^2 / (T + \bar{T})^3 \quad \longrightarrow \quad \eta_T = \epsilon_T > 1$$

Ellis et al. JCAP'06

T dynamics dominates and makes inflation impossible

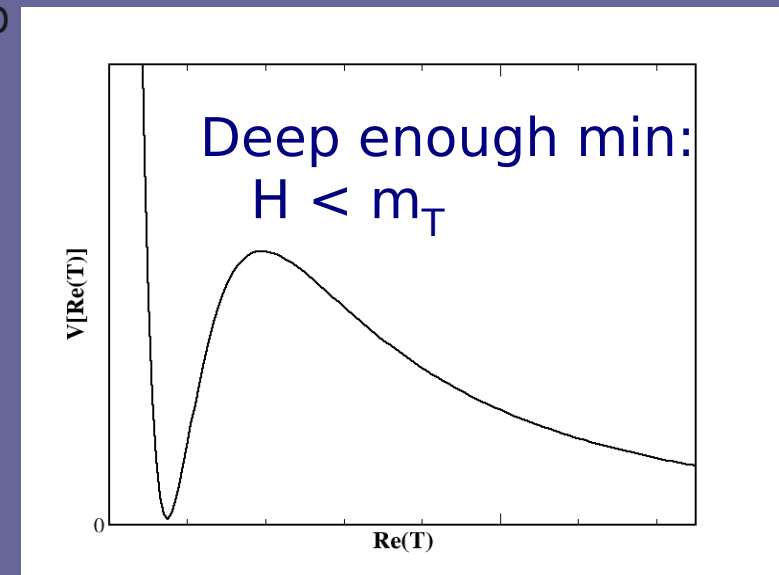
- Generating a potential minimum for T:

F-term stabilisation  $W(T)_{\text{KKLT}} = W_0 + A e^{-aT}$  Kachru et al. PRD '03  
 Fluxes      instanton effects

(a) AdS minimum : add  $V_{\text{lift}} \sim K_T^2 / \text{Re} T^p$   
 $\longrightarrow$  dS minimum

(b)  $W(T)_{\text{KL}} = W_0 + A e^{-aT} + B e^{-bT}$   
 $\longrightarrow$  Minimum  $W = W_T = 0$

Kallosch, Linde JHEP '07



# Sugra inflation + moduli

- Adding all together:  $K = K(\phi - \phi^*) + K(T)$ ,  $W = W(\phi) + W(T)$

$$V \simeq V_0(\Phi) + C(T)\Phi + M(T)^2 \Phi^2/2$$

- T stabilised at  $T_0$ : check  $M(T_0) < H$

- Check dynamics:  $V \simeq V_{\text{inf}}(T_0) + \delta V(\delta T)$

Hybrid (shift sym.) + KKLT/ KL:

$$\eta \sim -6$$

Brax et al. JCAP '06

(Adding Kahler functions:  $G = K(\phi) + K(T) + \ln W(T) W(\phi)$   $|\eta| < 1$ )

Achucarro, Hardeman, Sousa PRD '08

Davis & Postma JCAP'08

Chaotic (shift sym.) +  $W = M \phi^X$ :

quadratic chaotic model

# Sugra inflation + Heisenberg sym.

- Heisenberg symmetry protects flat directions in  $V$

$$\delta T = \epsilon \Phi, \quad \delta \Phi = \epsilon$$

Gaillard, Murayama, Olive '95

Gaillard, Lyth, Murayama PRD '98

Kahler invariant under Heisenberg sym. :

$$K = K(\rho), \quad \boxed{\rho = T + \bar{T} - |\Phi|^2} \quad (K = -3 \ln \rho) \quad (\text{no scale})$$

Sym broken in the superpotential :  $W = W(\Phi)$

$$V = e^{K(\rho)} \left[ \left( K_\rho^2 / K_{\rho\rho} - 3 \right) |W|^2 - |W_\Phi|^2 / K_\rho \right] \quad (V = |W_\Phi|^2 / (3\rho^2)) \quad (\text{no scale})$$

The modulus  $\rho$  must be stabilised during inflation:  $V_\rho(\rho_0) = 0$

$$M^2(\rho_0, \Phi) \simeq \begin{pmatrix} V_{\rho\rho} & -\Phi V_{\rho\rho} \\ -\bar{\Phi} V_{\rho\rho} & |\Phi|^2 V_{\rho\rho} \end{pmatrix} \Rightarrow \text{Det} M^2(\rho_0, \Phi) = 0$$

# “Kahler” moduli stabilisation

Antusch et al. JCAP'09

Murayama et al. PRD '94

Hybrid model + Heisenberg symmetry:  $\rho = T + \bar{T} - |N|^2$

No-scale: an example

- Matter content:

- T : modulus , N: inflaton

- S: singlet  $|F_s|^2 \neq 0$ ,  $S=0$   
(vacuum energy)

- H, waterfall field,  $H=0$

-  $m_N < H$

- All other masses  $> H$

-  $\kappa_S < 0$

$\kappa_S$  : S mass contribution

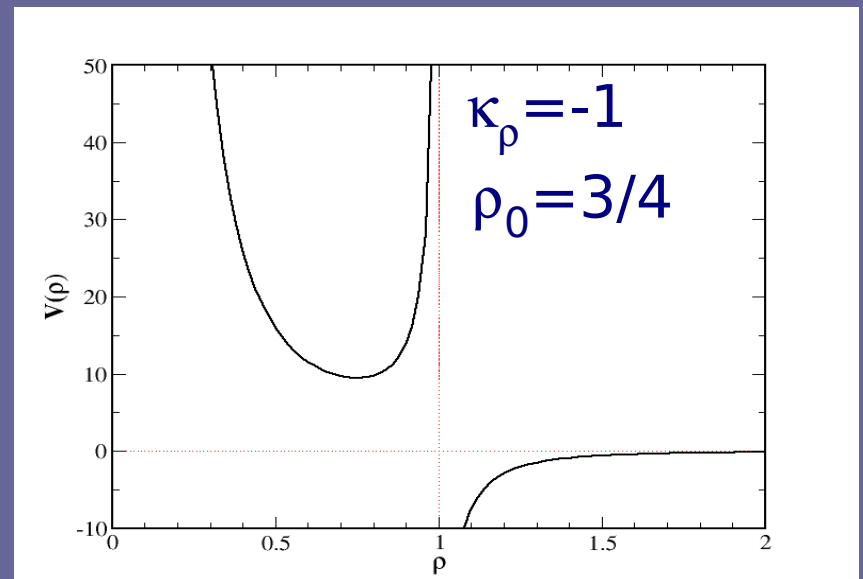
$\kappa_\rho$  : minimum  $\rho$  direction

$$K = -3 \ln \rho + (1 + \kappa_S |S|^2 + \kappa_\rho \rho) |S|^2 + |H|^2$$

$$W = \kappa S (H^2 - M^2) + \lambda N^2 H^2 / \bar{M}$$

$$V = \kappa^2 M^4 e^{K(\rho)} / (1 + \kappa_\rho \rho)$$

$\rho$  settles quickly at its minimum

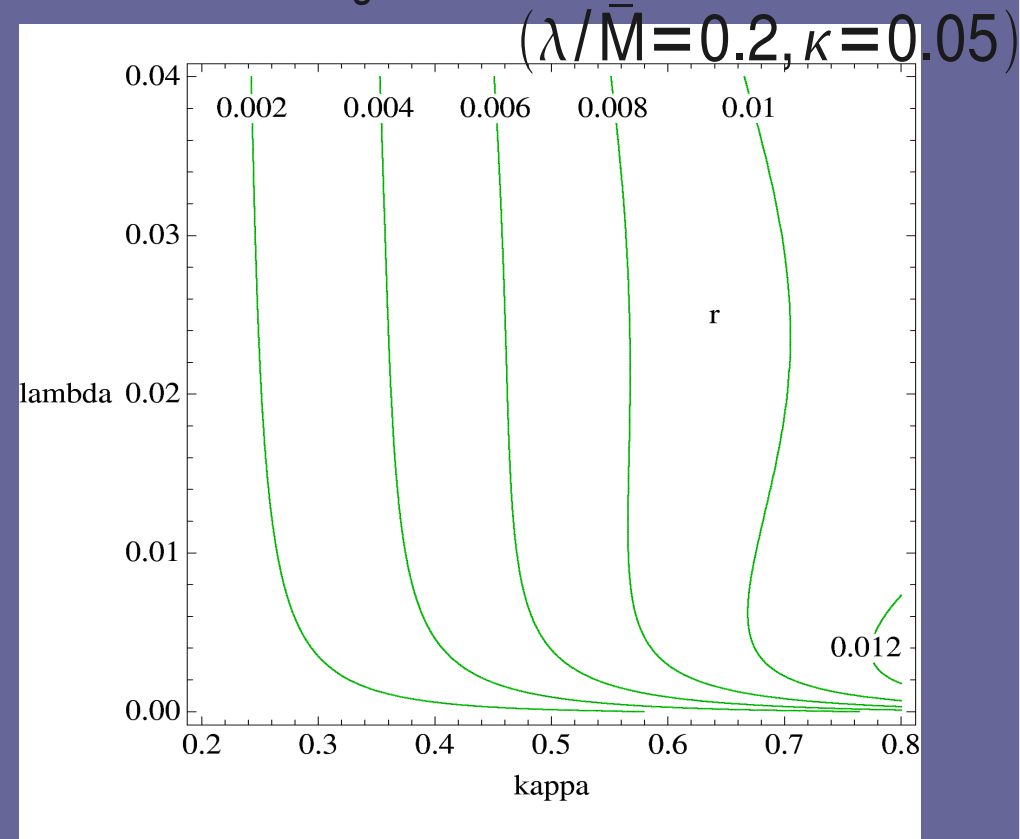
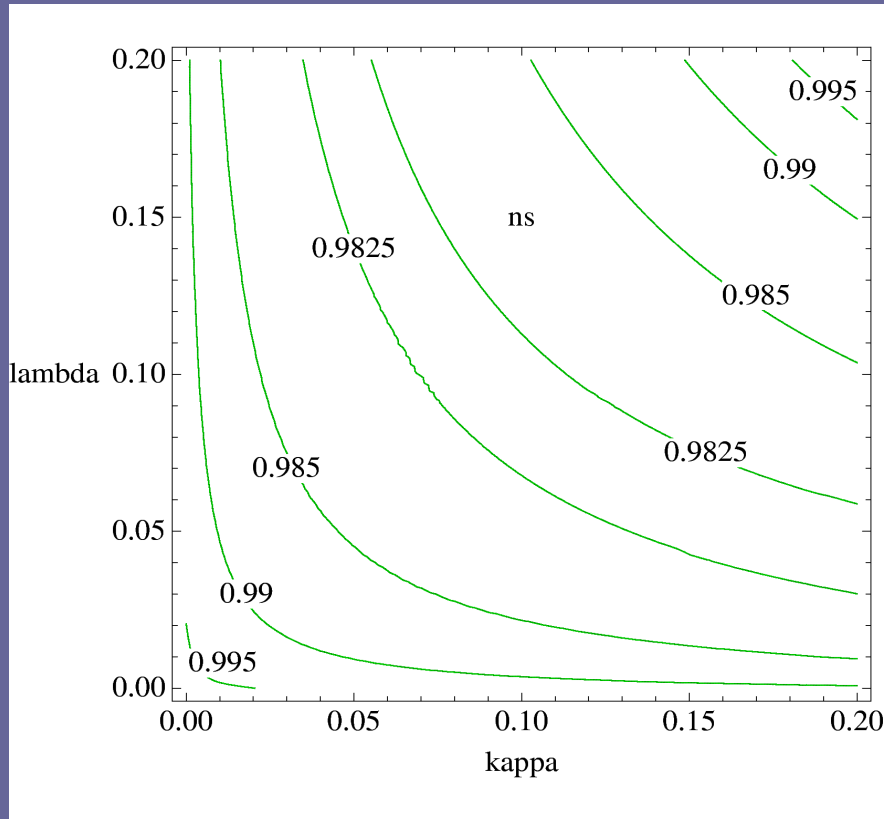


-  $V(N)$  lifted by 1-loop corrections

# Inflationary predictions

Similar to standard susy hybrid inflation:

$$n_s \simeq 0.98, \quad M \simeq 3 \times 10^{-3}, \quad r \simeq 10^{-4}, \quad dn_s/d\ln k \simeq -10^{-3}$$



- no blue-tilted spectrum

- $\kappa, \lambda$  can be larger than in standard susy hybrid inflation:

$$\kappa \sim 0.7, \quad \lambda/\bar{M} \sim 0.05 \quad \Rightarrow \quad r \sim 0.01 \quad (\text{larger tensor contribution})$$

- $\kappa_{SH} \sim 10 \quad \Rightarrow \quad n_s \sim 0.95$

# “Kahler” moduli stabilisation

Antusch et al. PLB'09

Chaotic model + Heisenberg symmetry:  $\rho = T + \bar{T} - |\Phi|^2$

Matter content:

- $T$  : modulus ,  $\Phi$ : inflaton
- $X$  :  $|F_X|^2 \neq 0$  (vacuum energy)

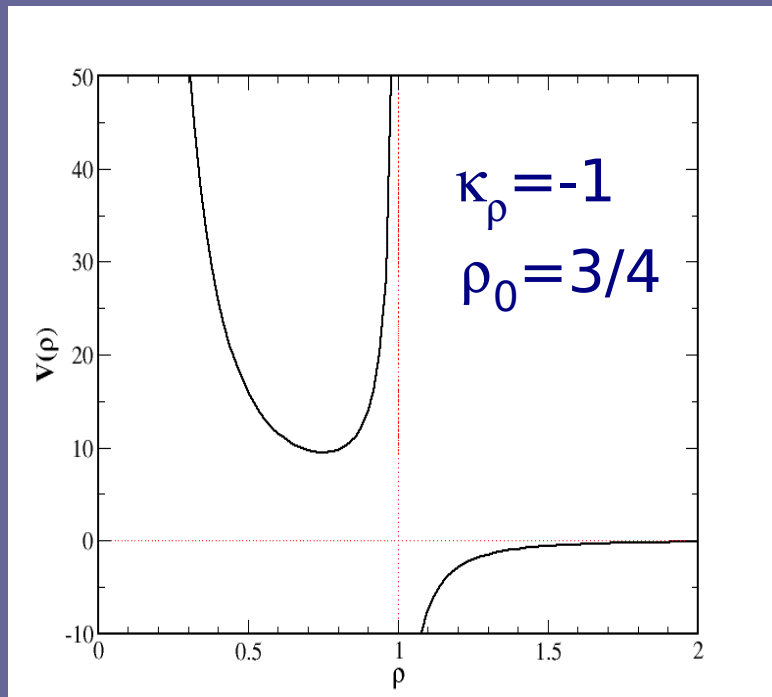
$$K = -3 \ln \rho + (1 + \kappa_X |X|^2 + \kappa_\rho \rho) |X|^2$$

$$W = M \Phi X$$

$\rho$  minimum

$$M_X > H, \quad \kappa_X < 0$$

$$V = M^2 |\Phi|^2 / (\rho^3 (1 + \kappa_\rho \rho))$$



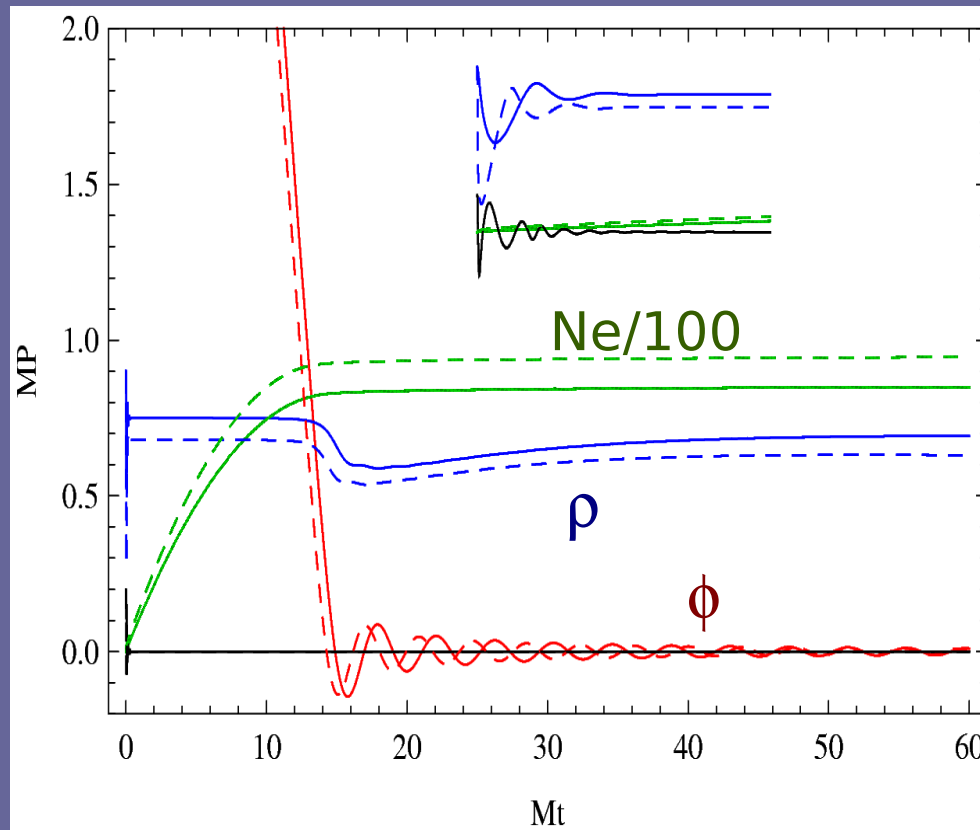
Quadratic model

$$n_s \simeq 0.97$$

$$M/m_P \sim 10^{-5}$$

$$r \simeq 0.13$$

# Chaotic model + Heisenberg symmetry: field evolution and 1-loop corrections



Solid: tree-level potential  
dashed: + 1-loop potential

## 1-loop effective potential

$$\Delta V^{(1)} = \frac{\Lambda^2}{32\pi^2} \text{Str } M_\alpha^2 + \frac{1}{64\pi^2} \text{Str } M_\alpha^4 \ln \frac{M_\alpha^2}{\Lambda^2}$$

$$\text{Str } M_\alpha^2 \neq 0$$

negligible

$$\Delta V^{(1)} \simeq M^2 \Phi^2 (C(\rho) - 12\kappa^X) / (\rho^3 (1 + \kappa_\rho \rho)^3)$$

$$V + \Delta V^{(1)} = m_{\text{eff}}^2(\rho_0) \Phi^2 / 2$$

1-loop correction: inflaton mass renormalization

# Summary

- Observations → Standard Model in Cosmology  
Primordial spectrum? Flatness? Horizon problem?

→ inflation

- Primordial power spectrum:

Adiabatic?

Scale invariant?

Tensors?

Non-gaussianity?

Planck

Higher order corrections

Renormalization issues?

# Summary

- Particle physics model of inflation?

Sugra inflation: “eta” & moduli problems

- Shift symmetry  $K(\phi - \phi^*)$
- Heisenberg symmetry  $K(\rho)$
- $G(H, I) = G(H) + G(I)$  adding Kahler functions
- ...

No detection of any (fundamental) scalar...yet...

The Higgs boson still missing  
(Electroweak symmetry breaking)

LHC will shed light on physics beyond the SM:

- Higgs physics? Supersymmetry? ...

# Summary

- Heisenberg symmetry  $K(\rho)$  protects flat directions

$$K=K(\rho), \quad \rho=T+\bar{T}-|\Phi|^2$$

Modulus  $\rho$  potential originates from Kahler corrections

$$K=-3\ln\rho+(1+\kappa_X|X|^2+\kappa_\rho\rho)|X|^2$$

$\kappa_X$ :  $X$  mass contribution,  $|F_X|^2$ : vacuum energy

$\kappa_\rho$  : minimum  $\rho$  direction

➡ Sugra corrections under control

➡ Hybrid model: no blue spectrum for large couplings

➡ Chaotic model: quadratic potential for  $W= M \Phi X$

No detection of any (fundamental) scalar...yet...

The Higgs boson still missing  
(Electroweak symmetry breaking)

**LHC** will shed light on physics beyond the SM:

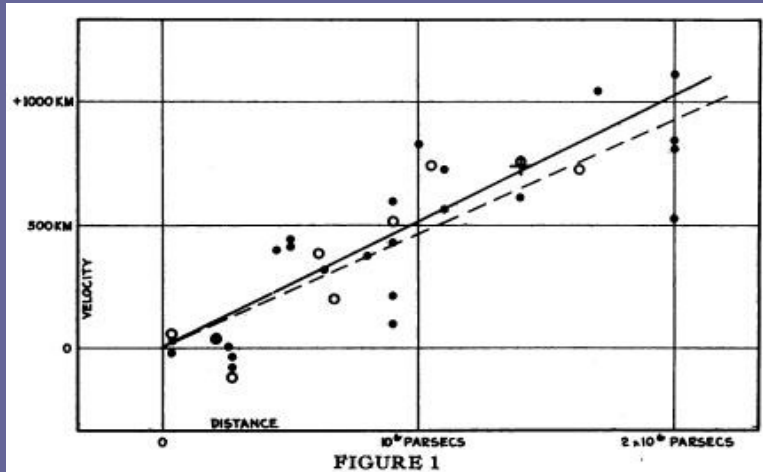
- Higgs physics?
- Supersymmetry?
- ...

**Planck** will improve our knowledge  
on the cosmological parameters

- Primordial spectrum?
- Polarization? Non-gaussianities?
- ...

# Expanding Universe

## Hubble's law, 1929



Velocity-Distance Relation among Extra-Galactic Nebulae.

Radial velocities, corrected for solar motion, are plotted against distances estimated from involved stars and mean luminosities of nebulae in a cluster. The black discs and full line represent the solution for solar motion using the nebulae individually; the circles and broken line represent the solution combining the nebulae into groups; the cross represents the mean velocity corresponding to the mean distance of 22 nebulae whose distances could not be estimated individually.

$$V = H_0 D(z)$$

$H_0$  = Hubble parameter

The rate of expansion ( $H_0$ ) depends on the matter content

## Einstein's Equations, 1917

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_N T_{\mu\nu}$$

geometry  $\longleftrightarrow$  "matter"

## Cosmological Principle:

The Universe is **homogeneous** and **isotropic** on very large scales

## FRW metric:

$$ds^2 = dt^2 - a(t)^2 d\bar{x}^2$$

( $a(t)$  = scale factor)

**Minimal + small corrections:**  $K = |\phi|^2 + \kappa_S \frac{|\phi|^4}{4m_P^2} + \kappa_{SS} \frac{|\phi|^6}{6m_P^4} + \dots$

$$V(\phi) \simeq \kappa^2 M^4 \left( 1 - \kappa_S \frac{\phi^2}{2m_P^2} + \gamma_S \frac{\phi^4}{8m_P^4} + \dots \right) + \Delta V^{(1)}$$

— Eta problem?

Spectral index:

1-loop potential

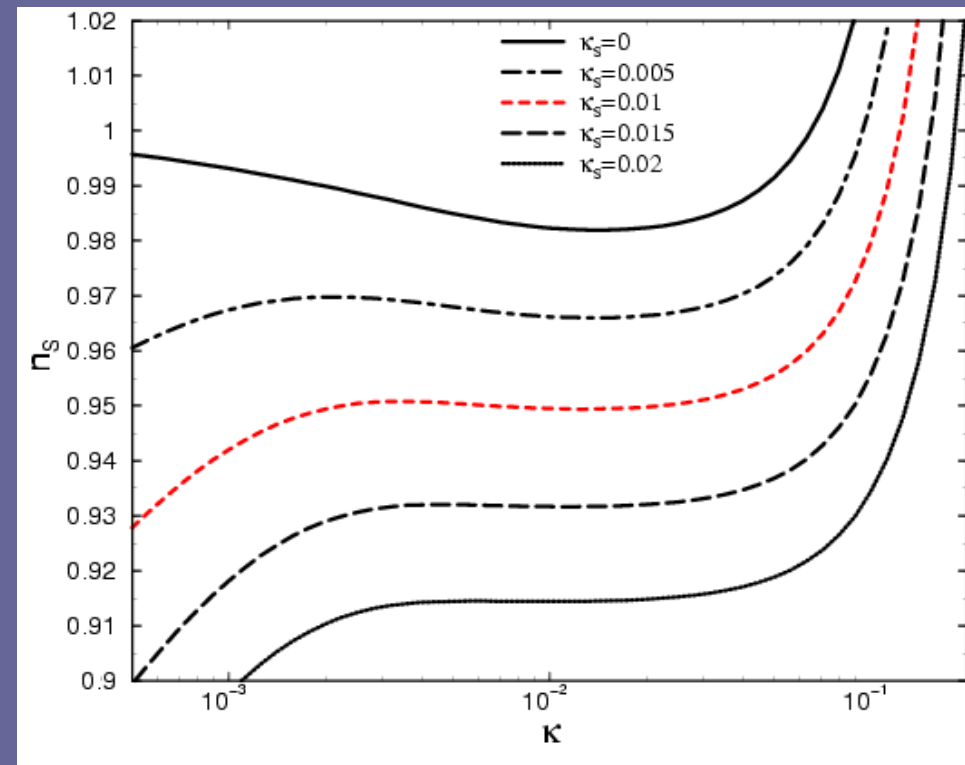
$$n_s \simeq 1 - 2\kappa_S - 2\delta$$

Kähler correction

Flatter potential:

→  $\phi_{N_e}$  smaller

→  $\delta \sim m_P^2 / \phi_{N_e}^2$  larger



$$n_s \simeq 0.95 \quad (\kappa_S \simeq \kappa \simeq 0.01)$$

# “Kahler” moduli stabilisation

ynamics during inflation:  $\rho$ ,  $N_R$

Non-canonical kinetic terms

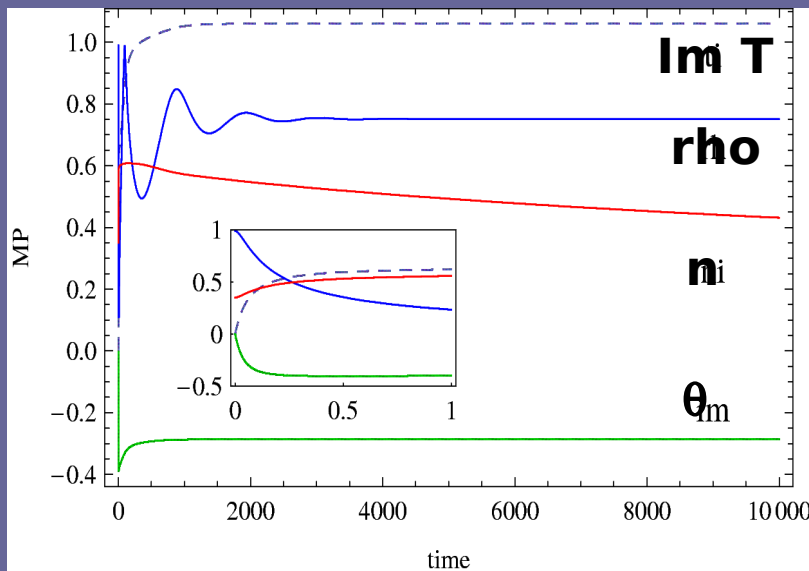
$$L_{\text{kin}} = \frac{K_{\rho\rho}}{4} (\partial_\mu \rho)^2 - \frac{K_\rho}{2} (\partial_\mu N)^2$$

$$V = \kappa^2 M^4 e^{\dot{K}(\rho)} / (1 + \kappa_\rho \rho)$$

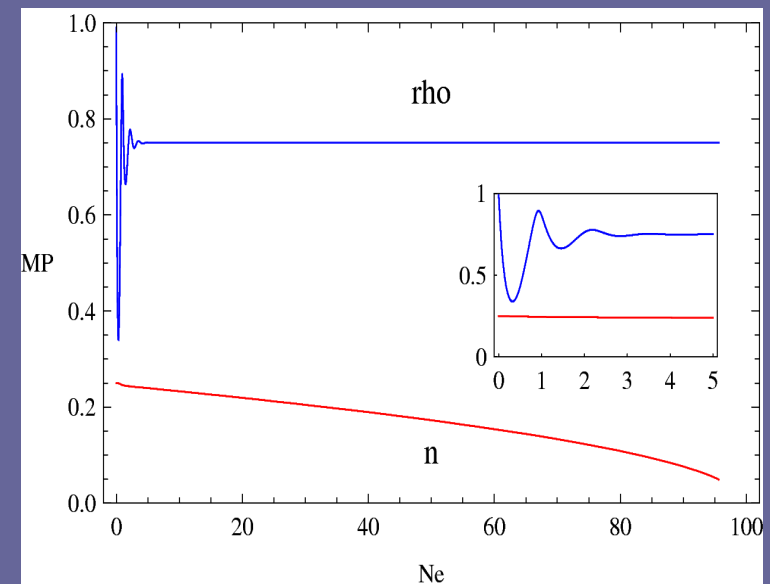
$$\ddot{N}_R + 3H\dot{N}_R - \frac{V_N}{K_\rho} = -\frac{K_{\rho\rho}}{K_\rho} \dot{N}_R \dot{\rho}$$

$$\ddot{\rho} + 3H\dot{\rho} + \frac{2V_\rho}{K_{\rho\rho}} = -\frac{K_{\rho\rho\rho}}{2K_{\rho\rho}} \dot{\rho}^2 - \dot{N}_R^2$$

stabilises quickly at its minimum



Phases set to a constant value at the start of inflation



# “Kahler” moduli stabilisation

Antusch et al. '09

Quartic model + Heisenberg symmetry:

$$\rho = T + \bar{T} - |\Phi|^2$$

Matter content:

-  $T$  : modulus ,  $\Phi$ : inflaton



$$V = M^2 e^{\dot{K}(\rho)} \left( A(\rho) \frac{\Phi^4}{m_P^2} + B(\rho) \Phi^2 \right)$$

$$K = -3 \ln \rho - \rho^\alpha$$

$$W = M \Phi^2$$


  
 $\rho$  minimum

Quartic model

$$n_s \simeq 0.95$$

$$M/m_P \simeq 10^{-7}$$

$$\underline{r \simeq 0.3}$$

$\rho_0$  evolves with  $\phi$   
but  $\eta_\rho, \epsilon_\rho < 1$

