APPLICATIONS OF KALMAN FILTERING IN SPACECRAFT ORBIT DETERMINATION

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PROBLEM STATEMENT

- Orbit Determination(OD):

  Tracking data: Vector from Ground station to satellite : **ρ** : 3x1 vector (ρ,Az.,Eln.)

  Given the Tracking Data (TD) and the system (orbital) dynamics , determine the the system state (orbital) parameters.

- State parameters:[Position & Velocity]$_{6x1}$
- Statevector(STV) = $[\mathbf{r},\mathbf{v}]^T$ at any time t
- If $[t_0,t_1]$ is the observation span:

  Smoothing if STV at $t = t_0$ is sought

  Filtering if STV at $t \in (t_0,t_1)$ is sought

  Prediction if STV at $t > t_1$ is sought

SOLUTION: LS

- Over Determined system:

  6x1 statevector is determined using 100s of observations.

  Obsvn.: TD from about 4 ground-stations per orbit; 13-15 minutes data per station; collected every 30 seconds.

- While smoothing (STV at t_0 is sought) Batch estimation: Off-line processing

  Kalman Filtering if in Real-time, STV is sought On-board.

BATCH: DIFFERENTIAL CORRECTION

- Least Squares (LS):

$$\Delta x_{LS} = (H^T H)^{-1} * H^T * \Delta z$$

- Weighted LS:

$$\Delta x_{WLS} = (H^T W H)^{-1} H^T W \Delta z$$

CONCERN : Accuracy of estimated position

ACCURACY

- IRS 1A   250 mts. Position error
- IRS 1C   100   mts. Position error
- IRS P6(RS-1) 80 mts. Position error
- Desired: < 50 mts. Position error

DYNAMICS FORCE MODEL

$$\ddot{\bar{r}} = -\frac{\mu}{r^3}\hat{r} + a_p$$

$$\ddot{\bar{r}} = -\frac{\mu}{r^3}\hat{r} + a_{non-sph} + a_{drag} + a_{sun-moon} + a_{SRP}$$

$$\mu = Gravitational\ \ constatnt$$

$$\bar{r} : radious\ \ vector\,(Earth\,center\ \ to\,satellite\,)$$

EARTH GRAVITY FORCE

$$\Phi = \mu / r \sum_{n=0}^{\infty} (\mathrm{R}_{\mathrm{e}} / r)^n \sum_{m=0}^{n} P_{n,m} \, Sin(\varphi) \, [C_{n,m} \, Cos\,(m\lambda) + S_{n,m} Sin(m\lambda)]$$

R_e : *Earth Radious*

(φ, λ) : *Latitude & Longitude of pt. in space*

$P_{n,m}$: *Langendre Polynmial of* deg. *n & order m*

$C_{n,m}$, $S_{n,m}$: *Geopotential Coefficients*

$J_n = -C_{n0}$: *Zonal Harmonic Coefficients*

FORCE DUE TO ATMOSPHERIC DRAG

$$a_{drg} = -\frac{1}{2} C_D \ \rho (\ A/M) v^2 \ U_v$$

$$C_D = Drag\ Coefficient$$

$$\rho = Atmospheric\ Density$$

$$A = Area\ of\ SpaceCraft$$

$$M = Mass\ of\ SpaceCraft$$

$$v = Velocity\ of\ SpaceCraft$$

$$U_v = Unit\ vector\ in\ the\ Velocity\ Direction\ U_v$$

FORCE DUE TO THIRD BODY ATTRACTION: (MOON)

$$a_{moon} = -\,\mu_{moon}\,[\,(\,r_{m-s} \,/\, r_{m-s}{}^{3}\,) - (r_{moon} \,/\, r_{moon}{}^{3}\,)]$$

μ_{moon} = *Gravitational constant of Moon*

r_{m-s} = *Radius vector from Moon to Satellite*

r_{moon} = *Radius vector of Moon*

SIMILAR FORCE MODEL FOR SUN

FORCE DUE TO :SRP
(Only for Geo Mission)

$SOLAR\ RADIATION\ PRESSURE$

$$a_{SRP} = \upsilon P_s \, (r_{sun^2} / r_{ss^2}) \, C_R \, (A / M) \, r_{ss}$$

$\upsilon = flag\ for\ eclipse$

$P_s = Ratio\ of\ Mean\ Solar\ Flux\ to\ the\ velocity\ of\ light$

$r_{sun} = Radius\ of\ sun$

$r_{ss} = Sun - Satellite\ vector$

$C_R = Coefficient\ of\ reflection$

DYNAMICS FORCE MODEL

$$\ddot{\bar{r}} = -\frac{\mu}{r^3}\hat{r} + a_p$$

$$\ddot{\bar{r}} = -\frac{\mu}{r^3}\hat{r} + a_{non-sph} + a_{drag} + a_{sun-moon} + a_{SRP}$$

$$\text{sec}\,ond\ \ order\ ODE.\ X = [r, v]^T$$

$$\dot{X} = [\dot{r}, \dot{v}]$$

MODEL

- $d\mathbf{X}/dt = \mathbf{f}(\mathbf{X}, t)$, $\mathbf{X}^T = [\mathbf{r}, \mathbf{v}]^T$

  $\mathbf{f}$, non-linear

- $\mathbf{Z} = \mathbf{h}(\mathbf{X}, \mathbf{X_s}, t)$: $\mathbf{h}$, nonlinear

$(\mathbf{z}: \boldsymbol{\rho} = \mathbf{r}_{s/c} - \mathbf{X_s}$; $\boldsymbol{\rho} = [\rho, Az, Eln]$;

$\boldsymbol{\rho}_{gps} = \mathbf{r}_{gps} - \mathbf{r}_{s/c}$)

Linearization

$\mathbf{X}_0$ reference state

$\delta \mathbf{x} = \mathbf{X} - \mathbf{X}_0$

$\mathbf{f}(\mathbf{X}(t), t) = \mathbf{f}(\mathbf{X}_0, t) + [\partial f(\mathbf{X}_0, t)/\partial \mathbf{X}^T]\, \delta \mathbf{X} + \ldots.$

$d(\boldsymbol{\delta}\mathbf{x})/dt = \mathbf{F}(\mathbf{X}_0, t)\delta \mathbf{X};\quad F = \partial f(\mathbf{X}(t), t)/\partial \mathbf{X}^{\mathbf{T}}$ at $\mathbf{X}_0$

$\delta \mathbf{z} = \mathbf{Z} - \mathbf{h}(\mathbf{X}_0, t)$

$\mathbf{h}(\mathbf{X}(t), t) = \mathbf{h}(\mathbf{X}_0, t) + [\partial \mathbf{h}(\mathbf{X}_0, t)/\partial \mathbf{X}^{\mathbf{T}}]\delta \mathbf{X} + \ldots$

$\delta \mathbf{z} = H(\mathbf{X}_0, t)\, \delta \mathbf{x}\quad H : \partial \mathbf{h}(\mathbf{X}(t), t)/\partial \mathbf{X}^{\mathbf{T}}$ at $\mathbf{X}_0$

Discretization

- Discretized State:

$$\delta \mathbf{x} = \boldsymbol{\Phi}\,(t_{k+1}, t_k)\,\delta \mathbf{x}_k + \boldsymbol{\omega}_k$$

Φ **:** State Transition Matrix (STM)

- Discretized Measurement Equation:

$$\delta \mathbf{z}_k = \mathbf{H}\,(\mathbf{X}(t_k), t_k)\,\delta \mathbf{x}_k + \boldsymbol{\upsilon}_k$$

DISCRETIZATION

- Discretize :

$\delta x_{k+1} = \Phi\ (t_{k+1}, t_k)\ \delta x_k$

This Φ matrix, state transition matrix STM, satisfies

$d\Phi(t, \tau)/dt = F(t)\ \Phi\ (t, \tau)$ for all t and τ.

$\Phi\ (\tau, \tau) = I$ for all τ

$\Phi(t, \tau) * \Phi(\tau, t_1) = \Phi(t, t_1)$ for all t , t_1 and τ

and is given by the expression

$$\Phi(t_{k+1}, t_k) = \exp[\ (t_{k+1} - t_k) * F(X_0(t_k), t_k)\ ]$$

Actual Model

Stochastic Difference Equations:

$$\delta \mathbf{x}_{k+1} = \Phi\,(t_{k+1}, t_k)\,\delta \mathbf{x}_{\mathbf{k}} + \boldsymbol{\omega}_{\mathbf{k}}$$

$$\delta \mathbf{z}_{k+1} = H\,(\mathbf{X_0}(t_{k+1}), t_{k+1})\,\boldsymbol{\delta}\mathbf{x}_{k+1} + \boldsymbol{\upsilon}_{\kappa+1}$$

$\{\boldsymbol{\omega}_{\mathbf{k}}\}, \{\boldsymbol{\upsilon}_{\mathbf{k}}\}$: State, measurement noise

- $E\{\omega_k\} = 0$; $E\{\omega_k \omega_j^T\} = Q_k\,\delta_{kj}$
- $E\{\upsilon_k\} = 0$; $E\{\upsilon_k \upsilon_j^T\} = R_k\,\delta_{kj}$
- $E\{\upsilon_k \omega_j^T\} = 0$

Initialization

- a priori state X_0
- error covariance $P_0 = \mathrm{E}[\, X_0 \, X_0^T]$ of the apriori state
- noise covariance matrices Q and R to be initialized

EXTENDED KF

- $X^p = X^e + \int f\,dt$
- $P^p = \Phi P^e \Phi^T + Q$

- $K = P^p H^T [H P^p H^T + R_{k+1}]^{-1}$
- $X^e = X^p + K [Z_{k+1} - h(X^e, t)]$
- $P^e = [I - KH] P^p [I - KH]^T + K R_{k+1} K^T$

Interest: Model Reduction

$$\ddot{\bar{r}} = -\frac{\mu}{r^3}\hat{r} + a_p$$

$$\ddot{\bar{r}} = -\frac{\mu}{r^3}\hat{r} + a_{j2}$$

$$X = \begin{pmatrix} r \\ v \end{pmatrix}$$

$$\dot{r} = v \; ; \; \dot{v} = a_m(t) + a_u(t)$$

$$\dot{v} = a_m + \varepsilon(t) \; ; \quad \dot{\varepsilon} = A\,\varepsilon(t) + u_\varepsilon(t);$$

$$A : \begin{bmatrix} -\frac{1}{\tau_x} & 0 & 0 \\ 0 & -\frac{1}{\tau_y} & 0 \\ 0 & 0 & -\frac{1}{\tau_z} \end{bmatrix}$$

$u_\varepsilon(t)$ *is zero mean, white Gaussian*

$$T^T = \begin{bmatrix} \tau_x & \tau_y & \tau_z \end{bmatrix}^T \; ; \; \dot{T} = 0 \; ;$$

idea

$Consider \quad \dot{\varepsilon} = -\frac{1}{\tau}\,\varepsilon\,(t) + u(t);\ where\ \tau\ is\ \ correletio n\ time\, const.$

$This\, eqn.\ is\ \ first\ \ \ order\ \ \exp onentially\ \ \ \ Correlated\ \ \ process\, noise;$

$where\ u\ is\ \ zero\, mean\, white\, Gaussian\ \ E\{u(t)u(\tau)\} = q_u\ \delta(t-\tau)$

$Solution\ \ :\ \varepsilon(t) = \exp\{-(t-\alpha)/\tau\}\,p(\alpha) + \int_{\alpha}^{t} \exp\{-(t-\zeta)/\tau\}u(\zeta)d\zeta$

$The\, \mathrm{var}\, iance\, of\ its\ \ solution\ \ \varepsilon\,(t)\ satisfies\ \ an\ ODE$

$d\ \left[\sigma_{\varepsilon}{}^{2}\right]/\,dt\ \ = -(2/\tau)\sigma_{\varepsilon}{}^{2}\ + q_u;$

$where\ \sigma_{\varepsilon}{}^{2}\ is\ \ E\{\varepsilon\,(t)^2\}$

$Assume:\ The\ process\ \ is\, operating\ \ indefenite\, ly.\ Then\ \ q_u\ \ = (2/\tau)\sigma_{\varepsilon}{}^{2}$

$Discrete\ \ version:\ \varepsilon_{j+1} = m_j \varepsilon_j\ + u_j\,;\ \ \ mj = \exp\{-\Delta t/\tau\}\ etc.$

$$\dot{r} = v \; ;$$

$$\dot{v} = a_m + \varepsilon(t) \; ;$$

$$\dot{\varepsilon} = A\,\varepsilon\,(t) + u_\varepsilon(t);$$

$$\dot{T} = 0$$

New State: Parameter Estimation

$$X^T = \begin{bmatrix} r^T & v^T & \varepsilon^T & T^T \end{bmatrix};$$

$$for \;\; t > t_j$$

$$r(t) = r(t_j) + v(t_j)\,\Delta t + \int_{t_j}^{t} a(r, v, \varepsilon, \tau)[t - \tau]\, d\tau$$

$$v(t) = v(t_j) + \int_{t_j}^{t} a(r, v, \varepsilon, \tau)\; d\tau$$

$$\varepsilon(t) = E(t)\varepsilon_j + u(t_j)$$

$$T(t) = T(t_j)$$

…New State: Parameter Estimation

$$E(t) = \begin{bmatrix} \alpha_x & 0 & 0 \\ 0 & \alpha_y & 0 \\ 0 & 0 & \alpha_z \end{bmatrix}; \ \alpha_x = \exp\{-(t - t_j)/\tau_x\} \ etc.$$

$$\theta = \begin{bmatrix} \sigma_x(\sqrt{1-\alpha_x{}^2})u_x \\ \sigma_y(\sqrt{1-\alpha_y{}^2})u_y \\ \sigma_z(\sqrt{1-\alpha_z{}^2})u_z \end{bmatrix}$$

$$X(t) = f\,(X_i\,, t_i\,, t) + \eta_i \qquad for\;\; t > t_i$$

$$where\;\; \eta^T\; = [\eta_r{}^T\,, \eta^T{}_v\,, \eta^T{}_\varepsilon\,, o];$$

$$\eta_r{}^T\; = \int_{tj}^{t} \theta_i\,(t - \tau)\,d\tau$$

$$\eta_v{}^T\; = \int_{tj}^{t} \theta_i\; d\tau$$

$$\eta_\varepsilon{}^T\; = \theta i_i$$

$$E[\eta_i] = \;0\;\;; \quad E[\,\eta_i\,\eta_i{}^T\,] = Qi\delta_{i\,j}\;\; where\; Q\;(12x12)$$

$$is\;\; given\; by\; ......$$

……Process noise

$$Q = \begin{bmatrix} S(\Delta t)^4/4 & S(\Delta t)^3/2 & S(\Delta t)^2/2 & O \\ S(\Delta t)^3/2 & S(\Delta t)^2 & S(\Delta t) & O \\ S(\Delta t)^2/2 & S(\Delta t) & S & O \\ O & O & O & W \end{bmatrix} \textit{where}$$

$$S = \begin{bmatrix} \sigma_x{}^2(1-\alpha_x{}^2) & 0 & 0 \\ 0 & \sigma_y{}^2(1-\alpha_y{}^2) & 0 \\ 0 & 0 & \sigma_z{}^2(1-\alpha_z{}^2) \end{bmatrix}$$

Results of all Three Procedures

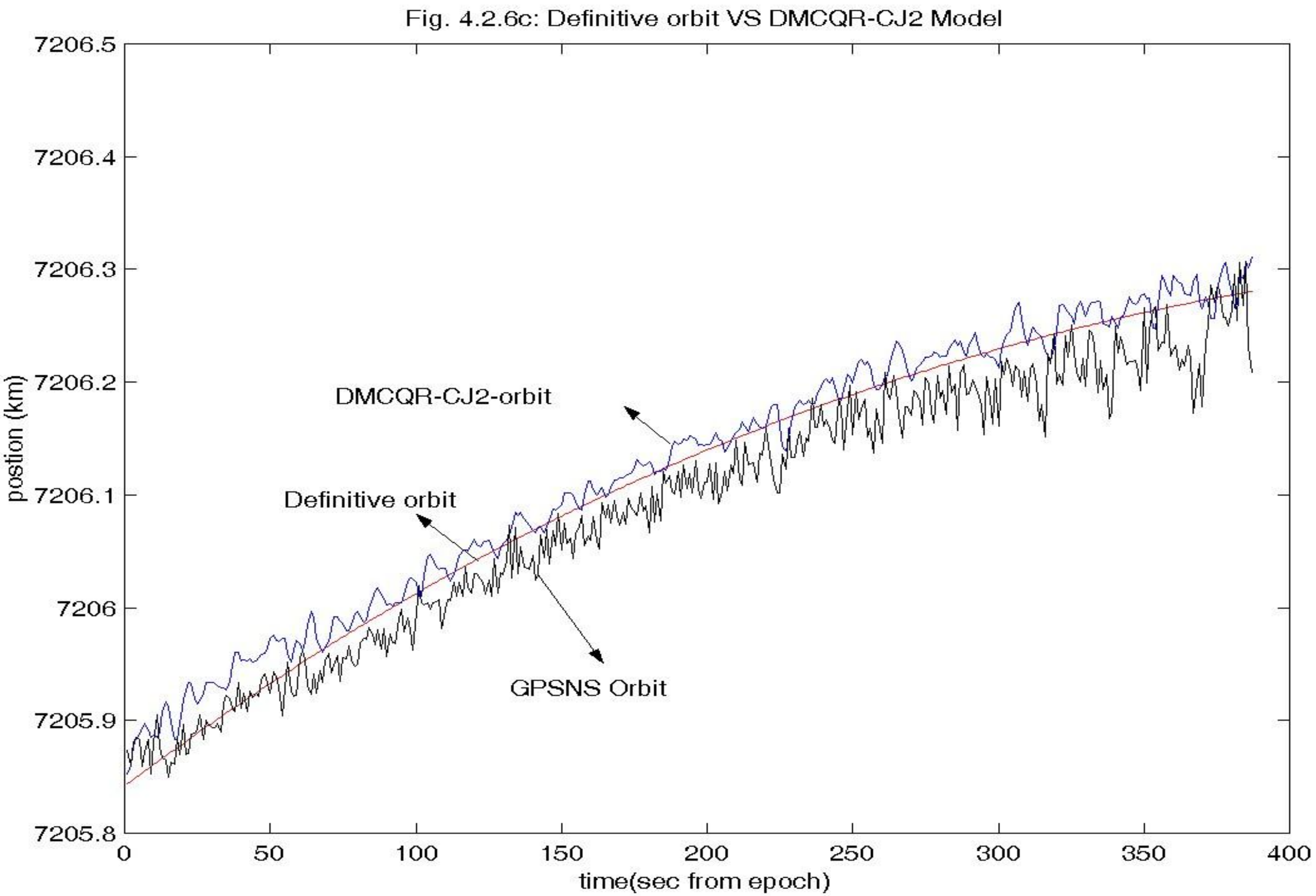


Fig. 4.2.6a:Difference in Position:DMCQR VS GPSNS

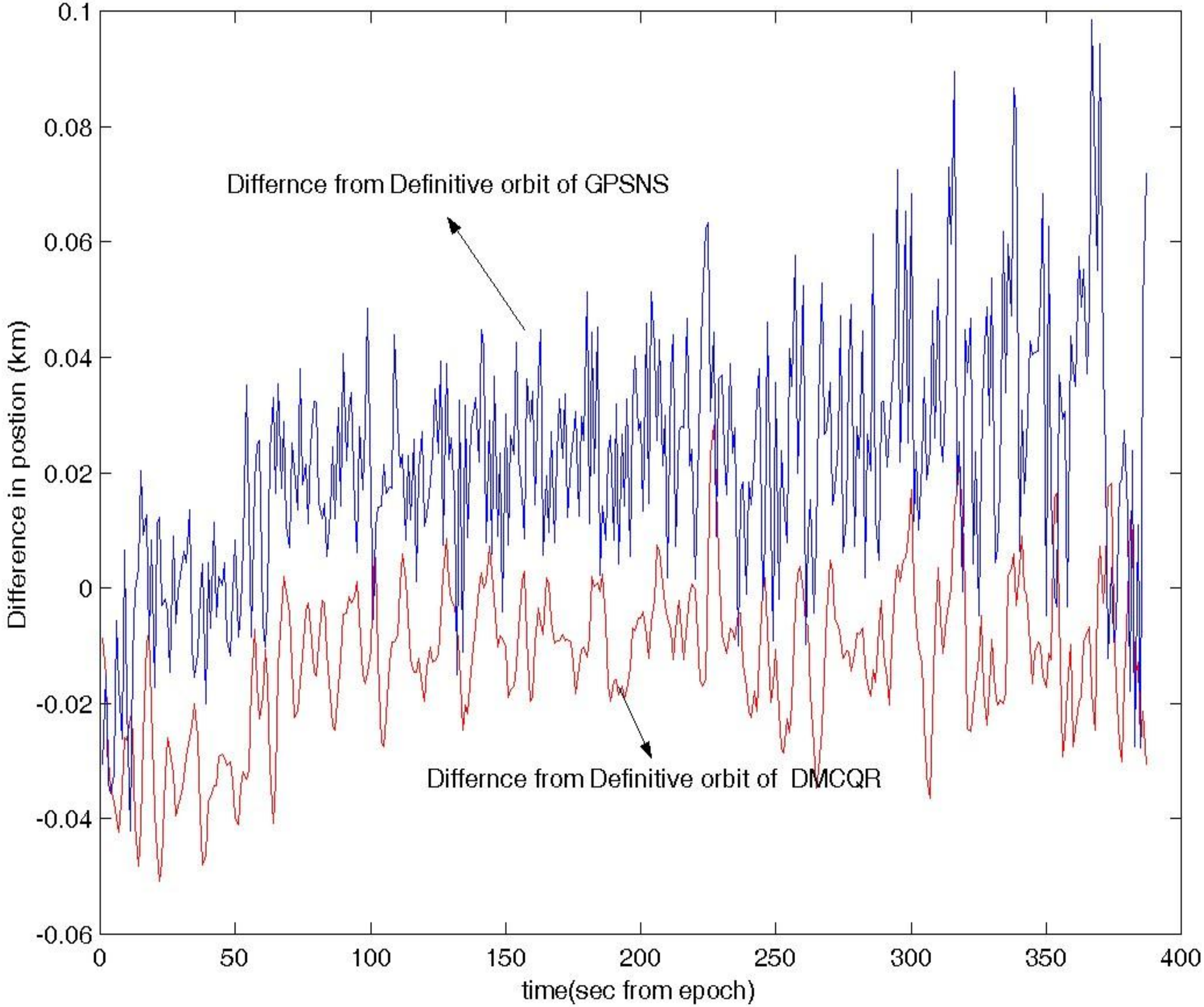
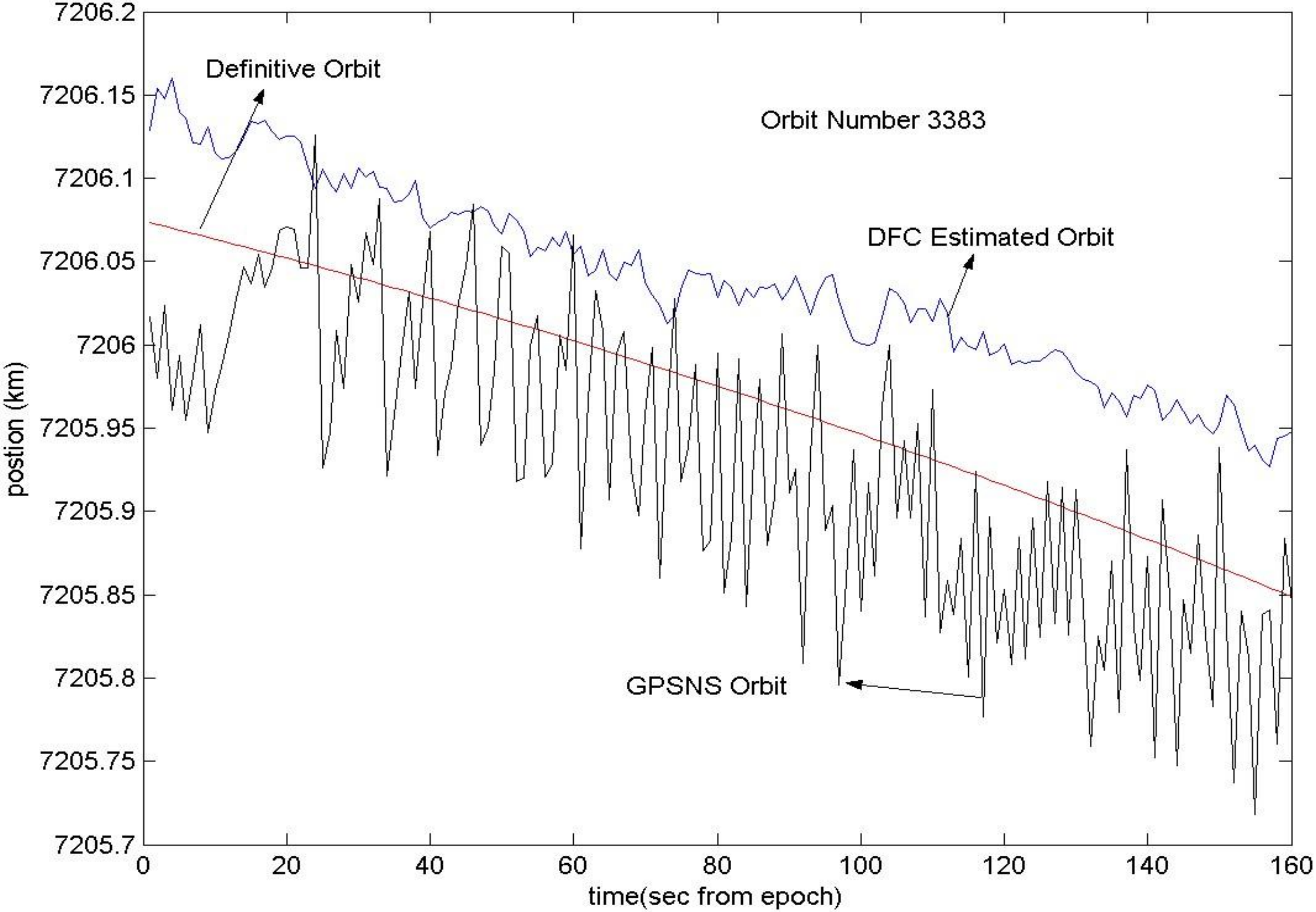
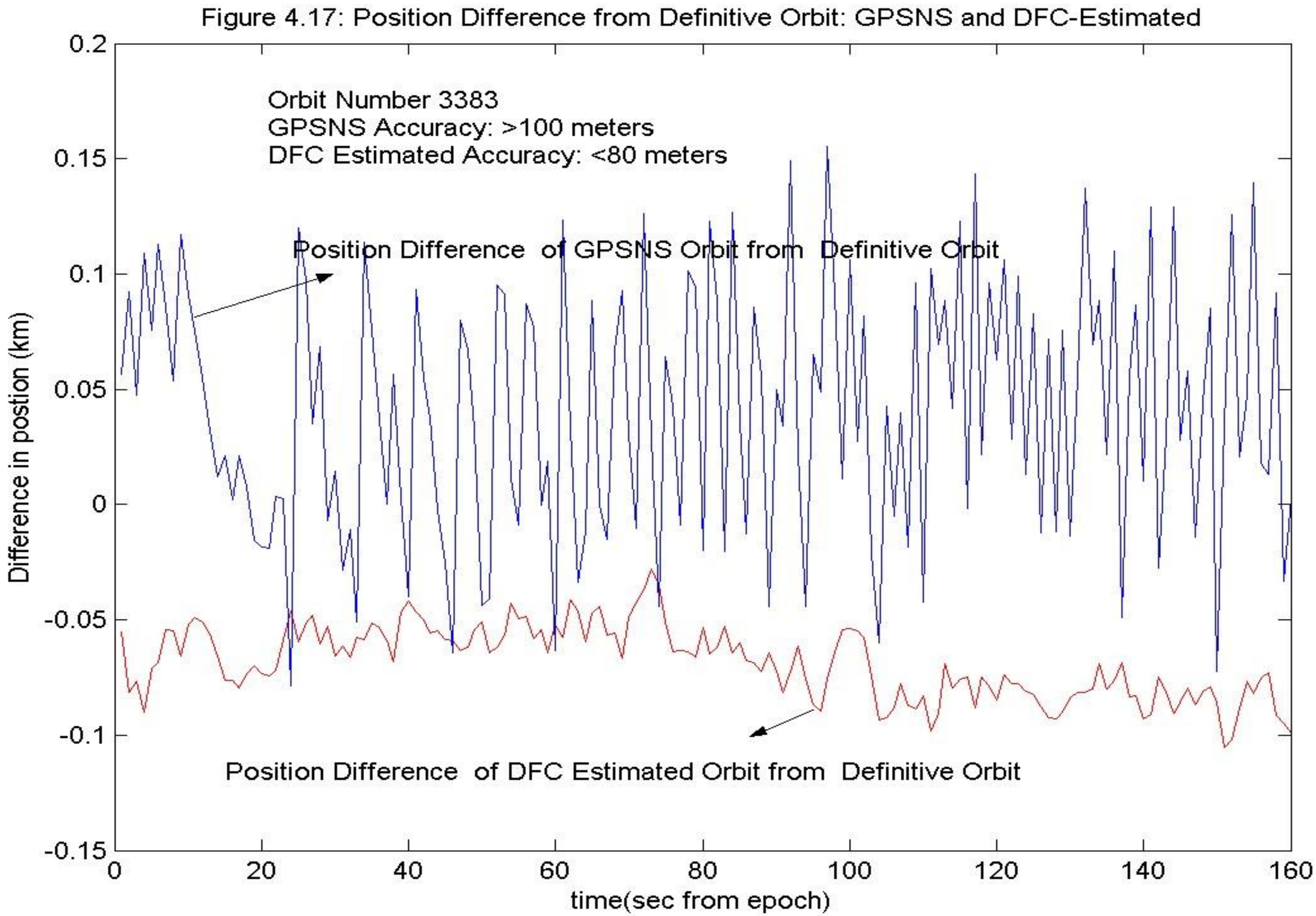


Figure 4.16: Positions of Definitive orbit, GPSNS,and Estimated orbit

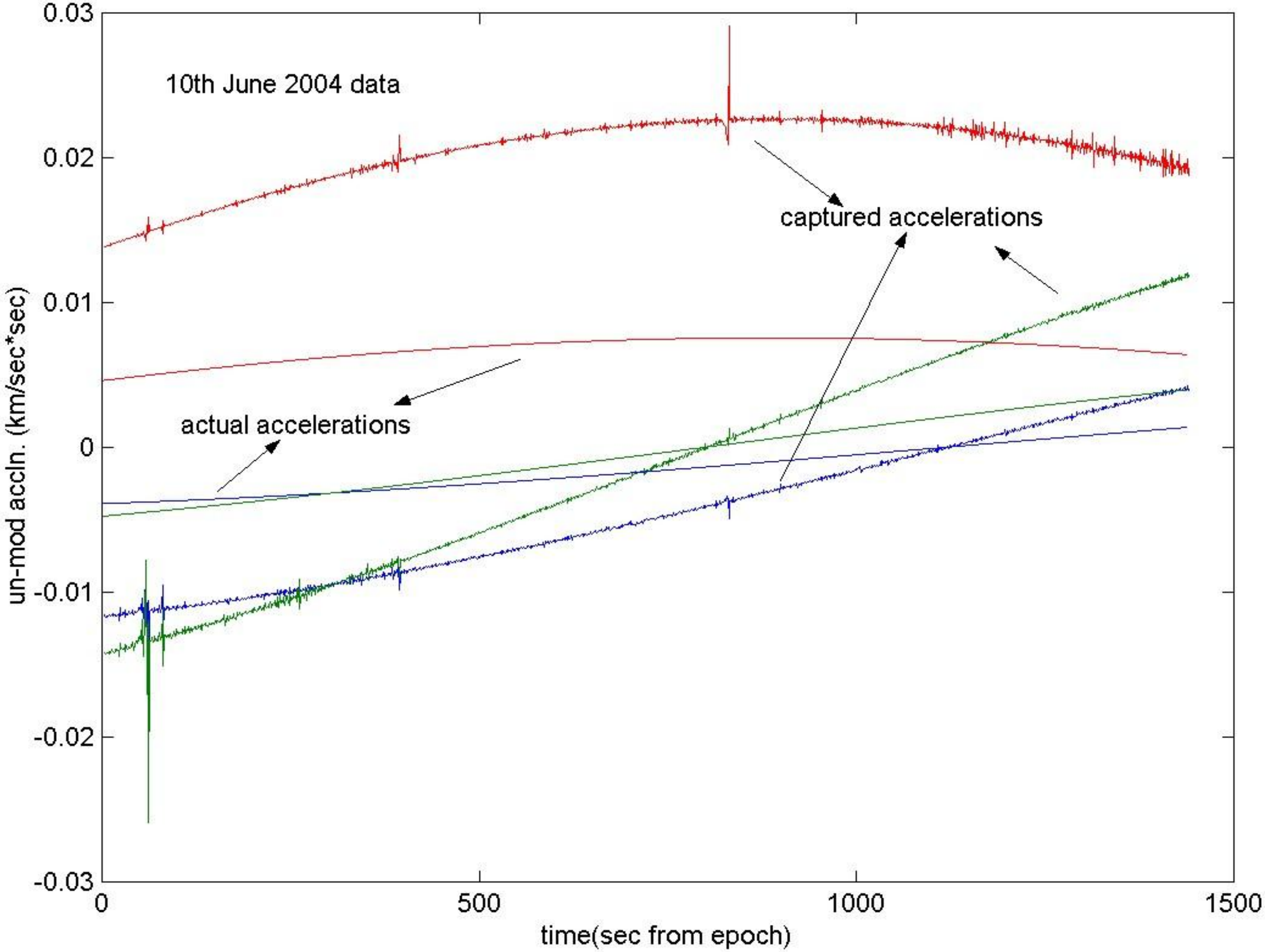




time(sec from epoch)

THANK YOU

Figure 4.10.1: Unmodelled accelerations: Captured and actual



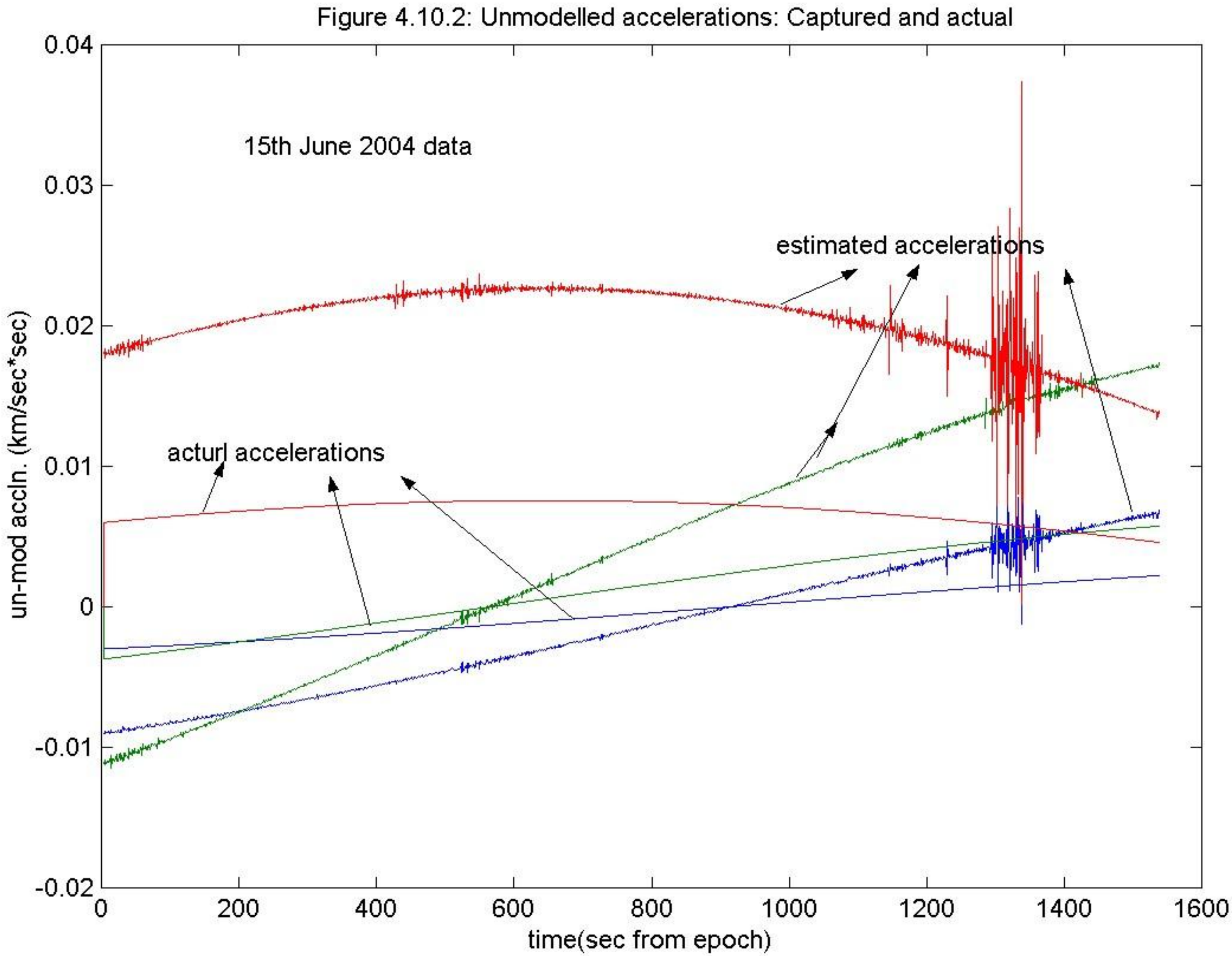


Figure 4.10.2: Unmodelled accelerations: Captured and actual
15th June 2004 data
estimated accelerations
acturl accelerations
un-mod accln. (km/sec*sec)
time(sec from epoch)
0.04
0.03
0.02
0.01
0
-0.01
-0.02
0
200
400
600
800
1000
1200
1400
1600