

This is a series of four lectures on modularity lifting, with emphasis on the ordinary case. Professor Balasubramanyam will give four lectures covering the necessary background on Hida theory. In this course, our lectures will be organised as follows:

1. Introduction to modularity lifting theorems. We will explain the general ideas behind the Taylor–Wiles–Kisin method for proving modularity of Galois representations.
2. Galois deformation rings. In the first lecture, we will see that the applicability of modularity lifting theorems is dependent on the structure of local Galois deformation rings. In practice, one is limited primarily by the local deformation rings at the primes dividing the residue characteristic of the field of coefficients (the  $l = p$  case). We will discuss some of what is known about these rings and some applications to modularity.
3. Potential diagonalizability. In this lecture we will discuss a technique introduced by Harris that allows one, in certain circumstances, to prove modularity lifting theorems even when the appropriate local deformation rings are very complicated. Hida families will play an important role here.
4. Recent developments. Depending on time remaining, we may discuss recent work of Thorne on modularity lifting in the residually reducible case (building on work of Skinner and Wiles). Again, Hida feature prominently in this work.