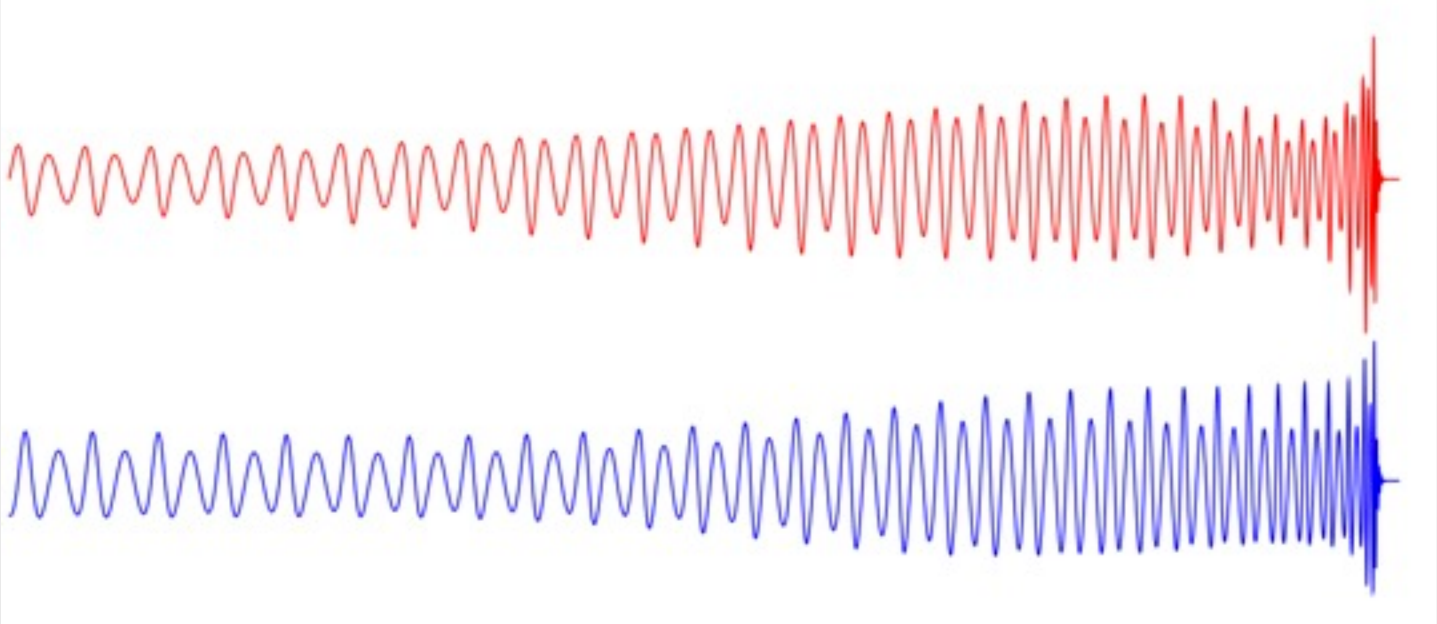
Current research in Numerical Relativity

Harald Pfeiffer, CITA

ICTS/TIFR Summer School in Numerical Relativity
June 21, 2013, Bangalore



Outline

- ❖ Motivation
- ❖ BH-BH
  - History
  - Recent technical improvements
  - The big picture
- ❖ Beyond vacuum GR
  - BH-NS
  - NS-NS
  - core-collapse SN
- ❖ The Future

Motivation for Numerical Simulations

❖ Investigate General Relativity in the dynamic, strong-field regime
  - compact object mergers
  - critical collapse
  - higher-dimensional gravity

❖ Astrophysics: What happens when
  - ... stars collapse?
  - ... compact objects collide?

❖ **Aid GW detectors**

LIGO's many Numerical Relativity needs

- ❖ **Signal detection**
  - Need template banks of that region of parameter space that is targeted in searches

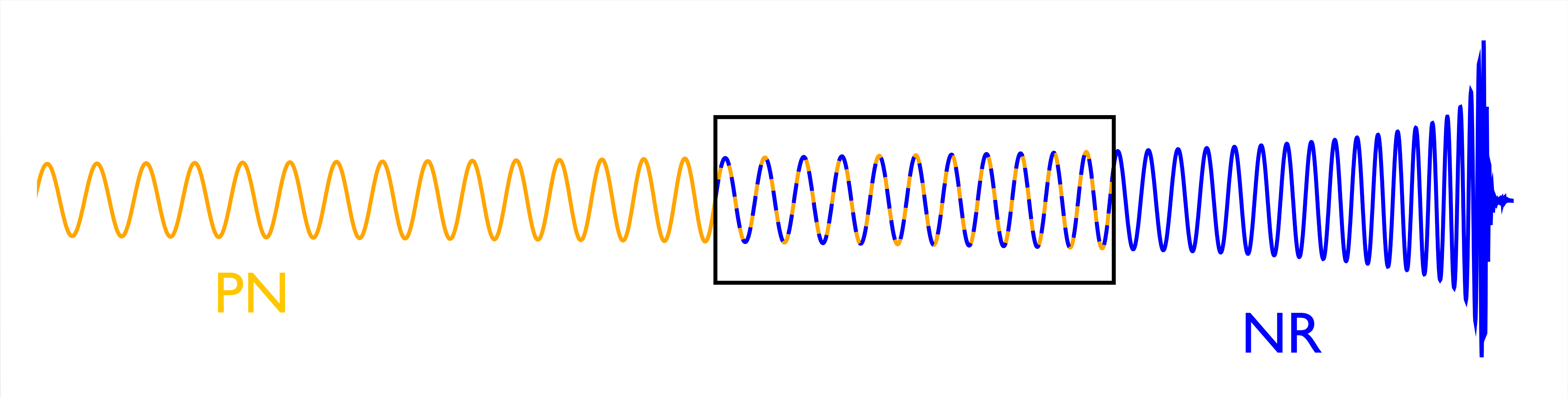
    e.g. aligned spin binaries
- ❖ Bounds on **event-rates** from non-detection
  - Some waveforms elsewhere in parameter space

    e.g. precessing systems; eccentric systems
- ❖ **Parameter estimation**
  - Especially accurate waveform models in all parameters being estimated

$$\vec{S}_1, \vec{S}_2, M_1, M_2, e, \ldots$$

- ❖ Properties of **electro-magnetic counterparts**
  - What should telescopes look for?

Tools for computing waveforms

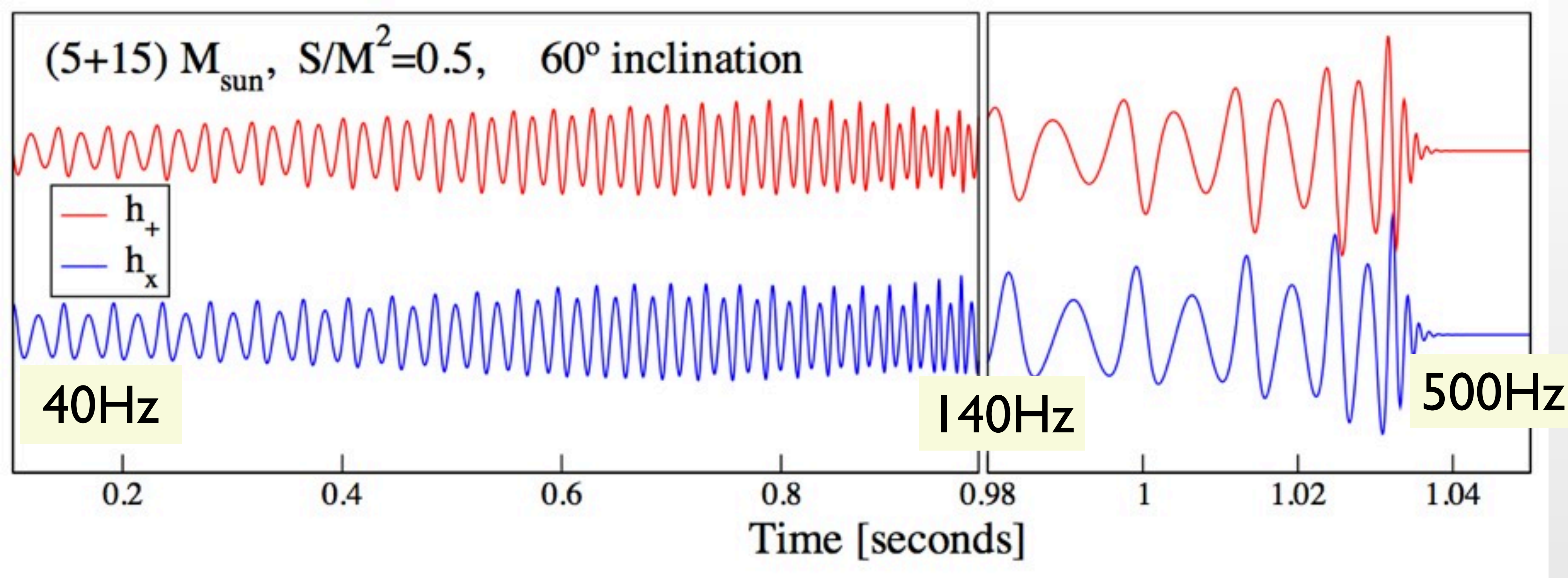


❖ Early inspiral
- Post-Newtonian calculations

❖ Late inspiral & Merger
- Computer simulations

❖ Ringdown
- Perturbation theory
- Computer simulations

More generic case



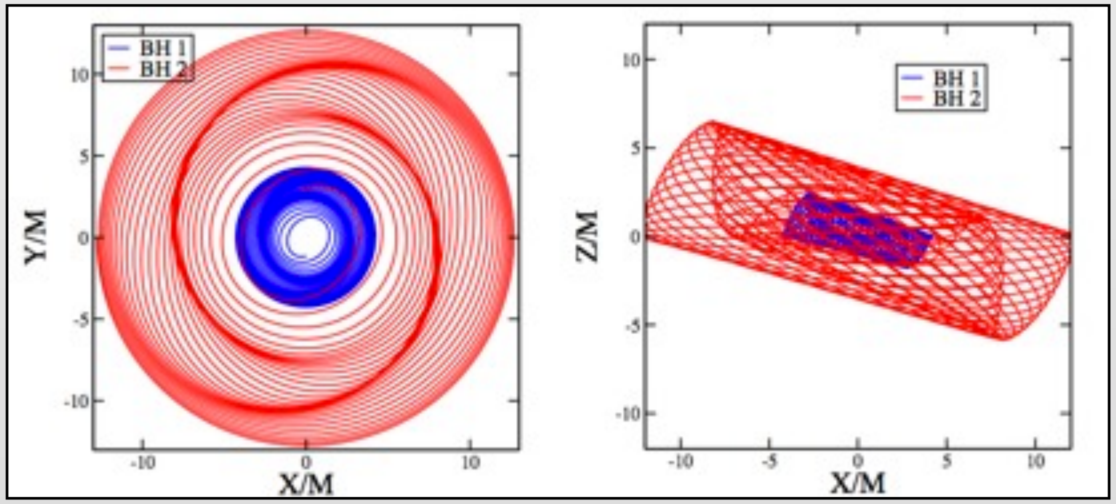
40Hz

140Hz

500Hz

Abdul Mroue (CITA)

- ❖ Modulated amplitude
- ❖ Higher temporal harmonics
- ❖ Dependence on inclination
- ❖ Modified phasing



50 Years of Vacuum Numerical Relativity

CITA–ICAT

1962 *(ADM)*
3+1 formulation

1964
Hahn-Lindquist
2 wormholes

1984
Unruh
excision

1994 *Cook*
Bowen-York
initial data

1997
Brandt-Brügmann
puncture data
(no excision)

1999-00
AEI/PSU
grazing collisions

~2000 *Choptuik; Schnetter;Brügmann*
mesh refinement

2000-04
AEI/UTB-NASA
revive crashing
codes (Lazarus)

2004
Brügmann et al
one orbit

2005 *Pretorius*
inspiral-merger-
ringdown (IMR)
w/ harmonic

2005-06
UTB/NASA
IMR with BSSN +
moving punctures

2006-08
Cornell/Caltech
IMR w/ spectral

2007-
Ajith, AEI, Jena
phenom GW models

2009-
UMD, SXS
EOB GW models

2011
Schmidt ea;
Boyle ea
Radiation aligned
frame

2011
Lousto ea
q=100

1975-77
Smarr-Eppley
head-on
collision

1989-95
Bona-Masso
modified ADM,
(hyperbolicity)

1992,3
Choptuik;
Abrahams+Evans
critical phenomena

1994-95
NCSA-WashU
improved
head-on collision

1999
BSSN
evolution
system

1999-2005 *J.W.York;*
Cornell, Caltech, LSU
hyperbolic formulations

1999 *York*
conformal thin
sandwich ID

2000-02
Alcubierre
gauge conditions

2000 *Ashtekar*
isolated horizons

2003-08
Cook, Pfeiffer ea
improved ID

2006,07
Baker et al
Gonzalez et al
non-spinning BBH
kicks

2007-11
RIT, Jena, AEI,...
BBH superkicks

2007 *SXS*
PN-NR
comparison

2008
all of NR
NINJA

2009-11
Bishop, ...
Cauchy
characteristic
extraction

2011
Lovelace ea
S/M^2=0.97

2013
GaTech, SXS
Precessing
parameter
studies

Courtesy Carlos Lousto,
updated by HP

LIGO

advanced LIGO

50 Years: The early days

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LIGO

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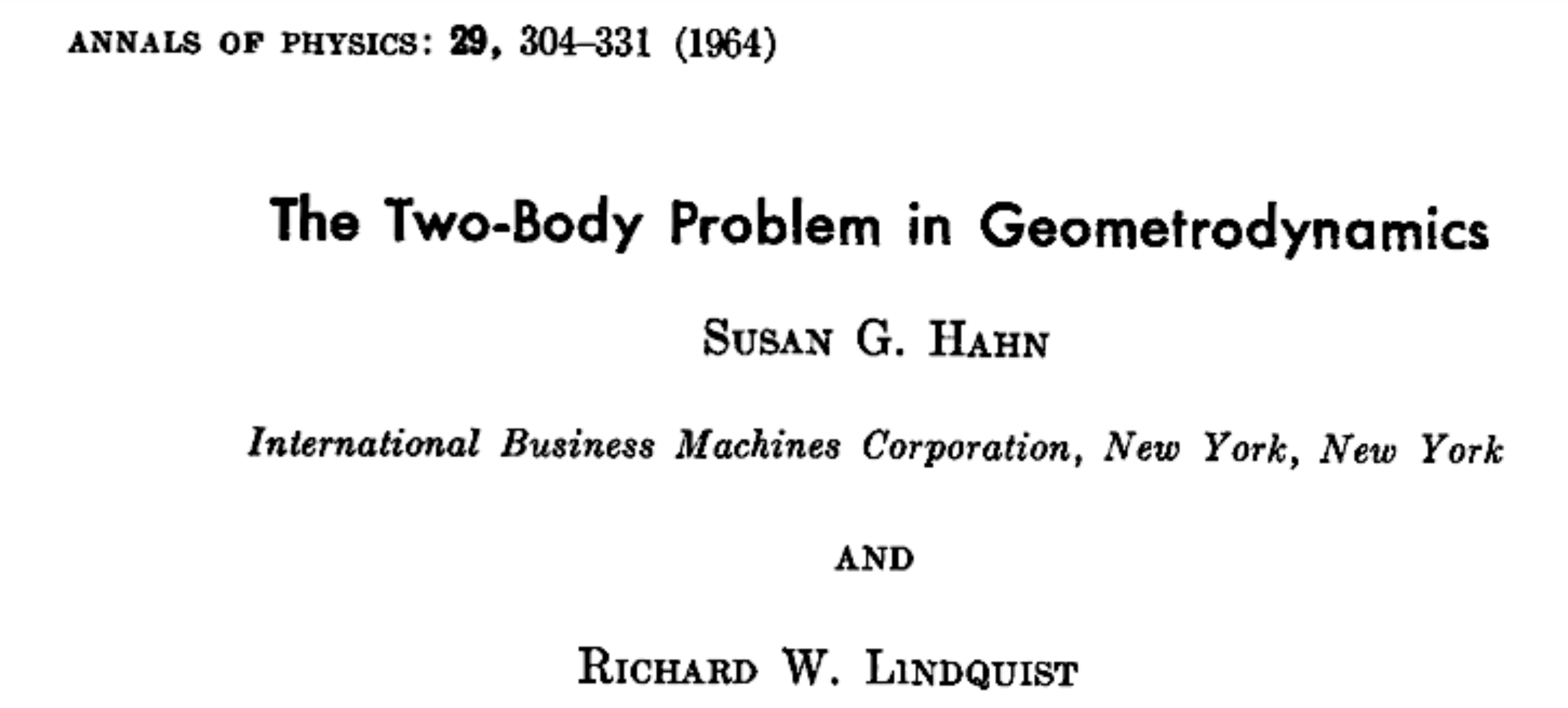
50 Years:

❖ These should also have been on the time-line

- 2005 Constraint Damping (Gundlach, Pretorius, Lindblom)
- 2005 Constraint-preserving outer boundary conditions for GH (Lindblom, Scheel)
- 2009 Unstable 5-D black strings (Lehner, Pretorius)

❖ Audience's additions:



ANNALS OF PHYSICS: **29**, 304–331 (1964)

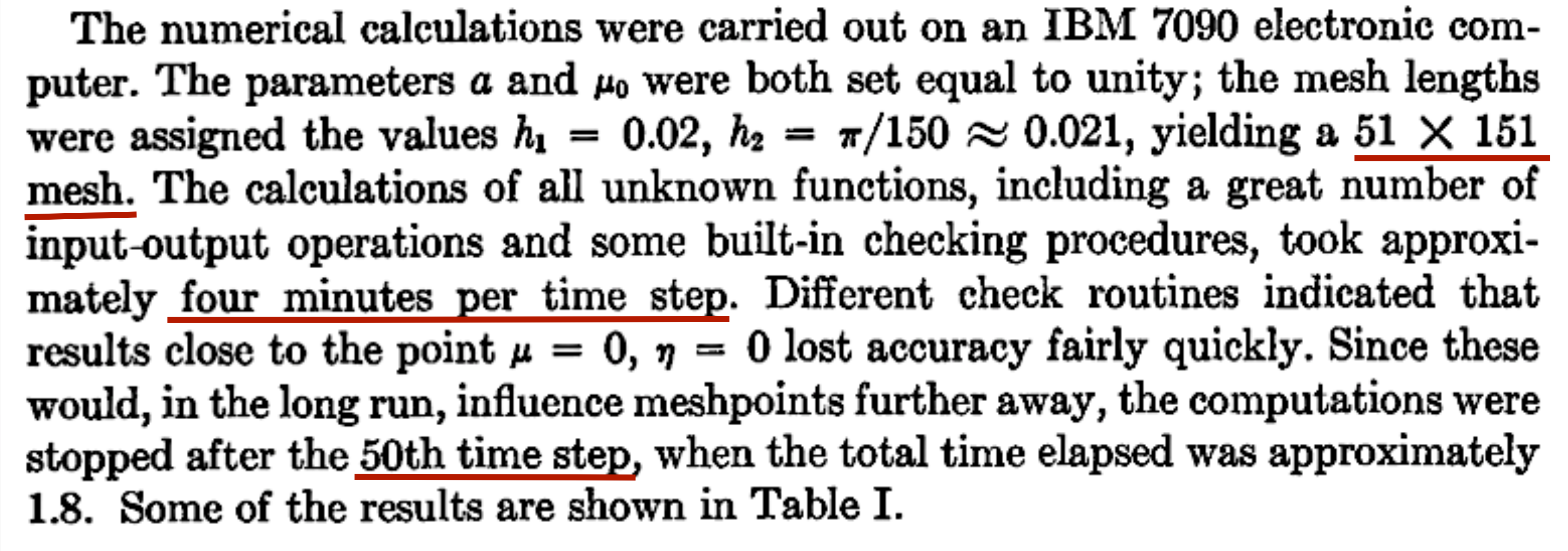
The Two-Body Problem in Geometrodynamics

SUSAN G. HAHN

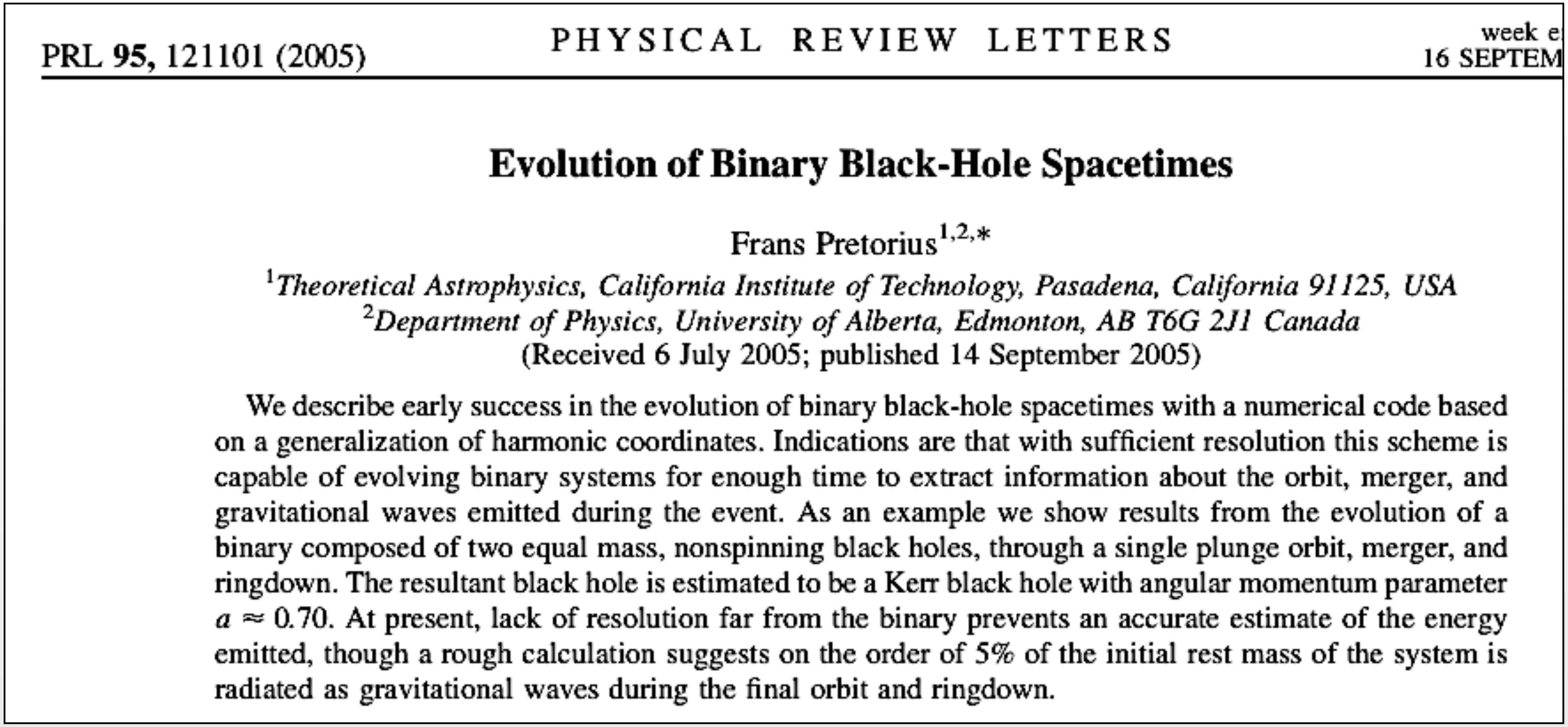
International Business Machines Corporation, New York, New York

AND

RICHARD W. LINDQUIST



The numerical calculations were carried out on an IBM 7090 electronic computer. The parameters a and μ_0 were both set equal to unity; the mesh lengths were assigned the values $h_1 = 0.02$, $h_2 = \pi/150 \approx 0.021$, yielding a 51×151 mesh. The calculations of all unknown functions, including a great number of input-output operations and some built-in checking procedures, took approximately four minutes per time step. Different check routines indicated that results close to the point $\mu = 0$, $\eta = 0$ lost accuracy fairly quickly. Since these would, in the long run, influence meshpoints further away, the computations were stopped after the 50th time step, when the total time elapsed was approximately 1.8. Some of the results are shown in Table I.



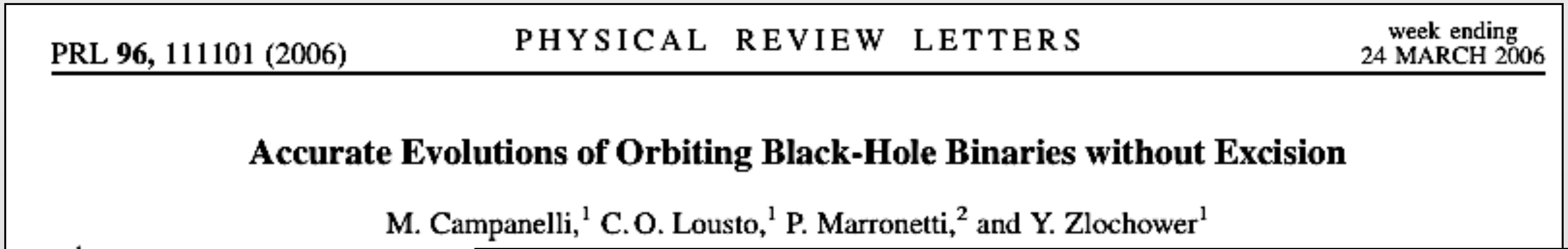
PRL **95,** 121101 (2005) PHYSICAL REVIEW LETTERS week e
16 SEPTEM

Evolution of Binary Black-Hole Spacetimes

Frans Pretorius[1,2,*]
[1]*Theoretical Astrophysics, California Institute of Technology, Pasadena, California 91125, USA*
[2]*Department of Physics, University of Alberta, Edmonton, AB T6G 2J1 Canada*

(Received 6 July 2005; published 14 September 2005)

We describe early success in the evolution of binary black-hole spacetimes with a numerical code based on a generalization of harmonic coordinates. Indications are that with sufficient resolution this scheme is capable of evolving binary systems for enough time to extract information about the orbit, merger, and gravitational waves emitted during the event. As an example we show results from the evolution of a binary composed of two equal mass, nonspinning black holes, through a single plunge orbit, merger, and ringdown. The resultant black hole is estimated to be a Kerr black hole with angular momentum parameter $a \approx 0.70$. At present, lack of resolution far from the binary prevents an accurate estimate of the energy emitted, though a rough calculation suggests on the order of 5% of the initial rest mass of the system is radiated as gravitational waves during the final orbit and ringdown.

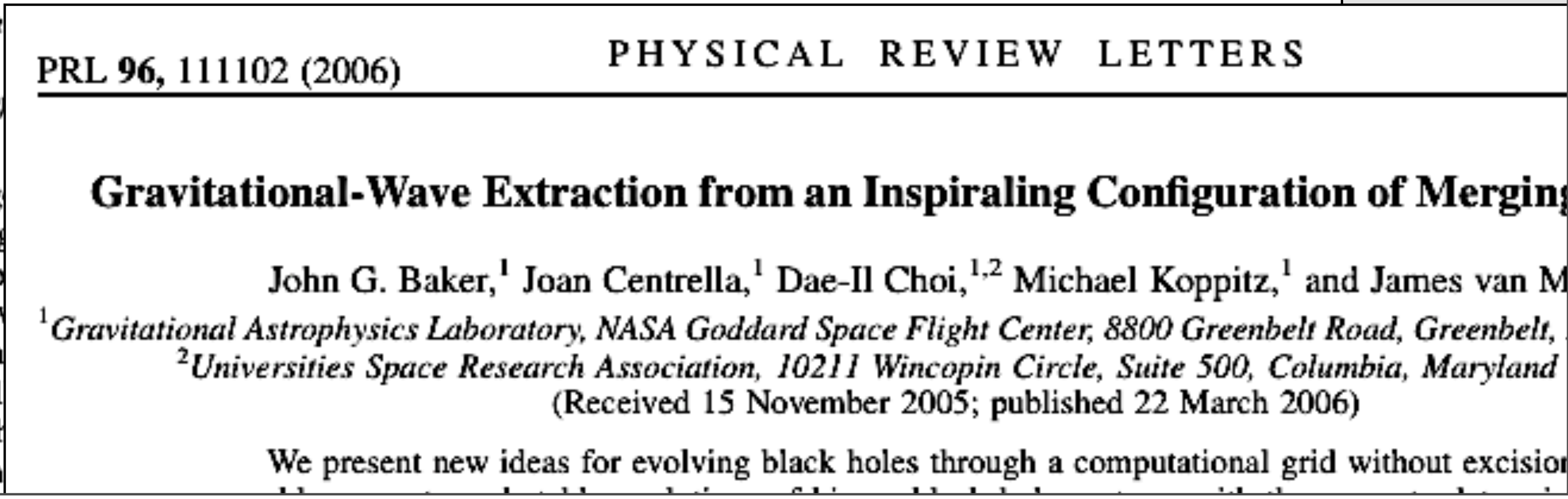


PRL **96,** 111101 (2006) PHYSICAL REVIEW LETTERS week ending
24 MARCH 2006

Accurate Evolutions of Orbiting Black-Hole Binaries without Excision

M. Campanelli,[1] C. O. Lousto,[1] P. Marronetti,[2] and Y. Zlochower[1]
[1]*Department of Physics and Astron*
[2]*Department o*

We present a new alg
corotating shift. Our alg
factor. This system, b
equations, when used
and remains nonsingula
use this technique to ful
regime. We show fourth
and angular momentum



PRL **96,** 111102 (2006) PHYSICAL REVIEW LETTERS

Gravitational-Wave Extraction from an Inspiraling Configuration of Mergin

John G. Baker,[1] Joan Centrella,[1] Dae-Il Choi,[1,2] Michael Koppitz,[1] and James van M
[1]*Gravitational Astrophysics Laboratory, NASA Goddard Space Flight Center, 8800 Greenbelt Road, Greenbelt,*
[2]*Universities Space Research Association, 10211 Wincopin Circle, Suite 500, Columbia, Maryland*

(Received 15 November 2005; published 22 March 2006)

We present new ideas for evolving black holes through a computational grid without excisio

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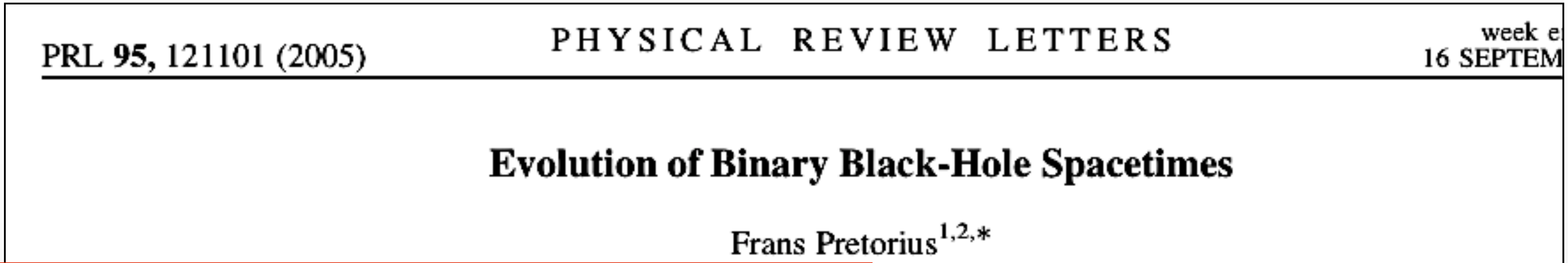

PRL **95,** 121101 (2005) PHYSICAL REVIEW LETTERS week e
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Evolution of Binary Black-Hole Spacetimes

Frans Pretorius[1,2,*]

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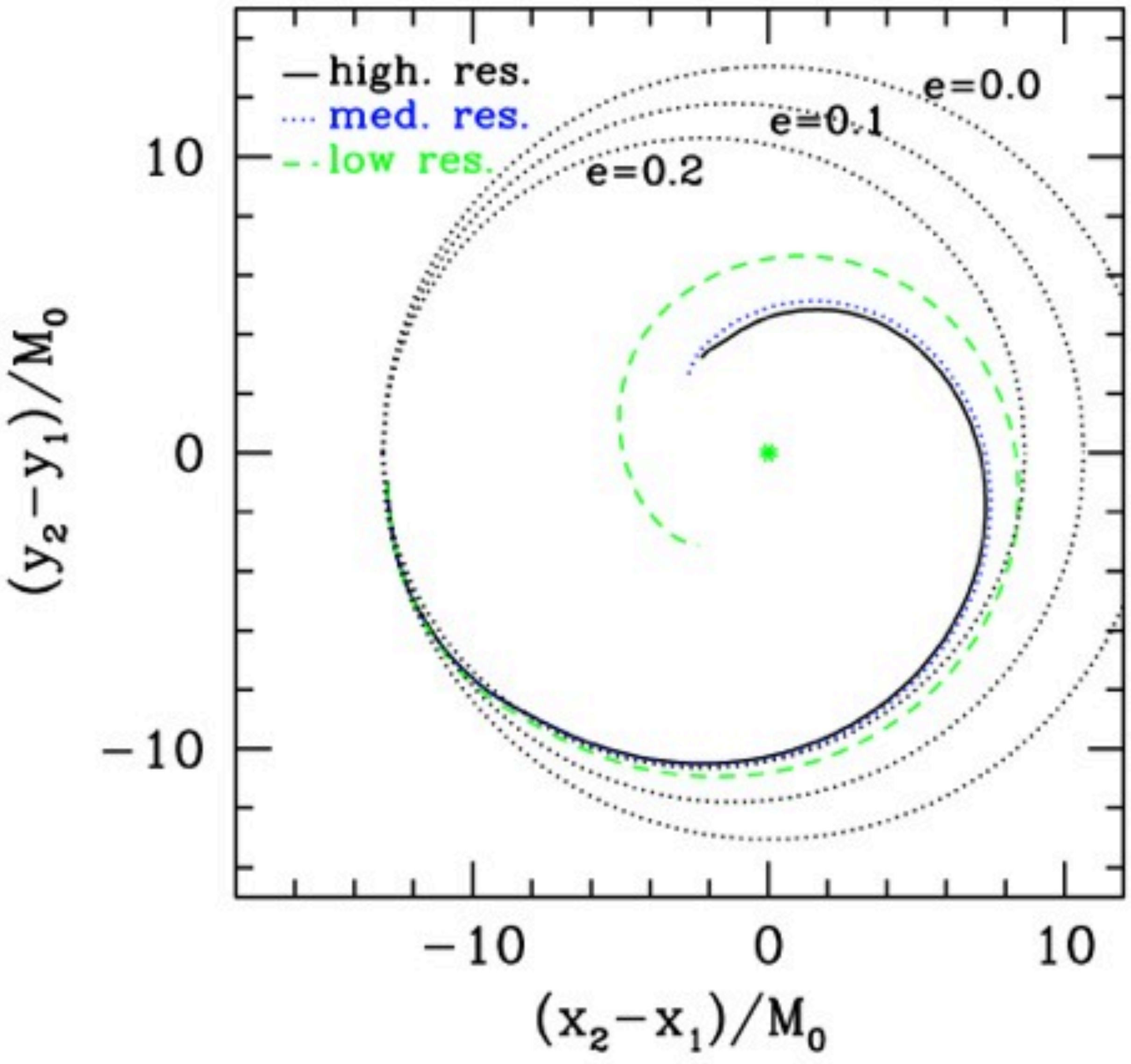
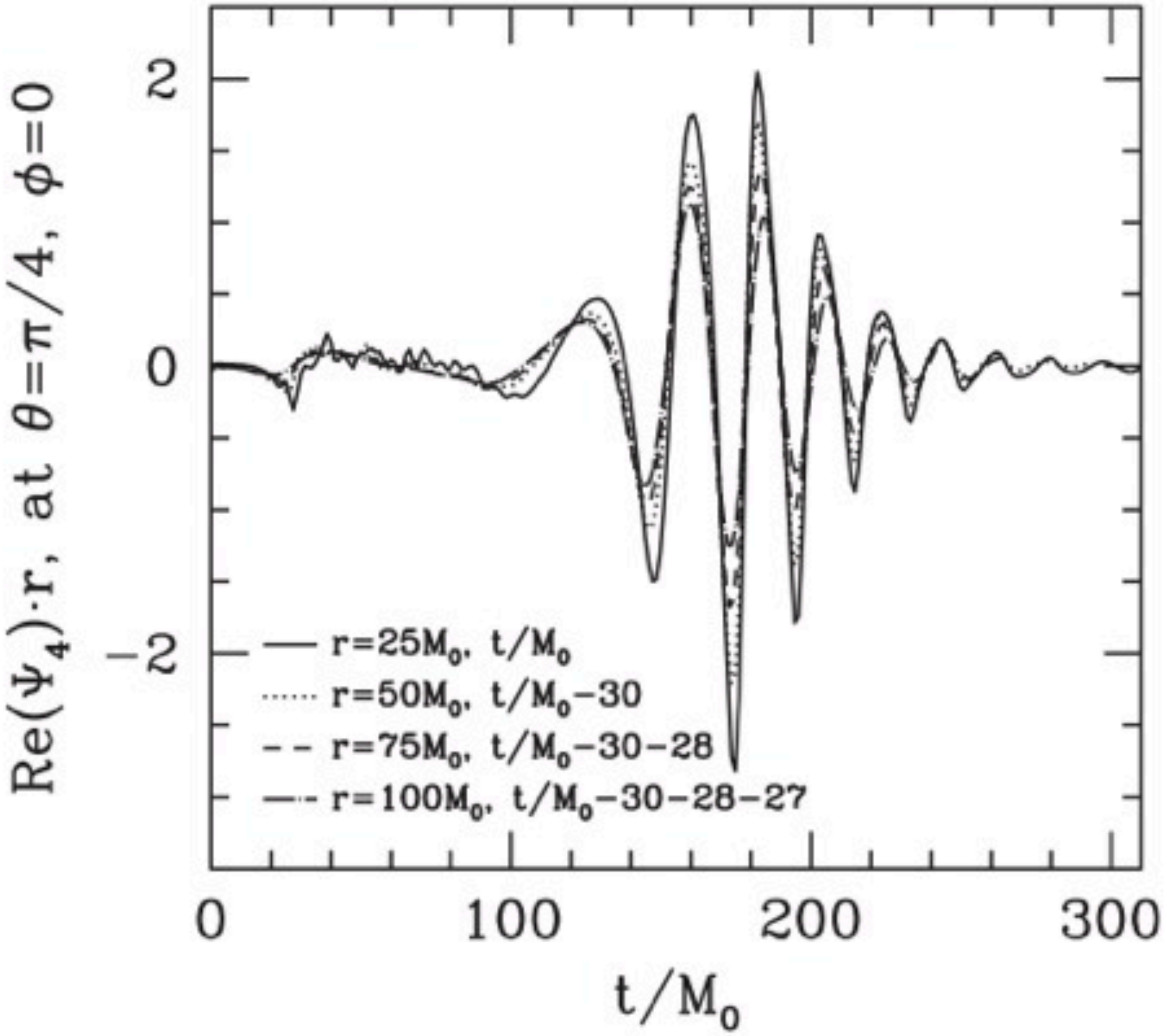
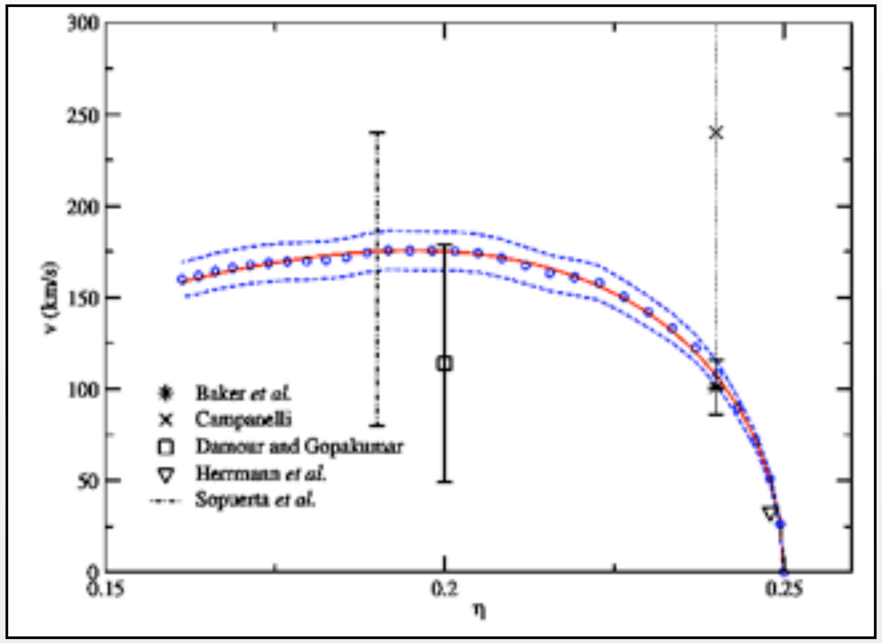




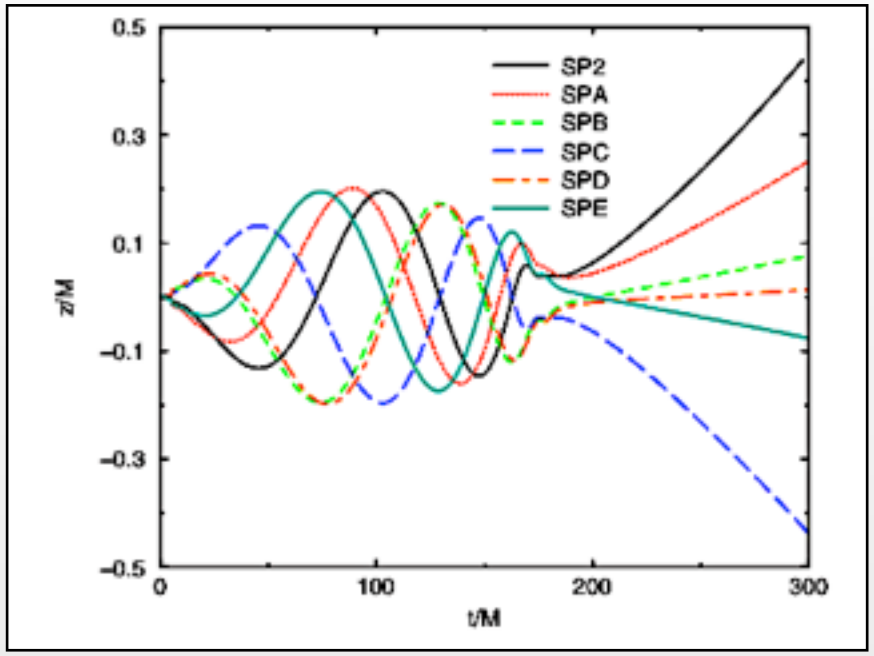
Early days of BH-BH sims



Spin=0 BH-BH kicks

Gonzalez, Sperhake, Brügmann, Hannam, Husa 07

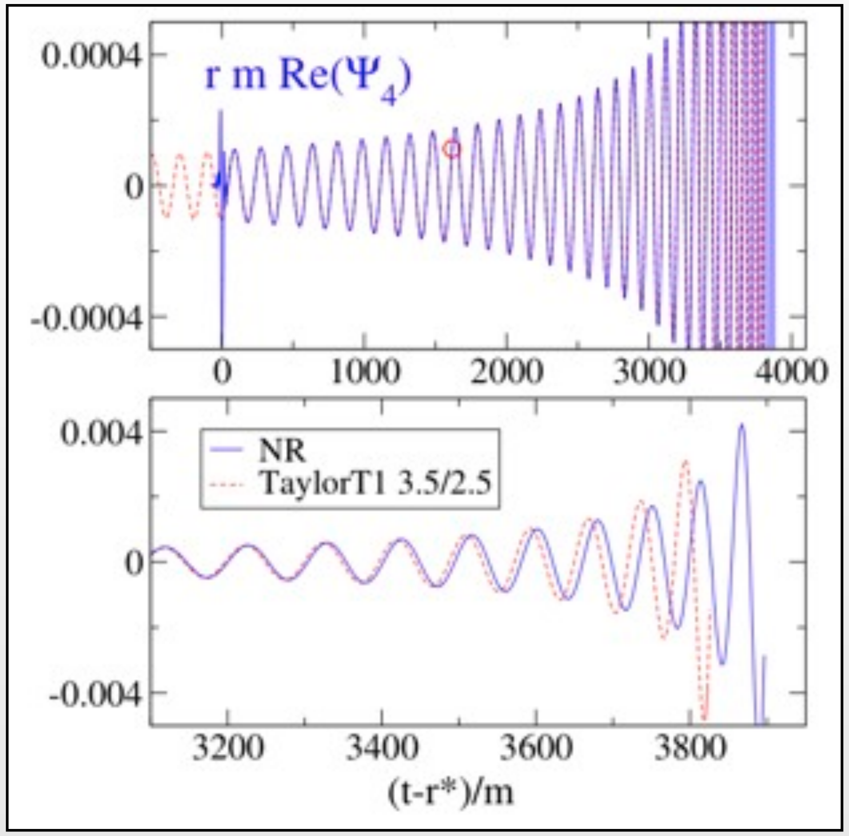
v_{max}=130km/sec



BH-BH superkicks

Campanelli ea 07

v_{max}~3500km/sec



(in-)validating PN

Boyle...HP ea 07

The two approaches to BH-BH

CITA–ICAT

Puncture initial-data

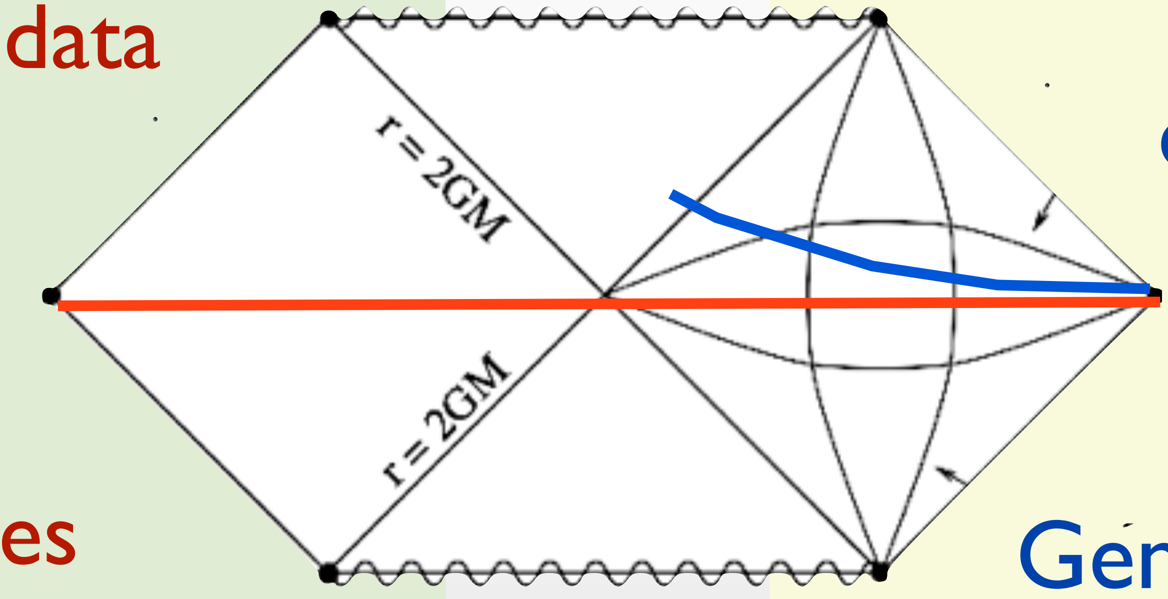
(Brandt&Brügmann 97)

BSSN w/
moving punctures

(Campanelli ea 06, Baker ea 06)

$$\begin{aligned} g_{ij} &= e^{4\phi}\tilde{g}_{ij}, \\ \tilde{\Gamma}^i &= \tilde{g}^{jk}\tilde{\Gamma}^i_{jk} \\ \partial_t\phi &= \dots \\ \partial_t\tilde{g}_{ij} &\approx -\tilde{A}_{ij} \\ \partial_t\tilde{A}_{ij} &\approx -\Delta\tilde{g}_{ij} \\ \partial_t\tilde{\Gamma}^i &= \partial_t\left(\tilde{g}^{jk}\tilde{\Gamma}^i_{jk}\right) \end{aligned}$$

Finite differences w/ AMR
(RIT, AEI, GATech,
Goddard, Jena, Palma,
Cardiff, Perimeter)



Quasi-equilibrium
excision initial-data

(Cook 02, Cook&HP 04)

Generalized Harmonic
w/ constraint damping

(Gundlach ea 05, Pretorius 05)

$$\Box g_{ab} = -2\nabla_{(a}H_{b)} + \gamma_0\left[t_{(a}C_{b)} - \frac{1}{2}g_{ab}t^cC_c\right] + \text{lower}$$

Multi-domain spectral
methods SpEC

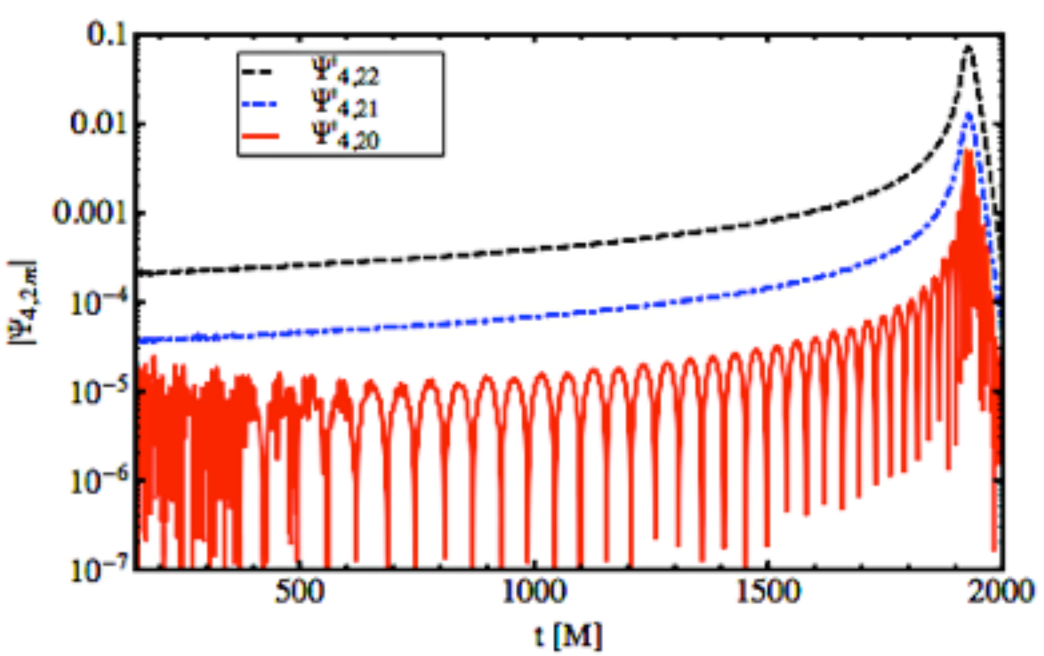
SXS collaboration (Cornell-Caltech-CITA-Washington State Univ-California State Univ Fullerton)

The two approaches to BH-BH

Finite differences w/ AMR
(RIT, AEI, GeorgiaTech, Jena, Palma, Cardiff, Perimeter)

Conventional wisdom:
-- Robust, "easy"
-- Many short simulations
-- Lower accuracy, higher cost

More recent:
-- longer and more accurate sims

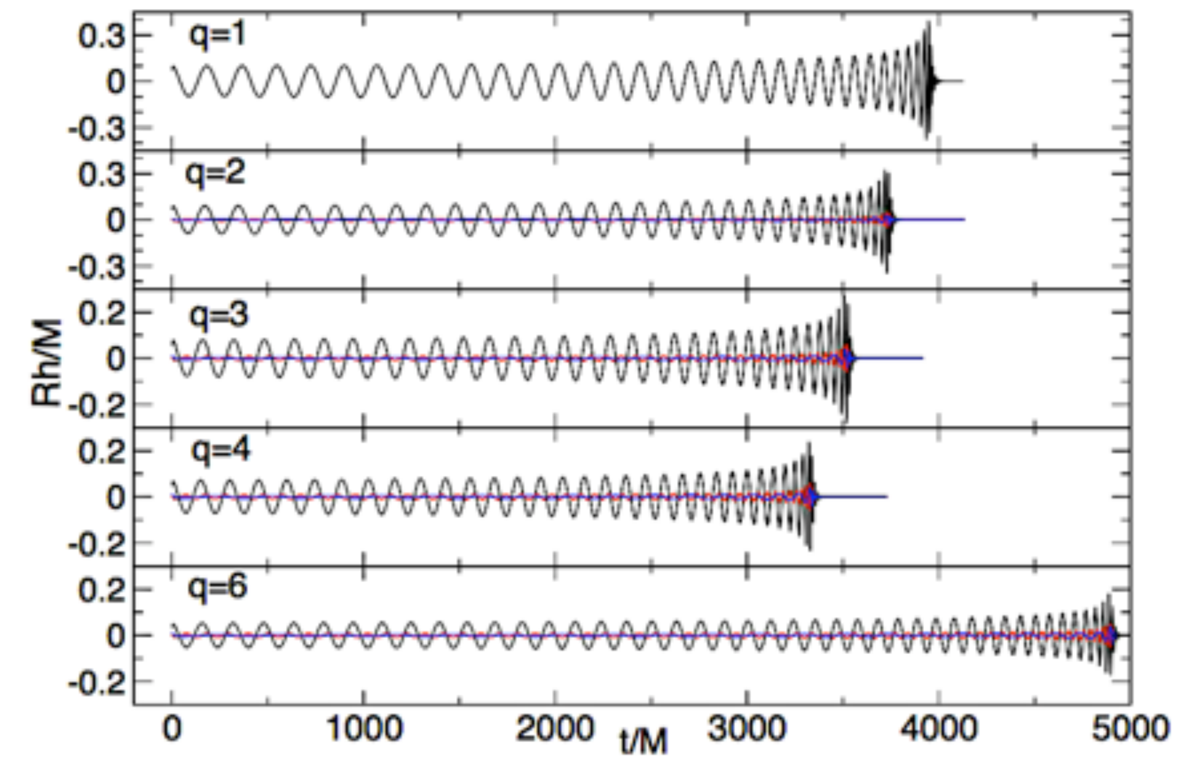


Schmidt ea 1012.2879

Multi-domain spectral methods
SpEC (Cornell-Caltech-CITA-WSU-CSUF)

Conventional wisdom:
-- Less robust, "difficult"
-- Few long simulations
-- Higher accuracy, lower cost

More recent:
-- mergers becoming routine



Buchman ea 1206.3015

Numerical Relativity Groups

CITA–ICAT

❖ Canada
- CITA/Univ. of Toronto (Pfeiffer)
- Perimeter Institute/Guelph University (Lehner)
- Vancouver (Choptuik)

❖ France
- Institute d' Astrophysique, Paris (Barausse)
- Meudon near Paris (Gourgoulhon)

❖ Germany
- AEI (Rezzolla)
- Garching (Janka, Müller)
- Jena (Brügmann, Ansorg)
- Tübingen (Kokkotas)

❖ Italy
- Trento (Giacomazzo)

❖ Japan
- Kyoto (Shibata)

❖ New Zealand
- Otago (Beyer, Fraundiener)

❖ Spain
- Barcelona (Cardoso)
- Palma de Mallorca (Husa)
- Valencia (Font)

❖ UK
- Cambridge (Sperhake)
- Cardiff (Hannam)
- Southampton (Hawke)

❖ United States
- Brigham Young Unviersity (David Neilsen)
- Cal State Fullerton (Lovelace, Read)
- Caltech (Ott, Scheel, Szilagyi)
- Cornell (Teukolsky, Kidder)
- Florida Atlantic Univ. (Marronetti, Tichy)
- Georgia Tech (Laguna, Shoemaker)
- Long Island (Liebling)
- NASA/Goddard (Baker)
- Oakland University (Garfinkle)
- Princeton (Pretorius, Burrows)
- Rochester Institute of Technology (Campanelli, Faber, Louso, Zlochower)
- Urbana Champaign (Shapiro)
- Washington State University (Duez)

Some recent BH-BH technical advances

Orbital eccentricity

- ❖ Initial data parameters Ω_0, v_r (or p_t, p_r) determine orbital eccentricity and phase at periastron

- ❖ Unique values for *zero* eccentricity

use PN parameters

Read off p_t, p_r from long post-Newtonian inspiral

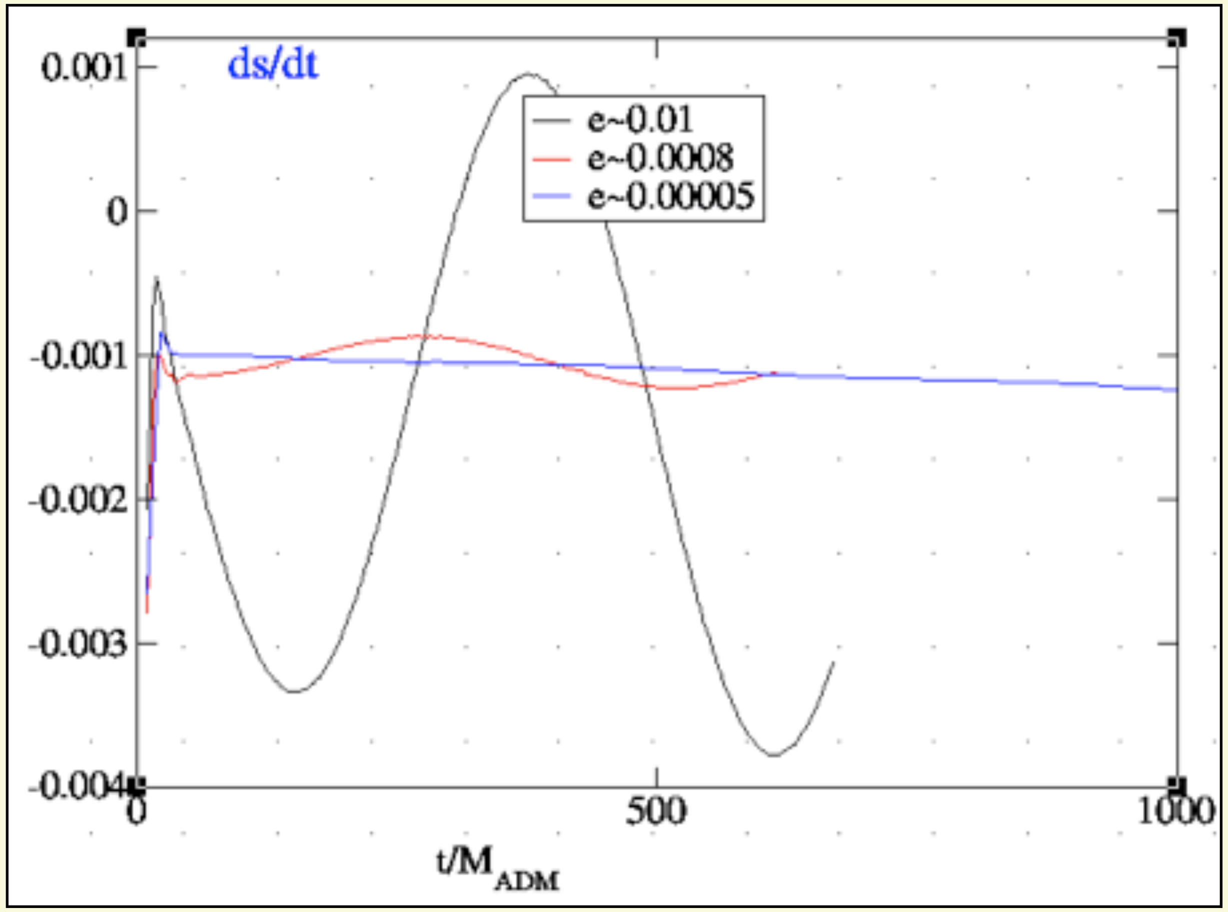
+ easy

+ works for cases w/o precession

=> e_{final} ~ few 10^{-3}

Husa ea 08, Hannam ea 10

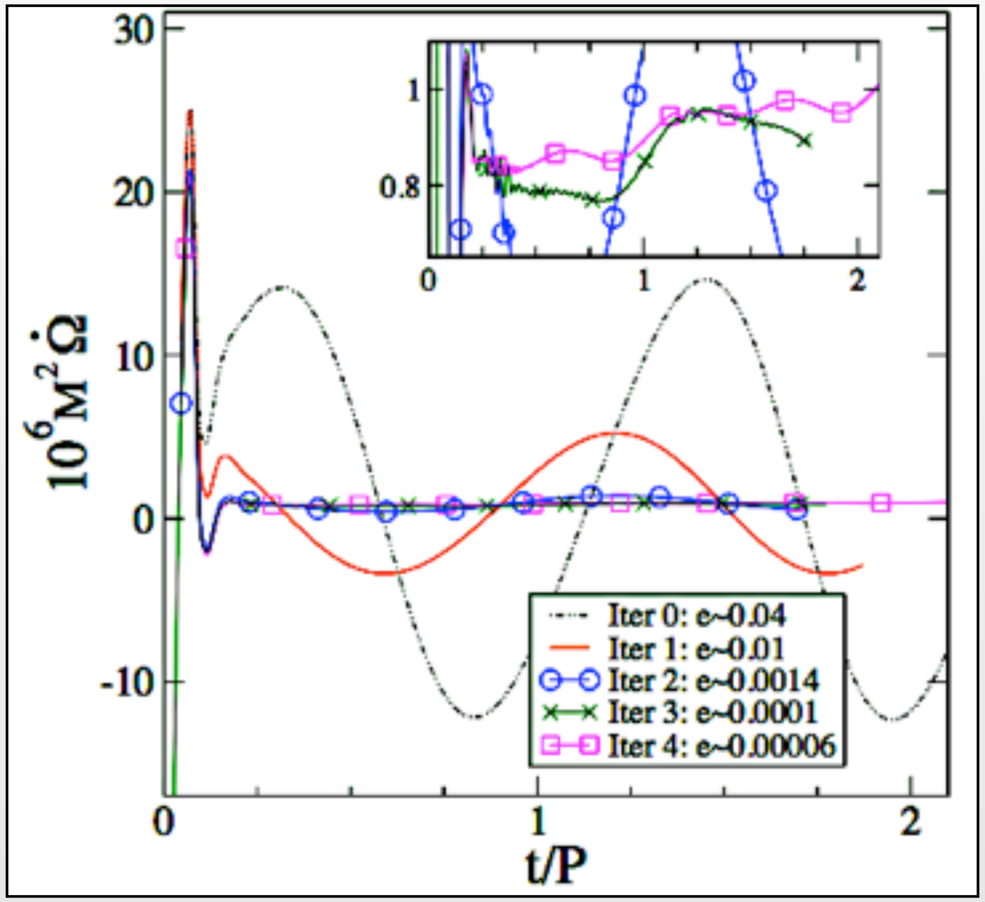
iterative ecc-removal

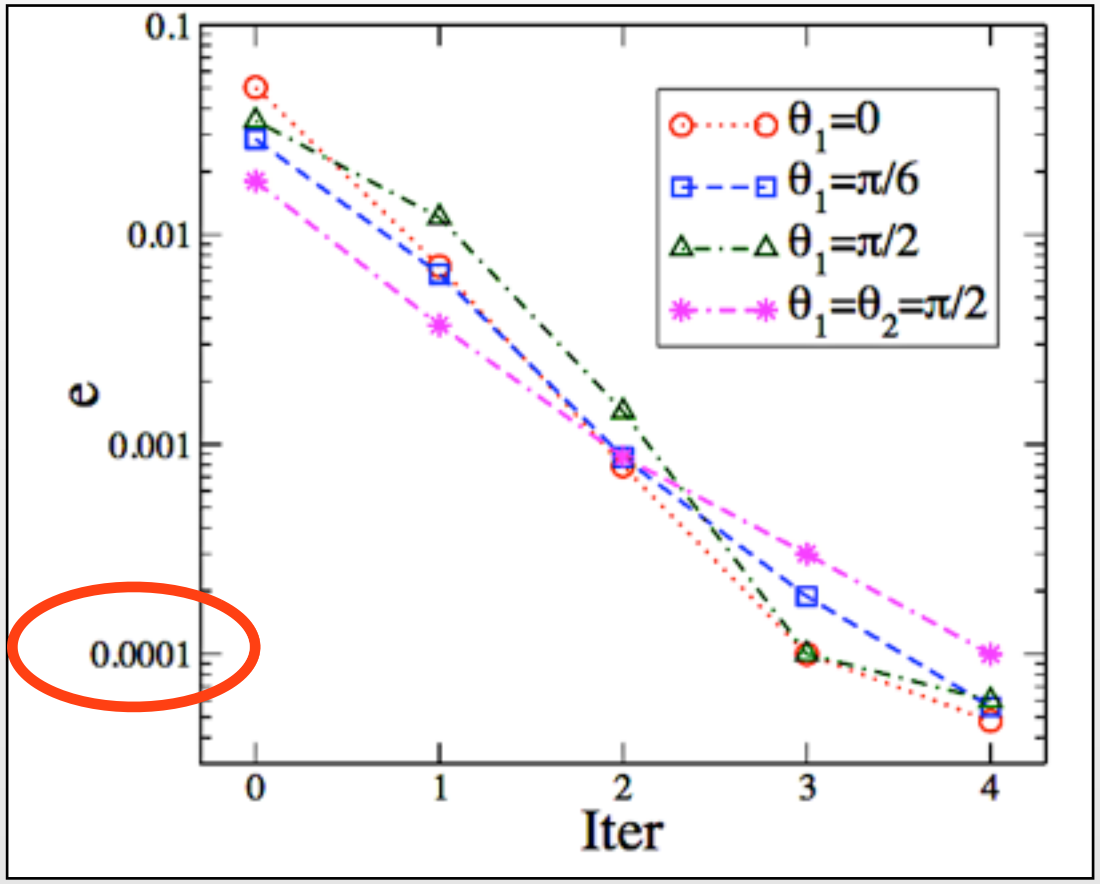


HP ea 07, Boyle ea 07, Pürrer ea 12

Eccentricity in precessing BH-BH

- With enough care, iterative eccentricity removal works!

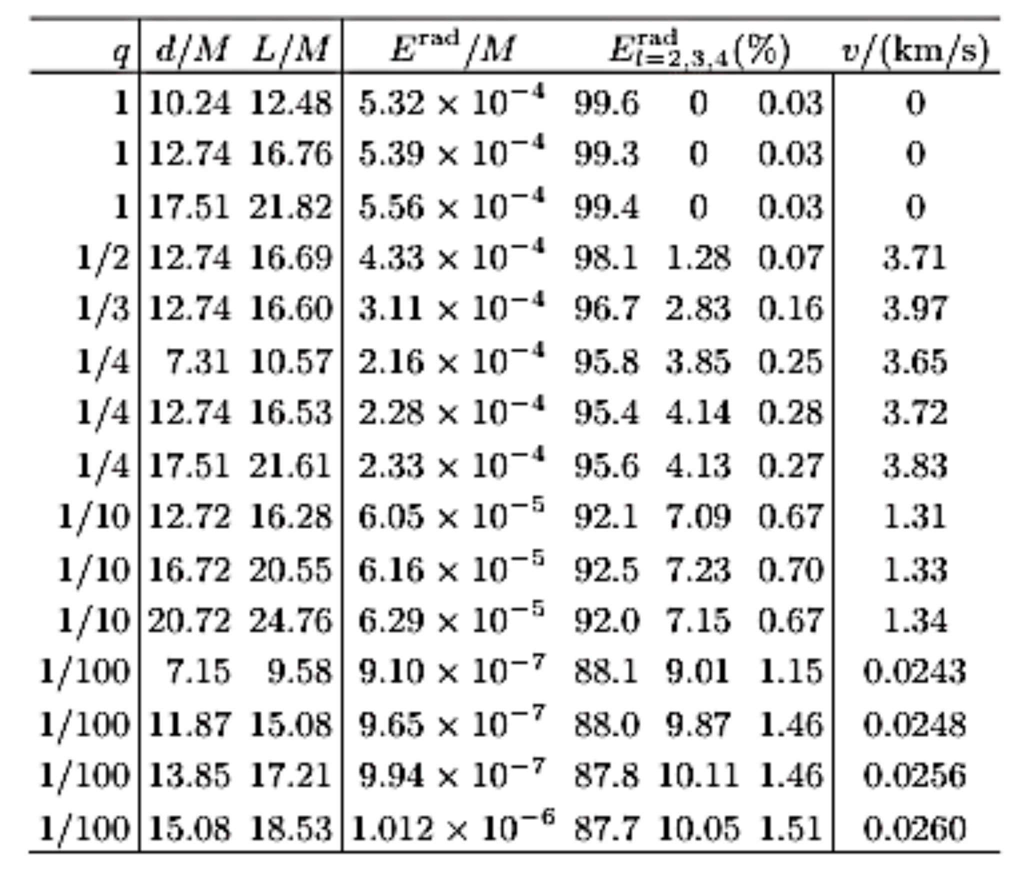




Buonnano, Kidder, Mroue, HP, Tarraccini, 10

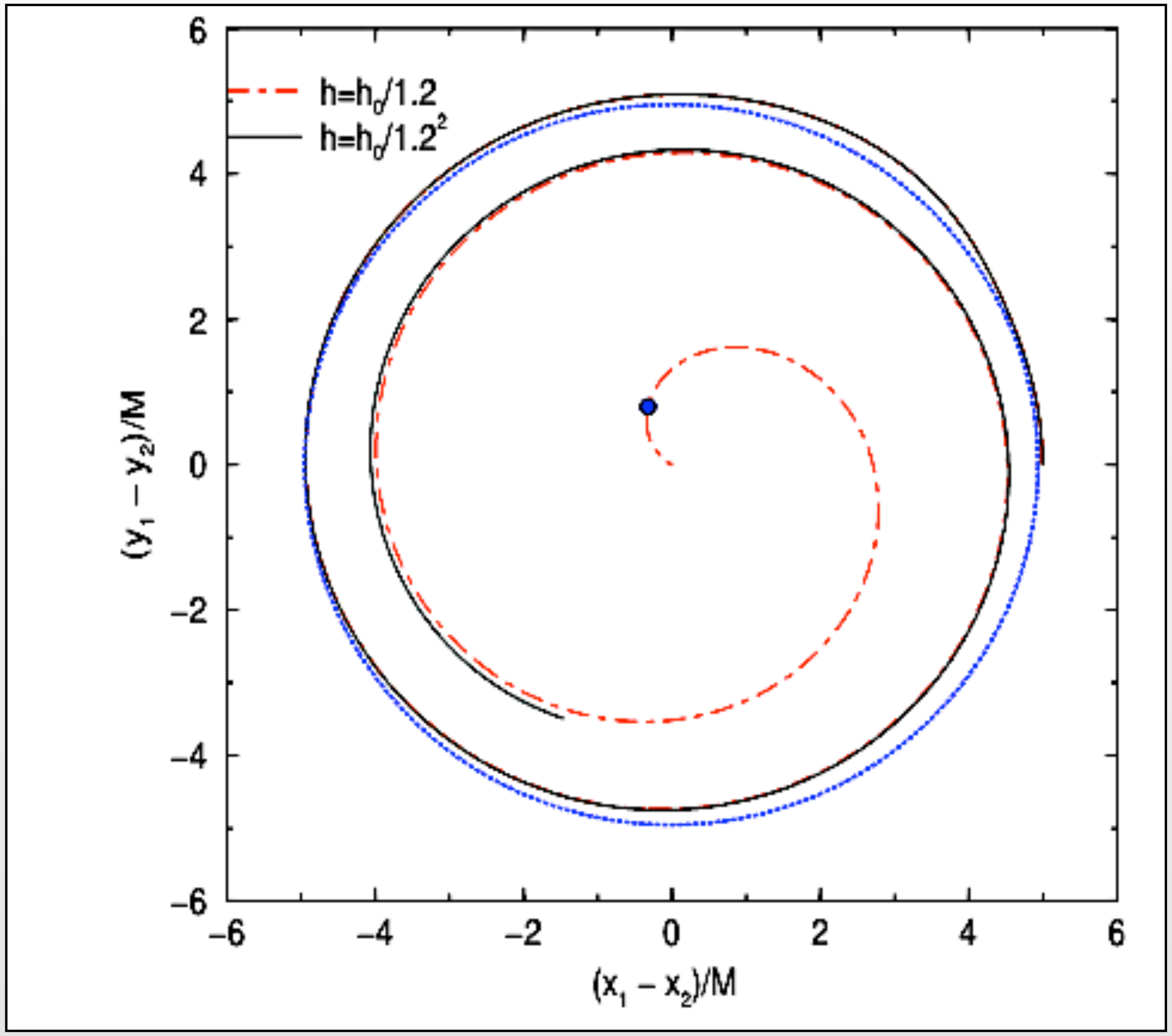
Pürrer, Husa, Hannam 2012: Iterative ecc. removal for moving punctures

Mass-ratio 1:100

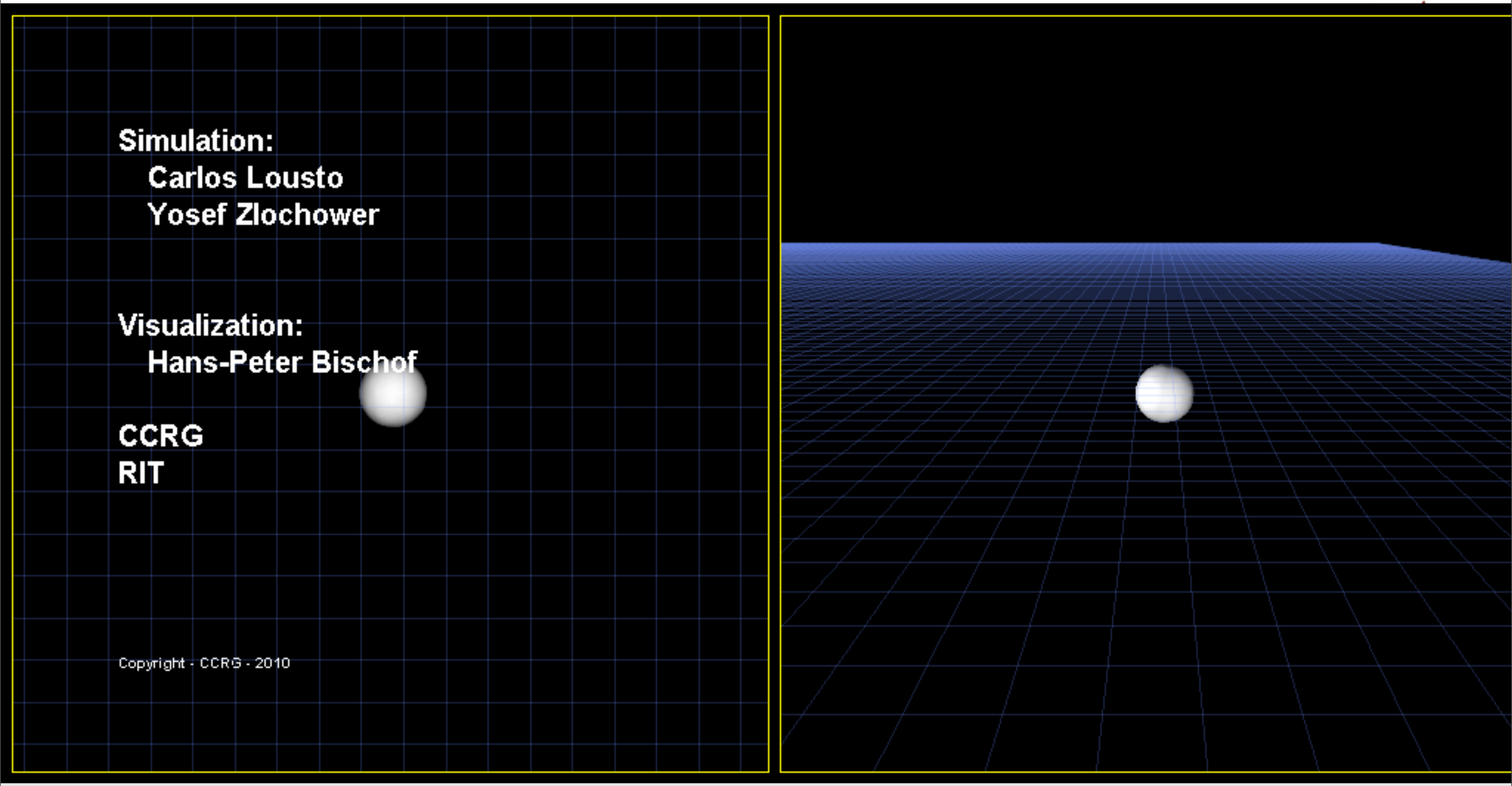


q	d/M	L/M	$E^{\rm rad}/M$	$E^{\rm rad}_{l=2,3,4}(\%)$			$v/({\rm km/s})$
1	10.24	12.48	5.32×10^{-4}	99.6	0	0.03	0
1	12.74	16.76	5.39×10^{-4}	99.3	0	0.03	0
1	17.51	21.82	5.56×10^{-4}	99.4	0	0.03	0
1/2	12.74	16.69	4.33×10^{-4}	98.1	1.28	0.07	3.71
1/3	12.74	16.60	3.11×10^{-4}	96.7	2.83	0.16	3.97
1/4	7.31	10.57	2.16×10^{-4}	95.8	3.85	0.25	3.65
1/4	12.74	16.53	2.28×10^{-4}	95.4	4.14	0.28	3.72
1/4	17.51	21.61	2.33×10^{-4}	95.6	4.13	0.27	3.83
1/10	12.72	16.28	6.05×10^{-5}	92.1	7.09	0.67	1.31
1/10	16.72	20.55	6.16×10^{-5}	92.5	7.23	0.70	1.33
1/10	20.72	24.76	6.29×10^{-5}	92.0	7.15	0.67	1.34
1/100	7.15	9.58	9.10×10^{-7}	88.1	9.01	1.15	0.0243
1/100	11.87	15.08	9.65×10^{-7}	88.0	9.87	1.46	0.0248
1/100	13.85	17.21	9.94×10^{-7}	87.8	10.11	1.46	0.0256
1/100	15.08	18.53	1.012×10^{-6}	87.7	10.05	1.51	0.0260

Head on collisions q=1...100
Sperhake ea 1105.5391



Two orbits, starting @ ISCO
Lousto, Zlochower 11



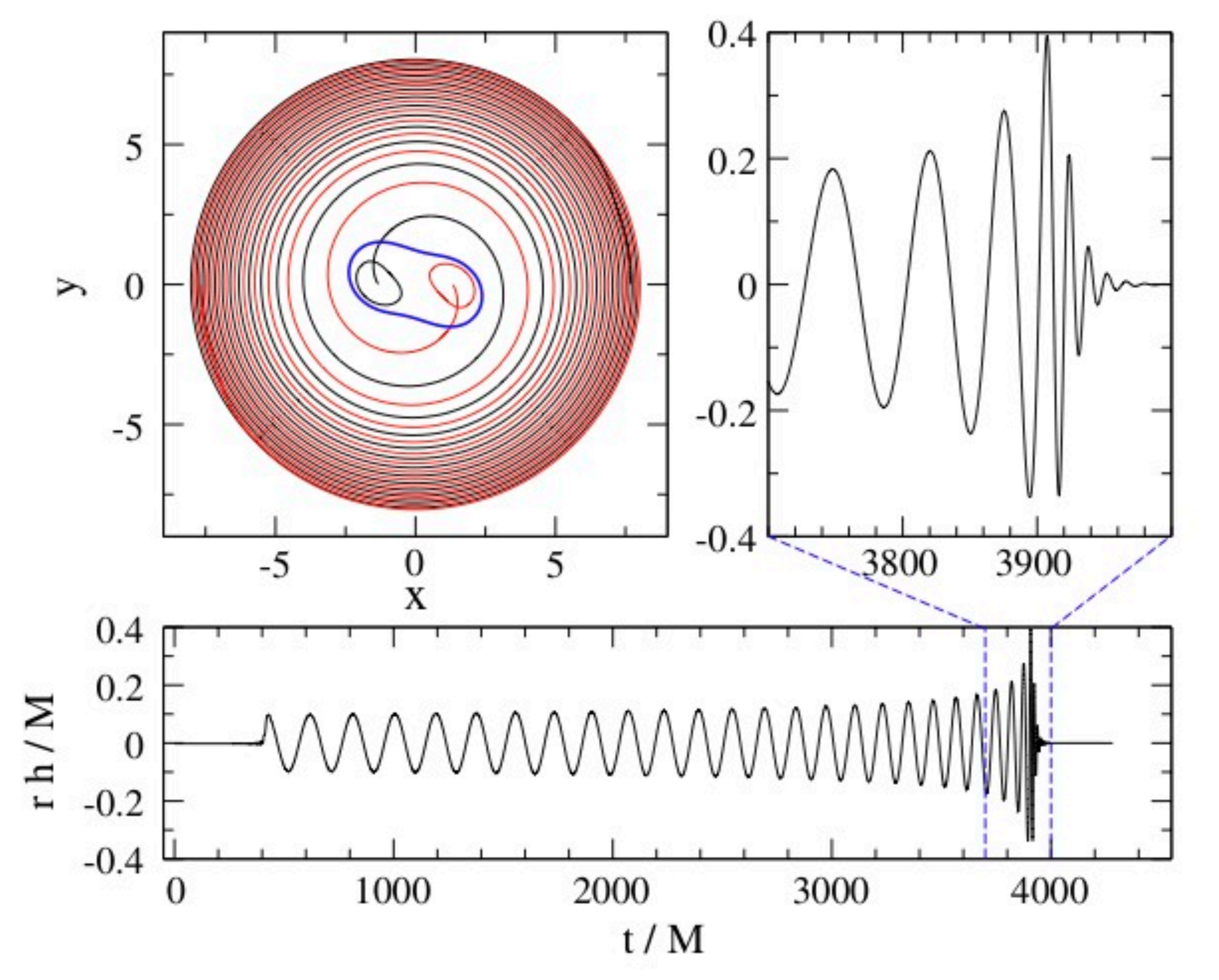
Simulation:
Carlos Lousto
Yosef Zlochower

Visualization:
Hans-Peter Bischof

CCRG
RIT

Carlos Lousto
Tue, 17:00

Spins above the Bowen-York limit



Lovelace, Scheel, Szilagyi 11,
Lovelace ea 11

- Puncture-data limit: **$S/M^2<0.93$**
- First complete BBH simulation above 0.93 limit!
  - Equal mass, equal spins anti-parallel to orbital L

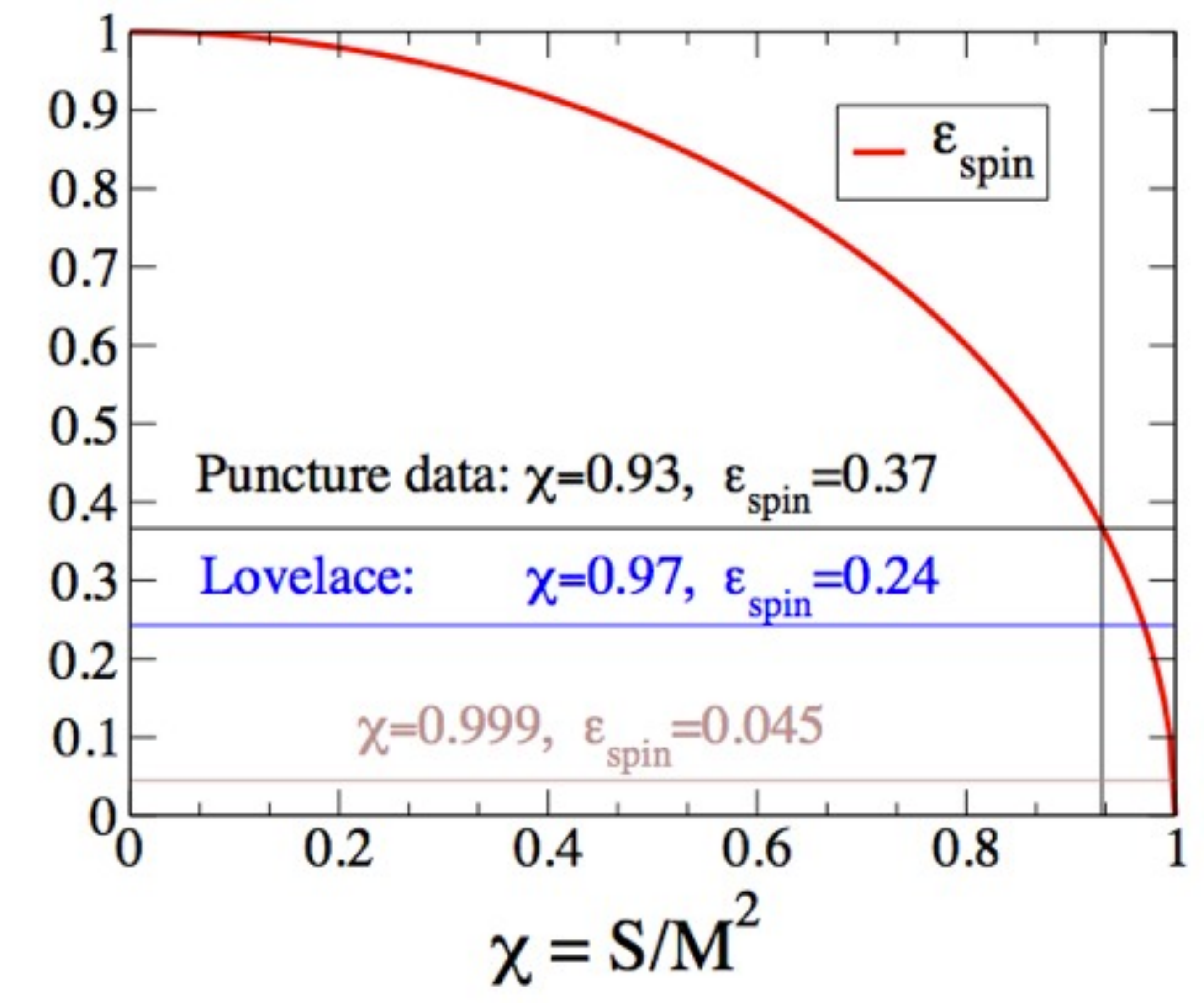
Importance of $S/M^2 > 0.93$

- ❖ Observational evidence for BH's with $S/M^2 \sim 0.998$

- ❖ Expansion parameter around extremality

$$\varepsilon_{\rm spin} \equiv \sqrt{1-\chi^2}$$

  - 0.93 is far from extremal!



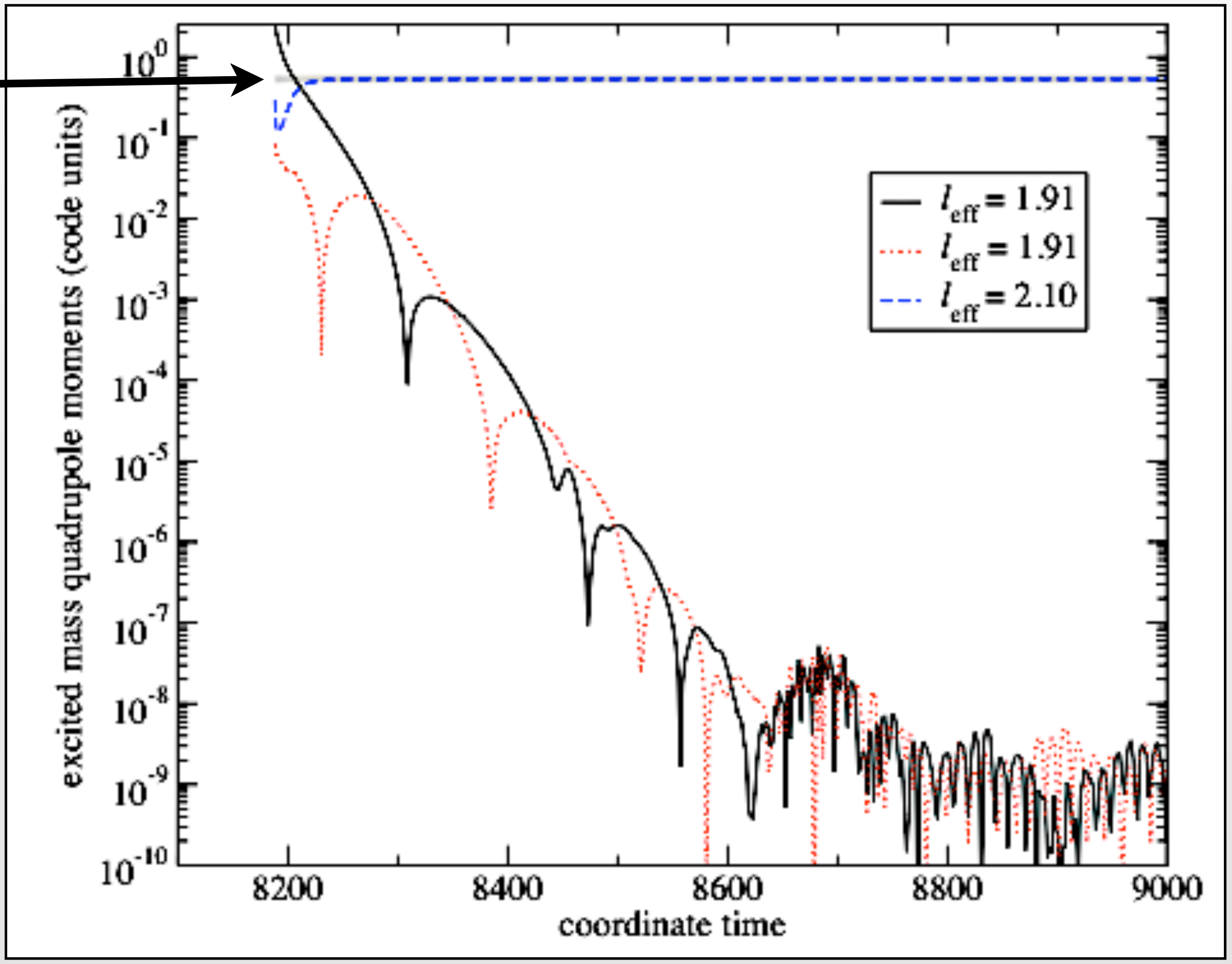
Merger Remnant approaches Kerr!

- ❖ Gauge invariant horizon structure

Robert Owen 09

(also Campanelli ea 09, Owen 10)

Kerr



Quadrupole moments

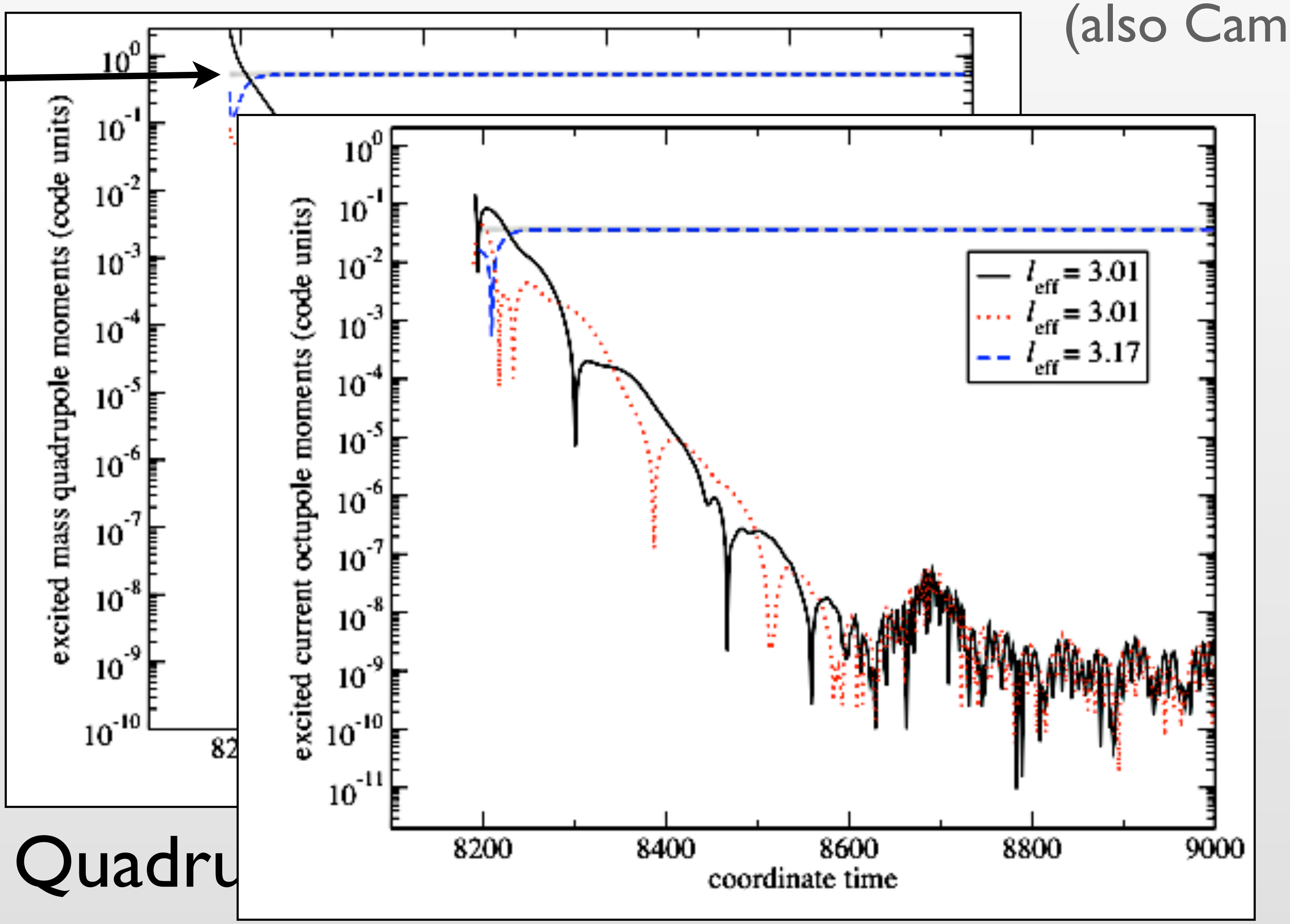
Merger Remnant approaches Kerr!

- ❖ Gauge invariant horizon structure

Robert Owen 09

(also Campanelli ea 09, Owen 10)

Kerr →



Quadru

Octupole

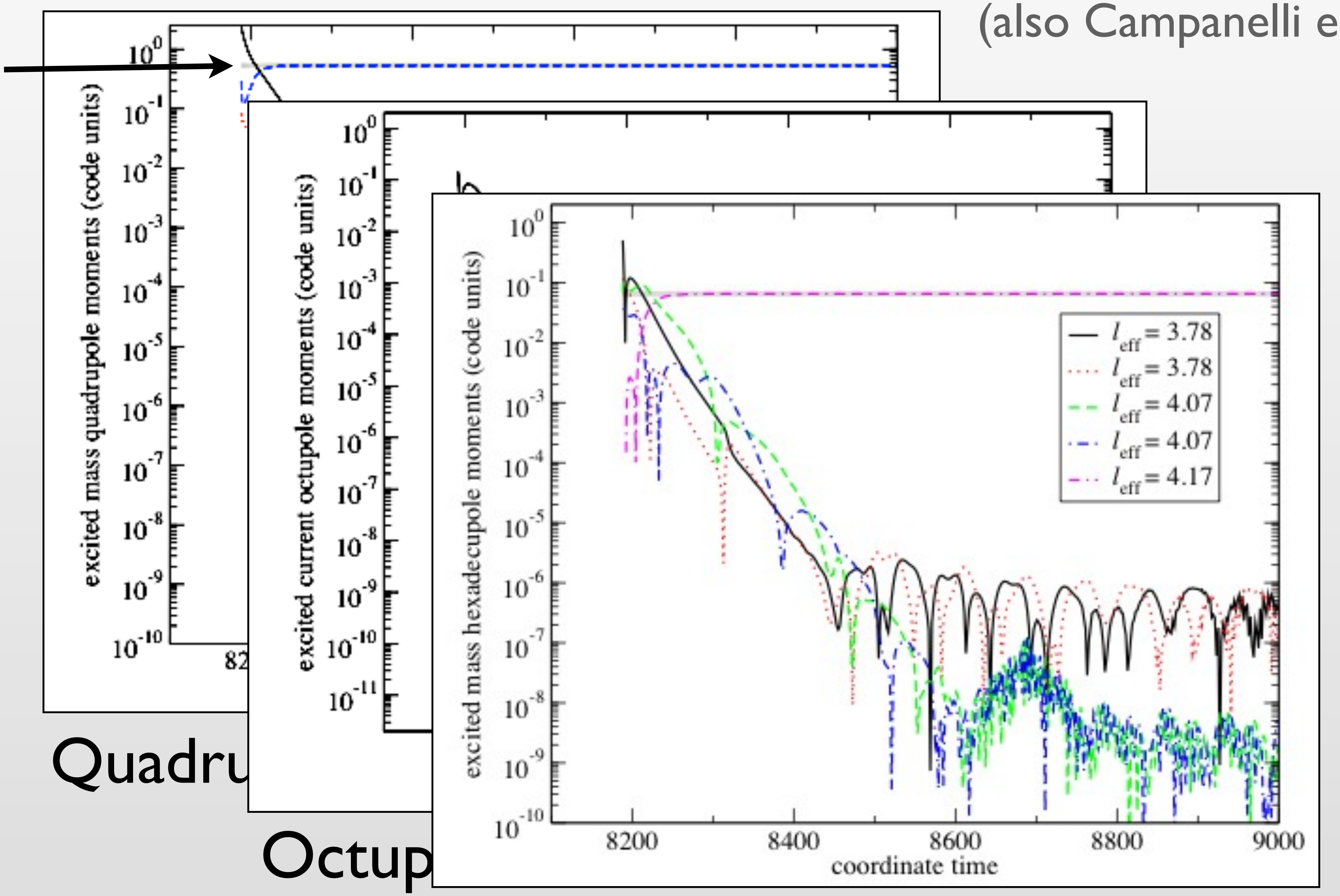
Merger Remnant approaches Kerr!

- Gauge invariant horizon structure

Robert Owen 09

(also Campanelli ea 09, Owen 10)

Kerr



Quadru

Octup

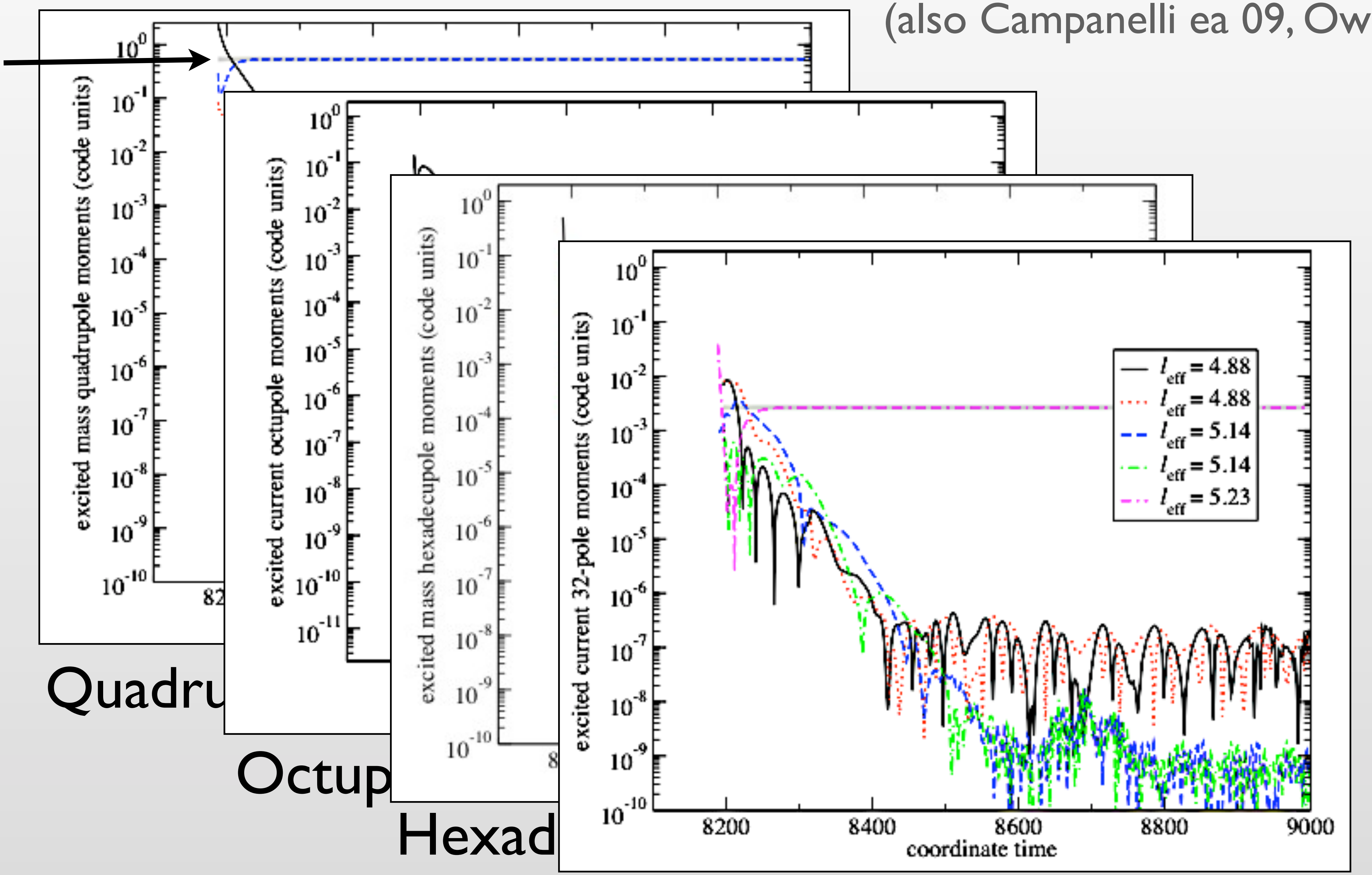
Hexadecupole

Merger Remnant approaches Kerr!

- ❖ Gauge invariant horizon structure

Robert Owen 09

(also Campanelli ea 09, Owen 10)



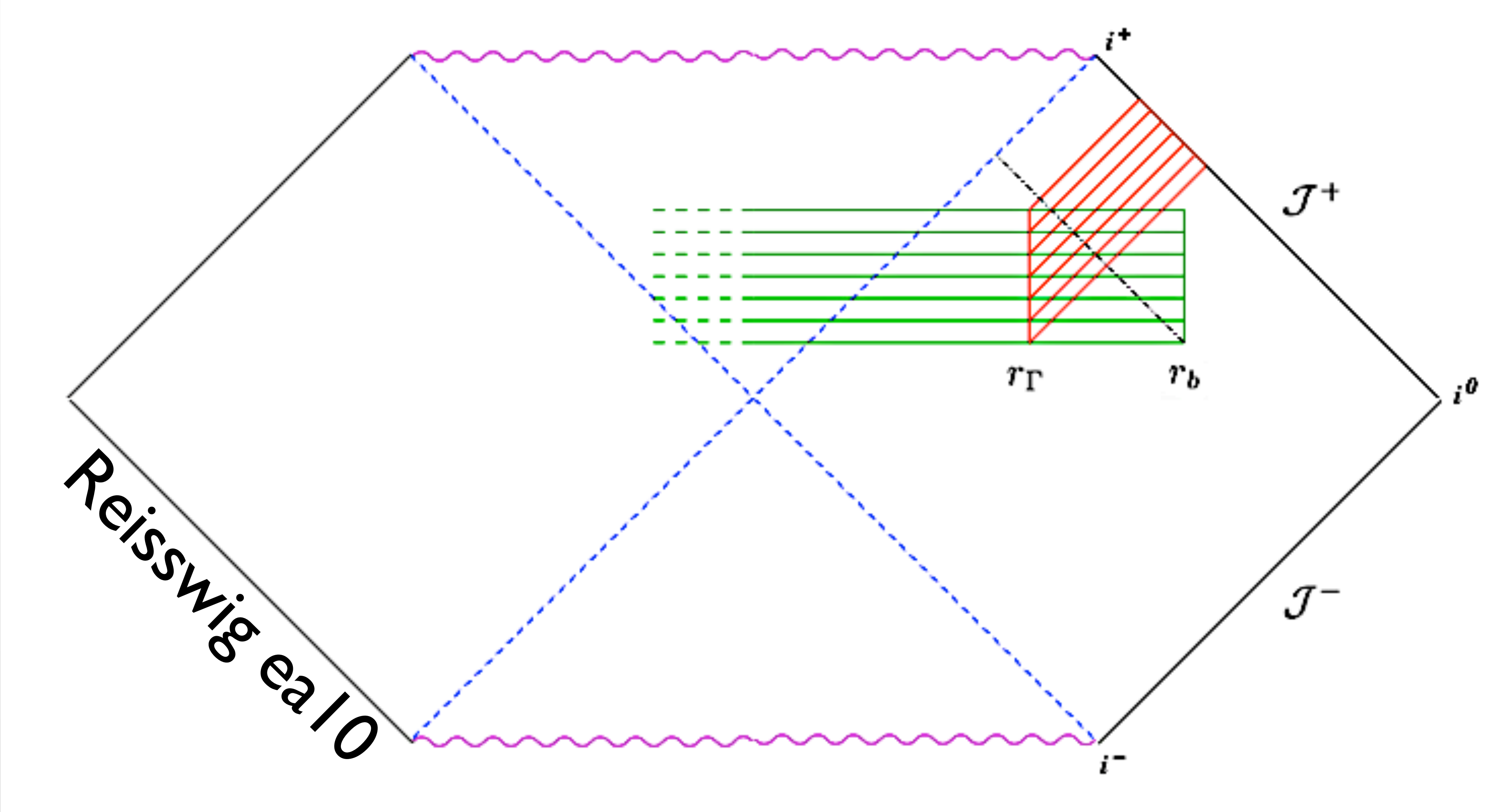
Quadru

Octup

Hexad

32-pole

Cauchy-characteristic Extraction



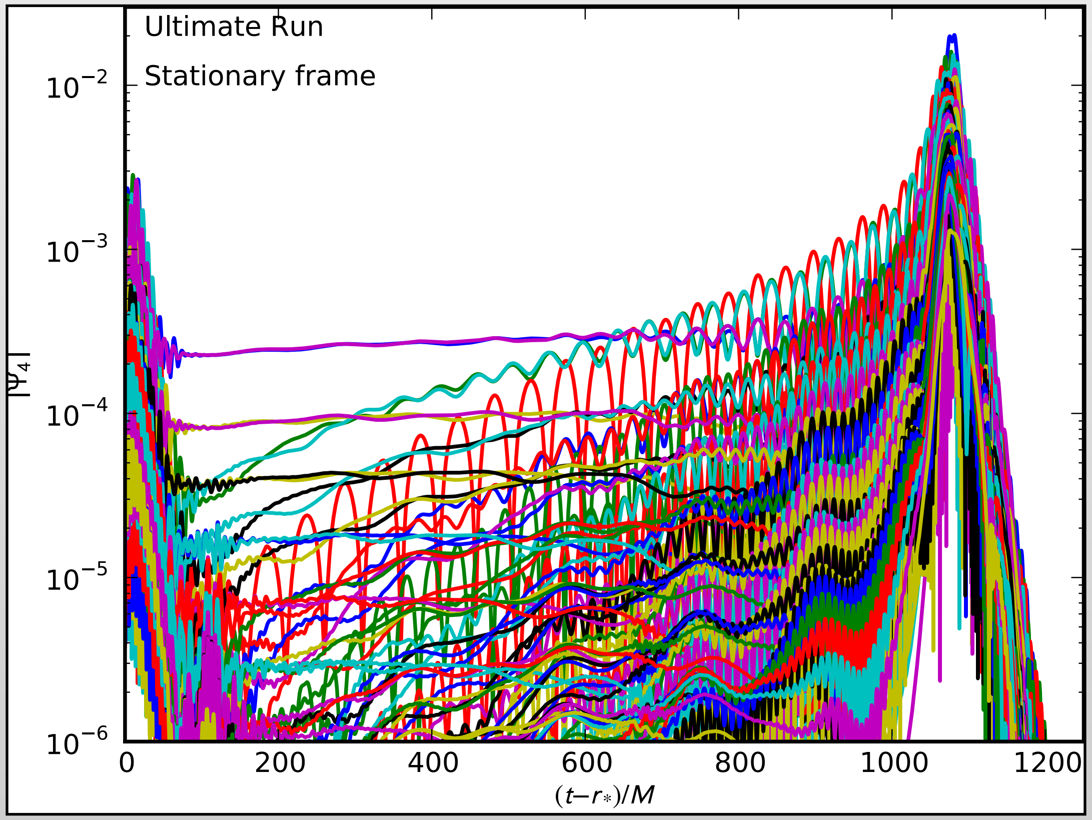
Reisswig ea10

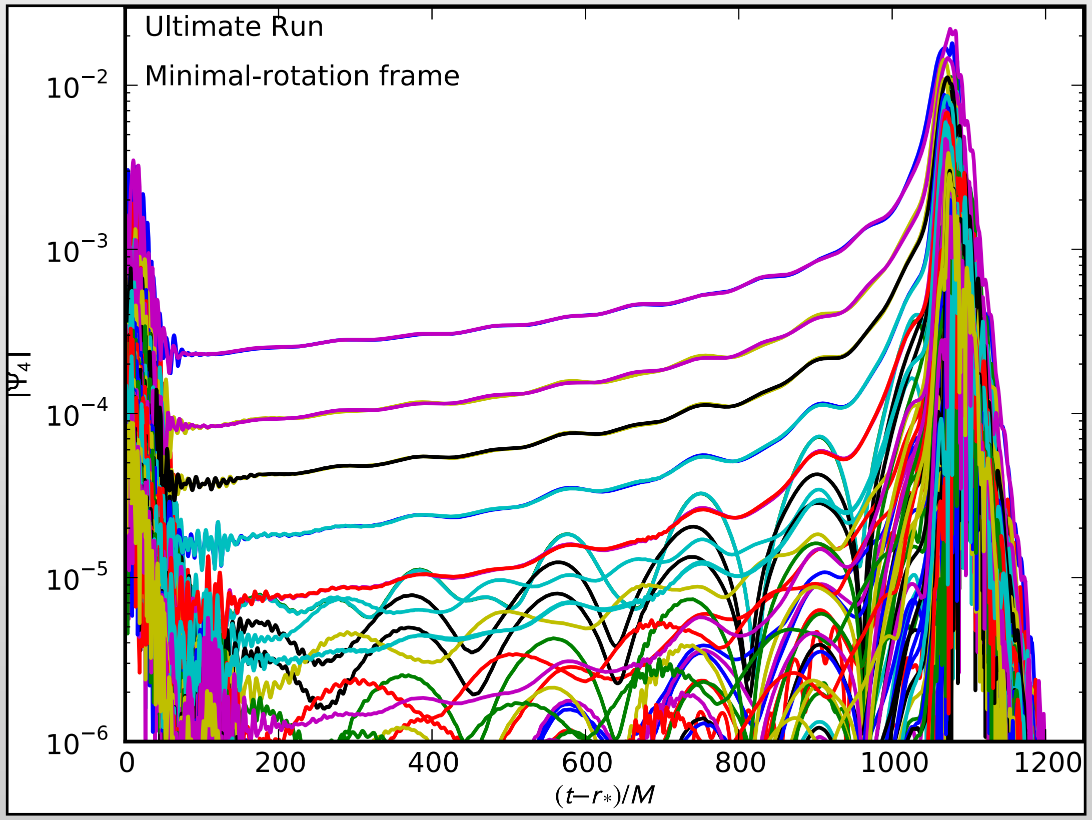
- h(t) **<u>at</u>** Scri+
- Post-processing tool for any Cauchy evolution (open source)

Reisswig ea 09, Reisswig ea 10, Babiuc ea 1011.4223, Babiuc ea 1106.4841

Radiation-aligned minimally-rotating frame

- ❖ Decompose radiation in a good frame, not an inertial frame
- ❖ Schmidt ea 2011, O'Shaugnessy ea 2011, Boyle ea 2011:
- Polar axis of Ylm-decomposition along dominant emission direction



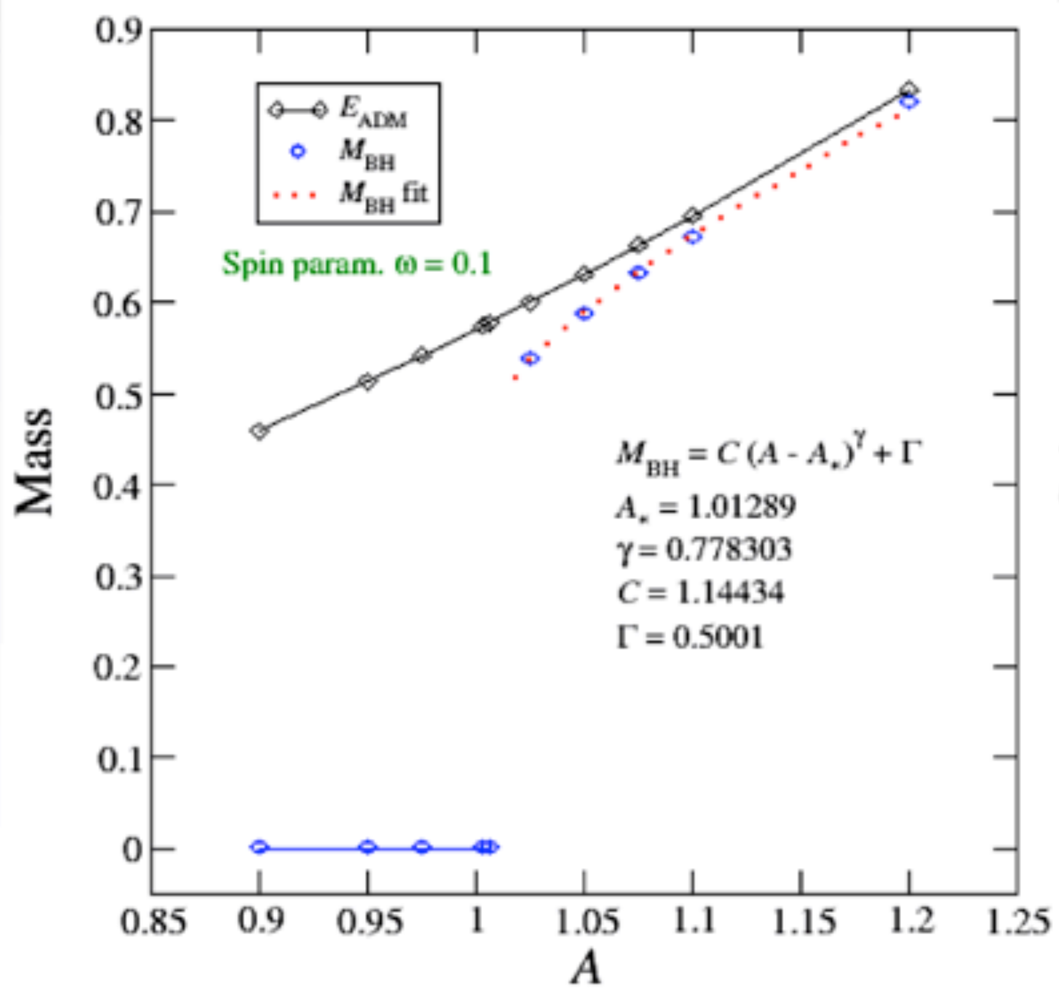


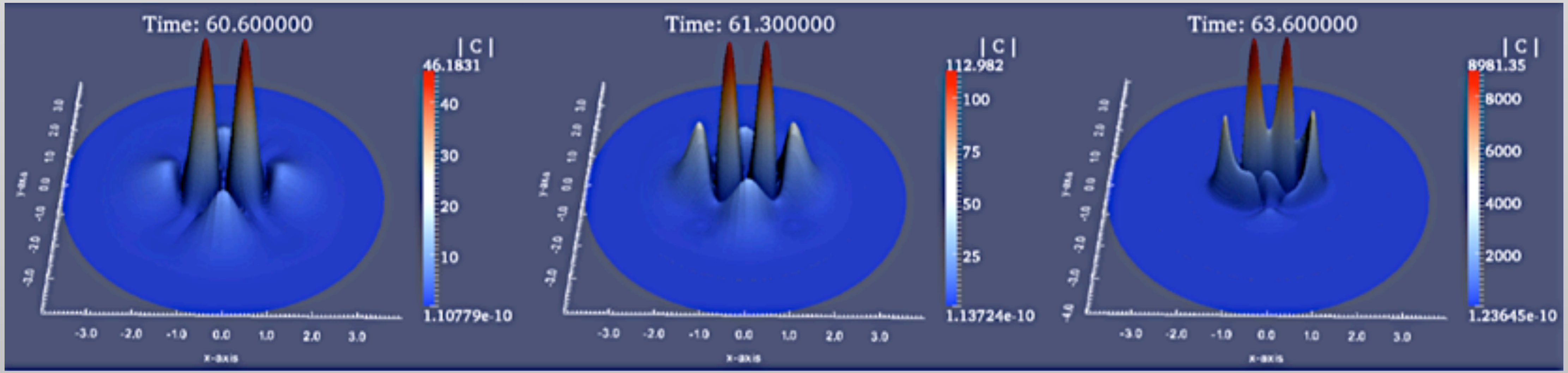
q=6, □A=0.9, □B=0.3, 8 orbits
Figures courtesy Mike Boyle & Larry Kidder

News on critical collapse

- ❖ Incoming grav wave *with* Angular momentum
- ❖ Small Amplitude
    - dispersal
- ❖ Large Amplitude
    - BH formation
    - signs of discrete self-similarity

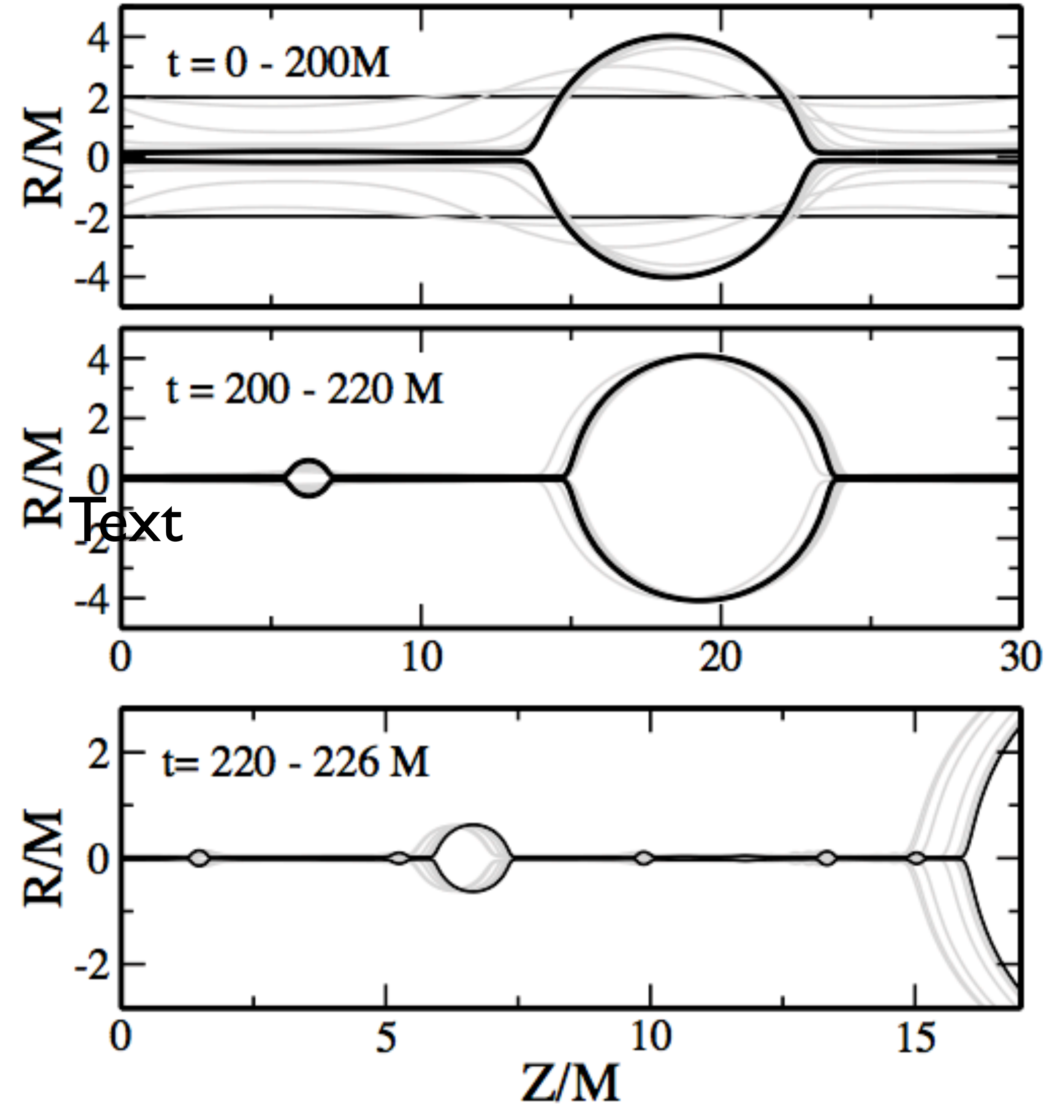
Tony Chu
CITA





News on 5-D black strings

- Lehner, Pretorius
  1106.5184



Some details about Spectral Einstein Code (SpEC)

Numerics I: Spectral methods

- Expand in basis-functions, solve for coefficients

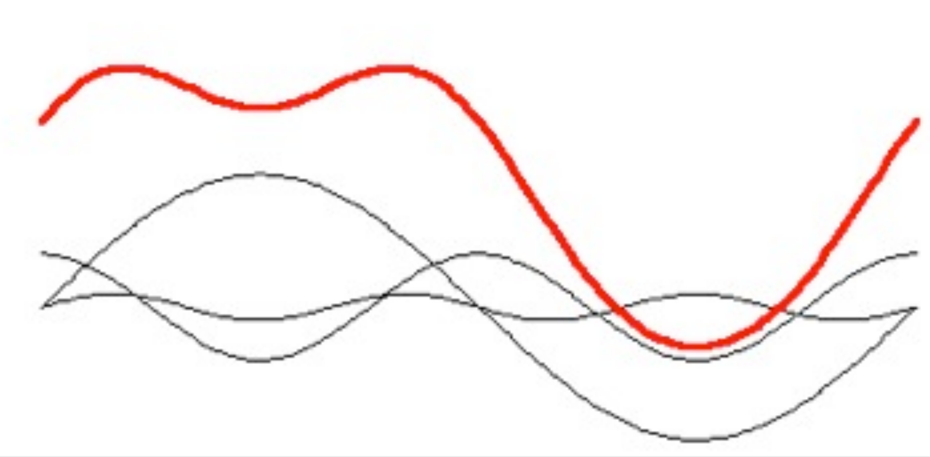
$$u(x,t) = \sum_{k=1}^{N} \tilde{u}(t)_k \Phi_k(x)$$

- Compute derivatives *exactly*

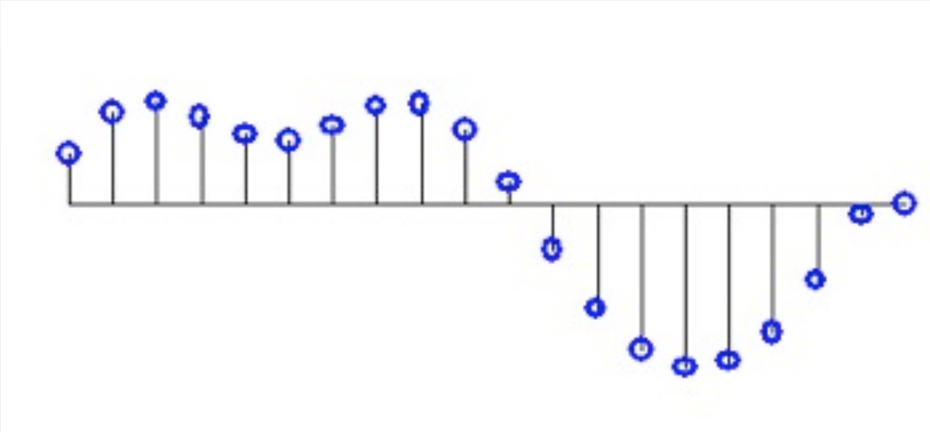
$$u'(x,t) = \sum_{k=1}^{N} \tilde{u}(t)_k \Phi'_k(x)$$

- Compute nonlinearities in physical space

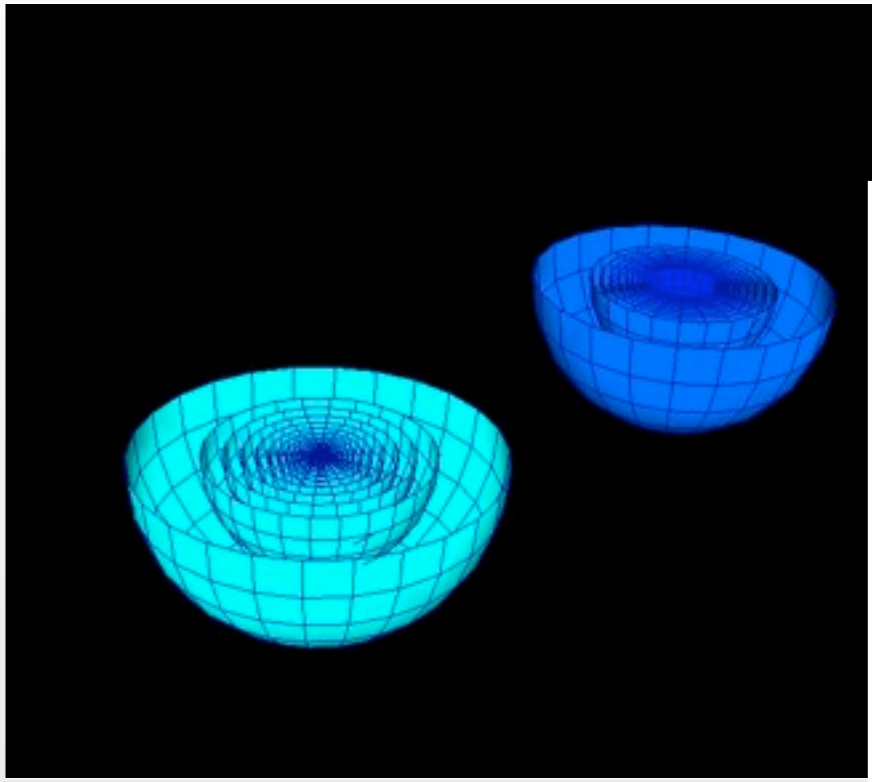
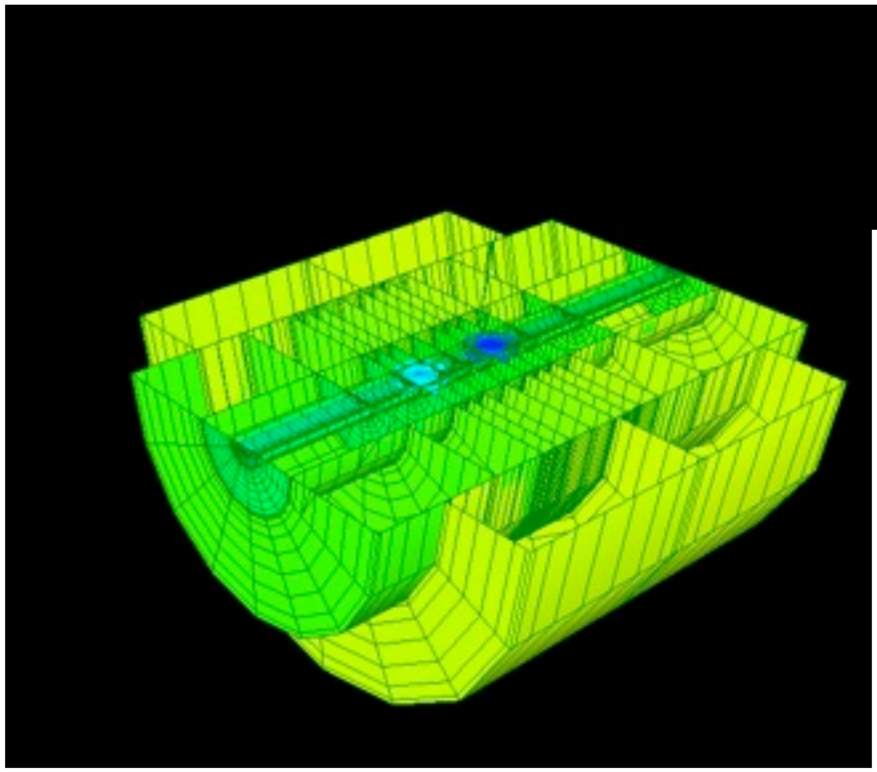
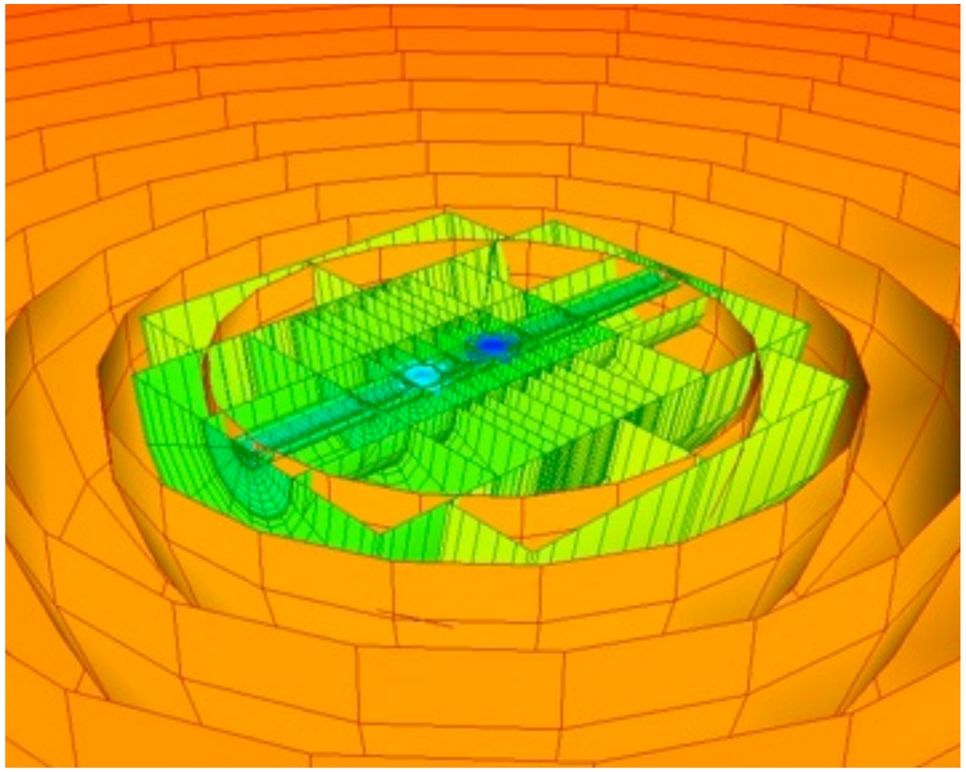
Spectral



Finite differences



Numerics II: Domain-decomposition

- ❖ Full resolution to outer boundary
- ❖ Can place resolution where needed

Spectral Einstein Code *SpEC* (Caltech-Cornell-CITA)
http://www.black-holes.org/SpEC.html

Einstein Constraint Equations

❖ Can be written as five coupled non-linear elliptic PDEs

- Unknowns $\psi, \vec{\beta}, \tilde{N}$
- Everything else know
- Derivative-operators on possibly non-flat 3-D (t=0) manifold

$$\tilde{\nabla}^2\psi - \frac{1}{8}\psi\tilde{R} - \frac{1}{12}\psi^5 K^2 + \frac{1}{8}\psi^{-7}\tilde{A}_{ij}\tilde{A}^{ij} = -2\pi G\psi^{-3}\tilde{\rho}.$$

$$\tilde{\nabla}_j\left(\frac{1}{2\tilde{N}}(\tilde{\mathbb{L}}\beta)^{ij}\right) - \tilde{\nabla}_j\left(\frac{1}{2\tilde{N}}\tilde{u}^{ij}\right) - \frac{2}{3}\psi^6\tilde{\nabla}^i K = 8\pi G\tilde{j}^i,$$

$$\begin{aligned}\tilde{\nabla}^2(\tilde{N}\psi^7) - (\tilde{N}\psi^7)\left[\frac{1}{8}\tilde{R} + \frac{5}{12}\psi^4 K^2 + \frac{7}{8}\psi^{-8}\tilde{A}_{ij}\tilde{A}^{ij} + 2\pi G\psi^4(\rho + 2S)\right]& \\ = -\psi^5\left(\partial_t K - \beta^k\partial_k K\right)&\end{aligned}$$

HP, Kidder, Scheel, Teukolsky, 2002

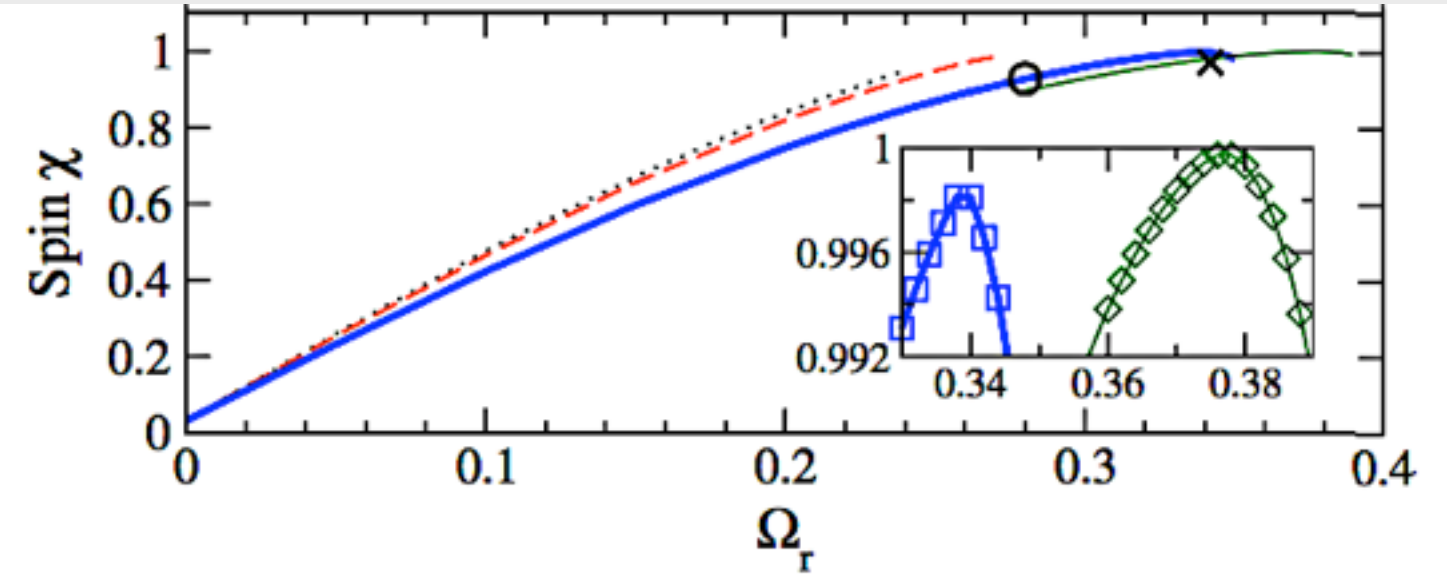
Recap: Solving the constraints

- ❖ Puncture Initial Data (Brandt, Brügmann, 1997)
    - conformal flat
    - analytical Bowen-York solution
    - disregard "5th equation"
    - Demand certain behavior inside BHs
  - Simple numerics, but *spins limited to <0.92*

- ❖ Solve all five equations
  - Tailor to spinning BH's

$$\tilde{g}_{ij} = g^A_{ij} + g^B_{ij} - \delta_{ij}$$

  - Use BH excision



Lovelace, ea 2008

Elliptic Solver I: Spectral discretization

- Elliptic equations:

$$\mathbf{F}[\mathbf{u}(\vec{x})] = 0$$

- Substitute in spectral expansion

$$\mathbf{u}(\vec{x}) = \sum \tilde{u}_i \Phi_i(\vec{x})$$

- nonlinear algebraic set of equations for $\tilde{u}_i$
- Linearize with Newton-Raphson
- Preconditioned fGMRES for linear solution

Evolution equations: Pretorius' breakthrough

- Einstein's equations

$$0 = R_{ab}[g_{ab}] = -\frac{1}{2}\Box g_{ab} + \nabla_{(a}\Gamma_{b)} + \text{lower order terms}, \qquad \Gamma_a = -g_{ab}\Box x^b.$$

- Generalized harmonic coordinates $g_{ab}\Box x^b \equiv H_a(x^a, g_{ab})$

  (Friedrich 1985, Pretorius 2005; $H = 0$ used since 1920's)

$$\Box g_{ab} = \text{lower order terms}.$$

$\Rightarrow$ Constraint $C_a \equiv H_a - g_{ab}\Box x^b = 0$

- Constraint damping (Gundlach, et *al.*, Pretorius, 2005)

$$\Box g_{ab} = \gamma\left[t_{(a}C_{b)} - \frac{1}{2}g_{ab}t^c C_c\right] + \text{lower order terms}$$

$$\partial_t C_a \sim -\gamma C_a.$$

First order reduction

Lindblom ea 2006

- ❖ Rewrite as first order symmetric hyperbolic system

$$\partial_t u^\alpha + A^{k\,\alpha}{}_\beta[u]\,\partial_k u^\beta = R^\alpha[u] \qquad u^\alpha = \{g, \partial_t g_{ab}, \partial_i g_a b\}$$

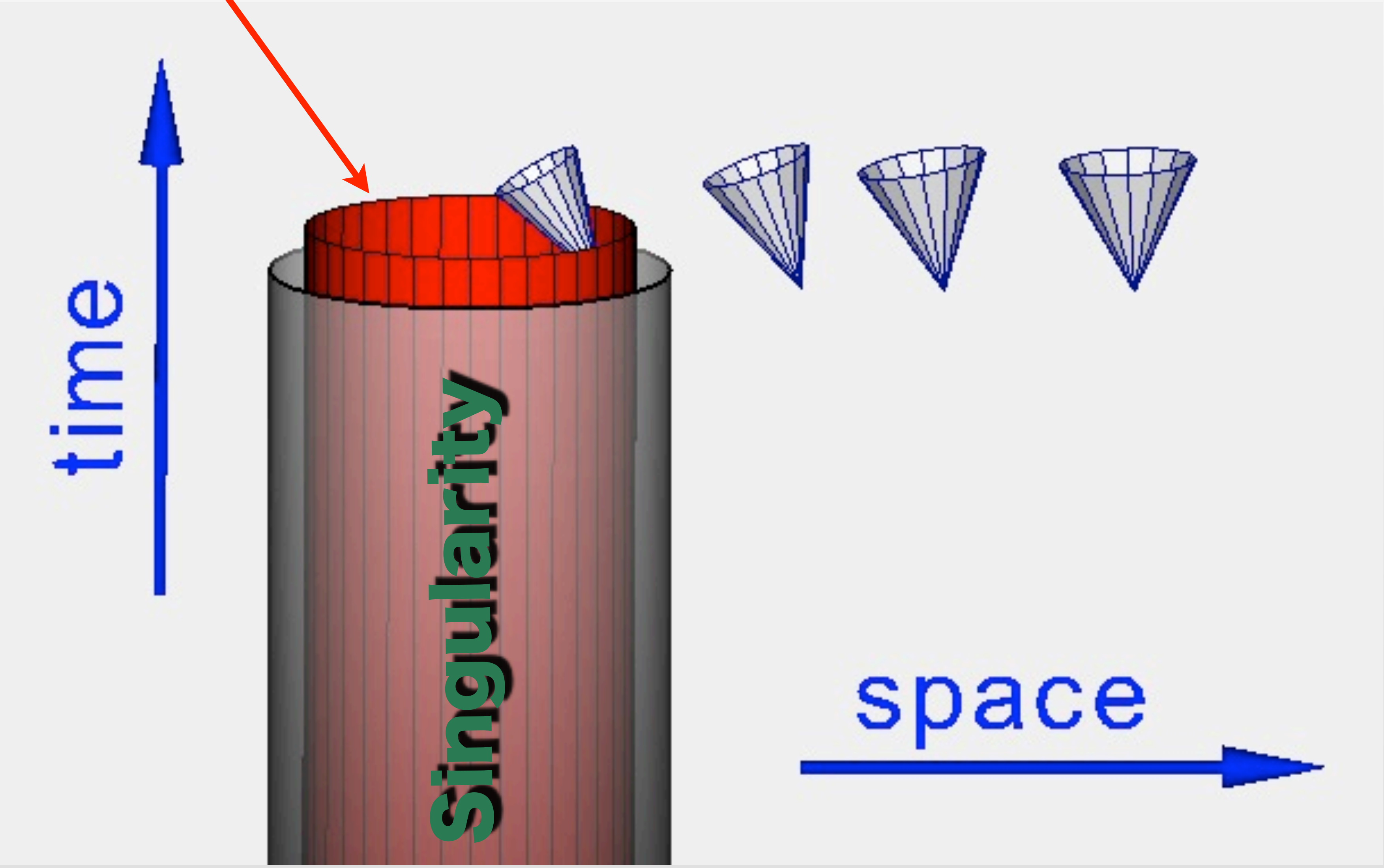
- ❖ Characteristic fields w.r.t. boundary normal n:

$$e^{\hat\alpha}{}_\alpha n_k A^{k\,\alpha}{}_\beta = v_{(\hat\alpha)} e^{\hat\alpha}{}_\beta$$

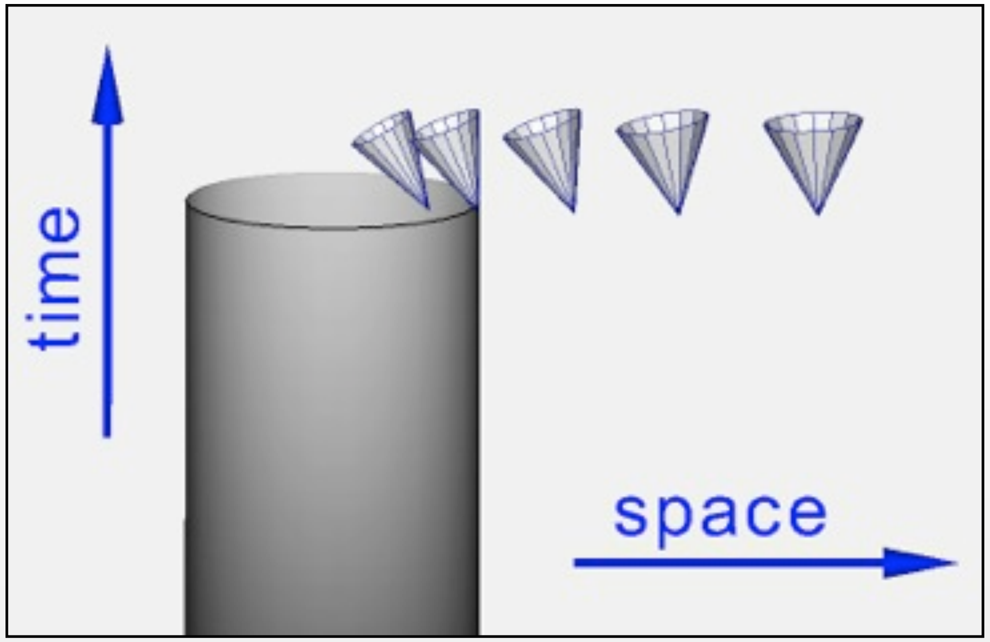
- ❖ Must impose BC on incoming fields $v_{\hat\alpha} < 0$
  - Internal boundaries:
    - Outgoing fields become incoming fields of neighbor
  - Outer boundary:
    - Incoming Constraints≡0 ⇒ conditions on some incoming fields
    - two further incoming fields represent GWs -> set to zero
    - final four incoming fields represent coordinates

Black Hole Excision

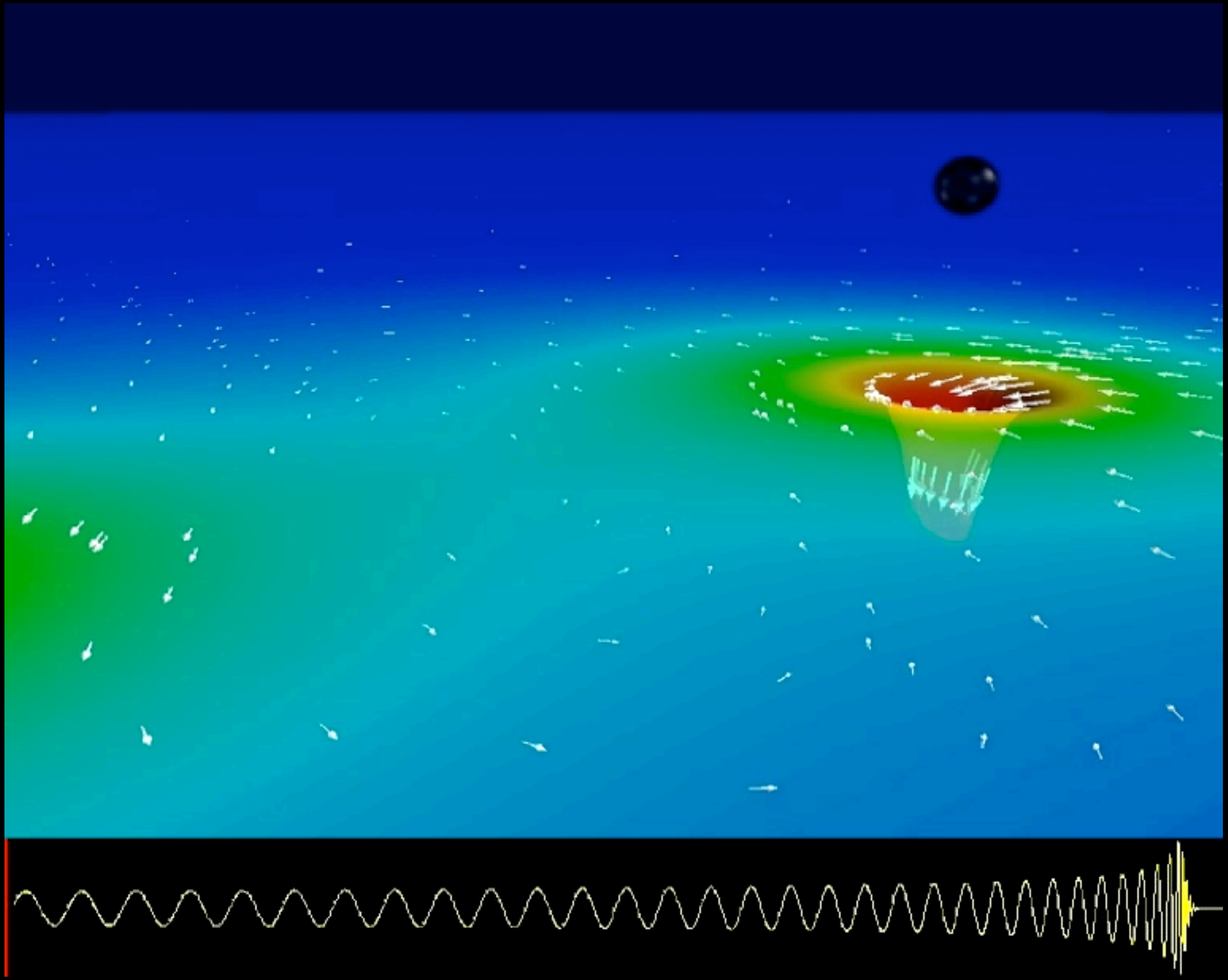
Artificial boundary
inside horizon



Technical details



- ❖ BH excision (no inner BCs)
- ❖ Non-reflective outer BCs (Lindblom, Rinne et al. 06)
- ❖ Wave-extraction & extrapolation (Boyle et al 07, Boyle & Mroue, 09)
- ❖ Coordinate conditions (Pretorius; Lindblom & Szilagyi, 09)
- ❖ Domain-decomposition follows BHs (Scheel, et al., 06)
- ❖ Switch domain-decomposition at merger (Scheel, et al., 08, Szilagyi et al 09)
- ❖ Construct initial data (Cook, HP 04-07, Lovelace et 08)
- ❖ Reduce orbital eccentricity (HP ea 2006, Buonnano, et al, 10)

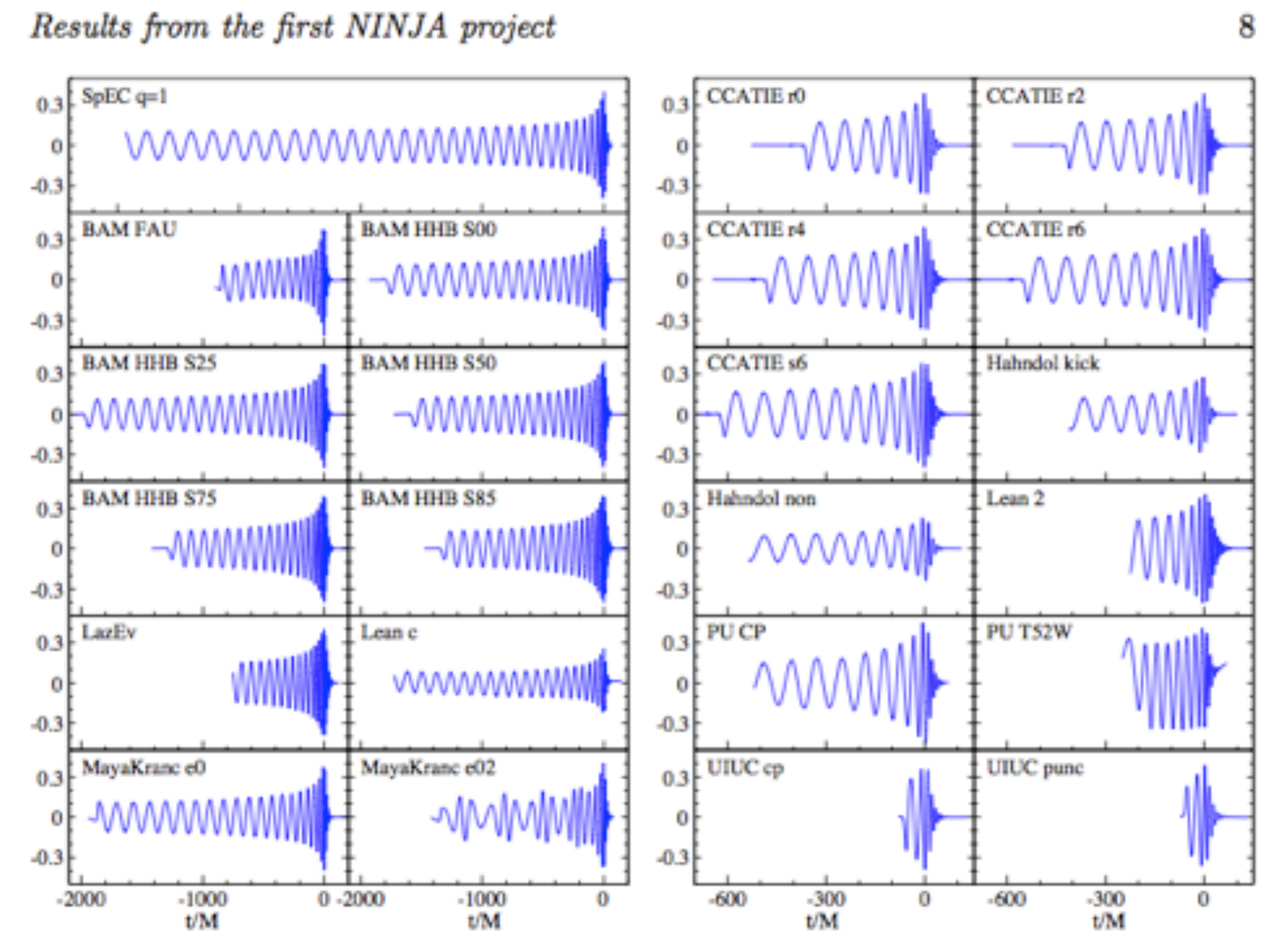


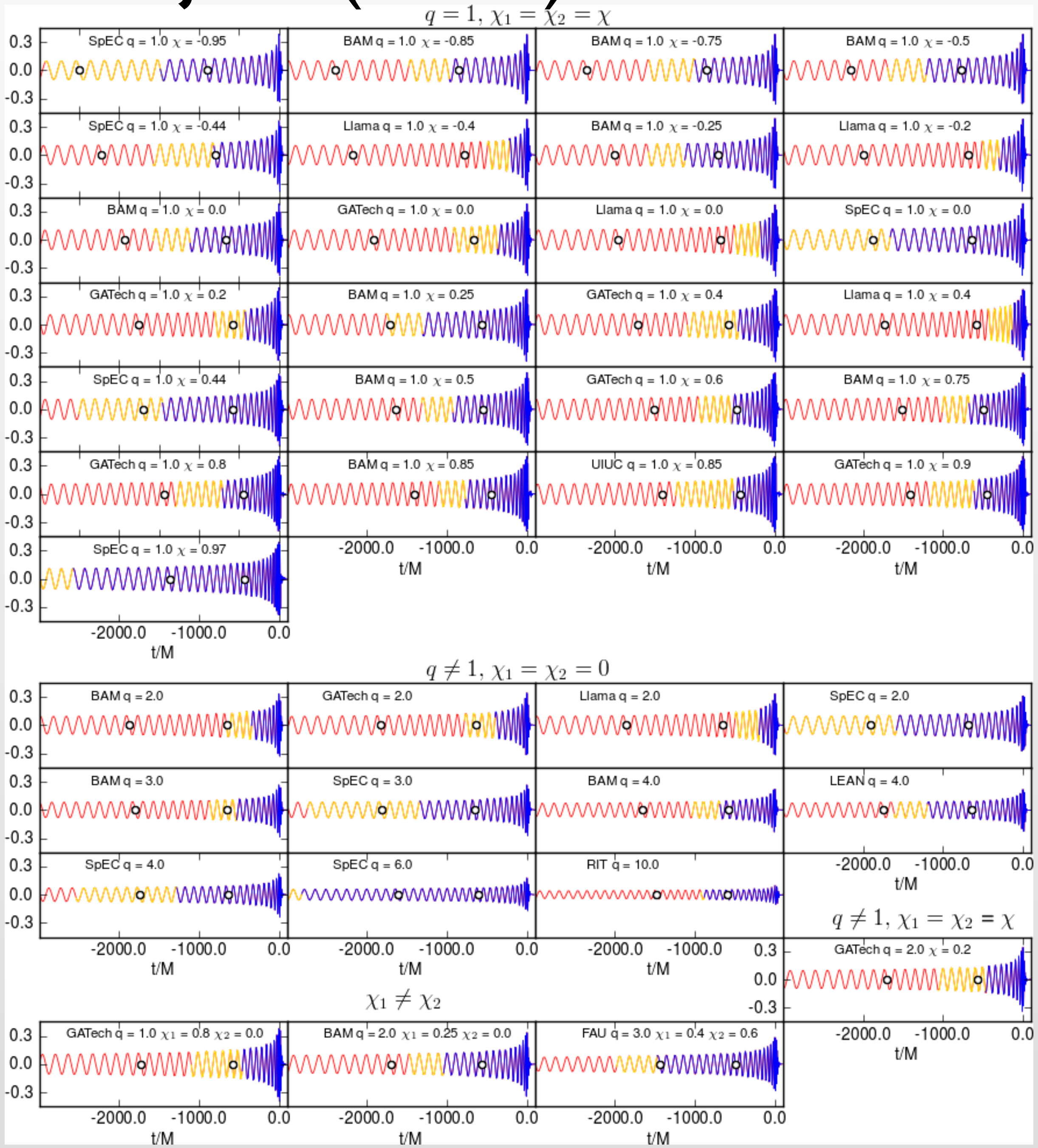
Exploring Parameter Space & Precession

Waveform Catalog Efforts

Ninja1 (2008)

Ninja2 (2012)

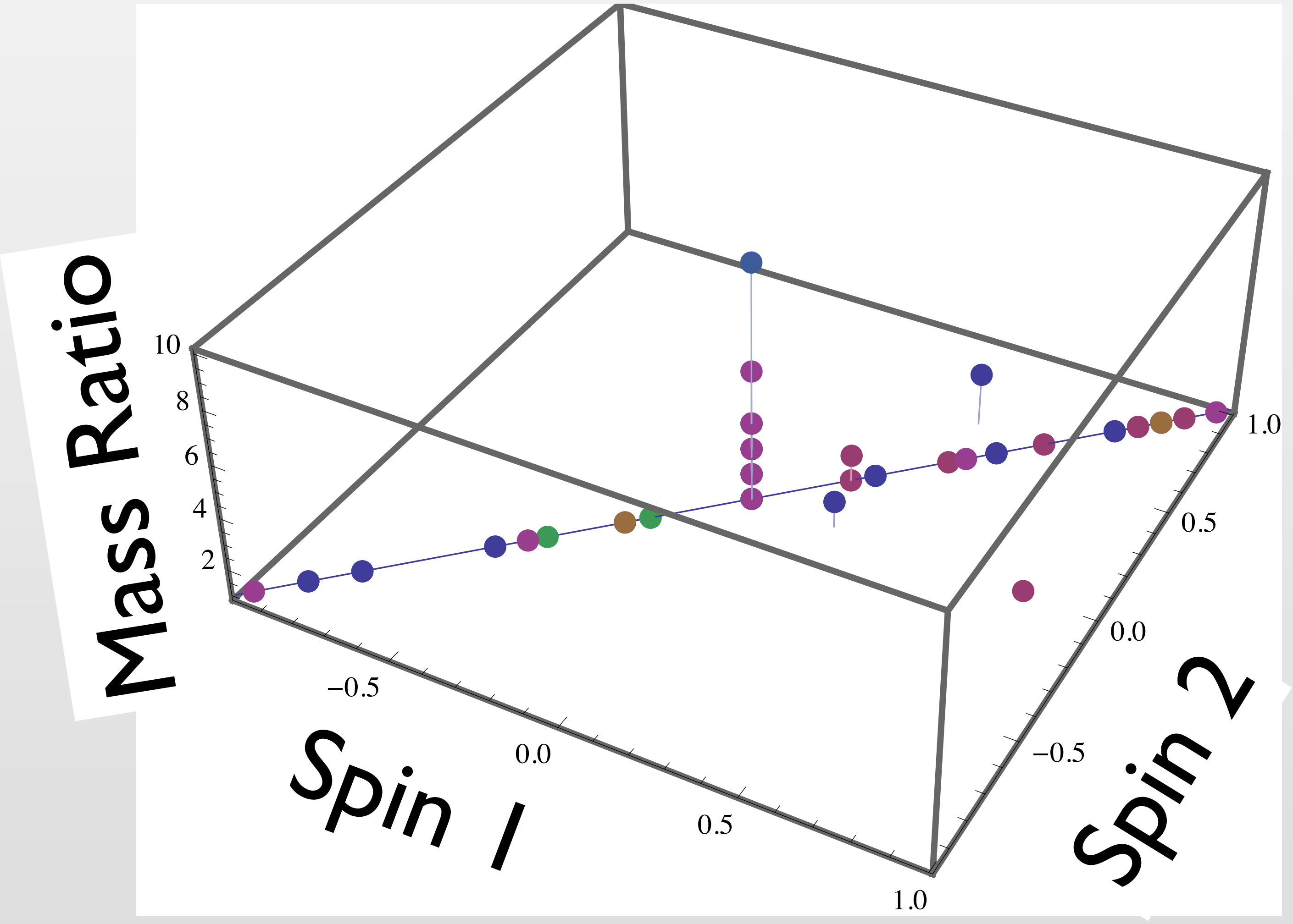


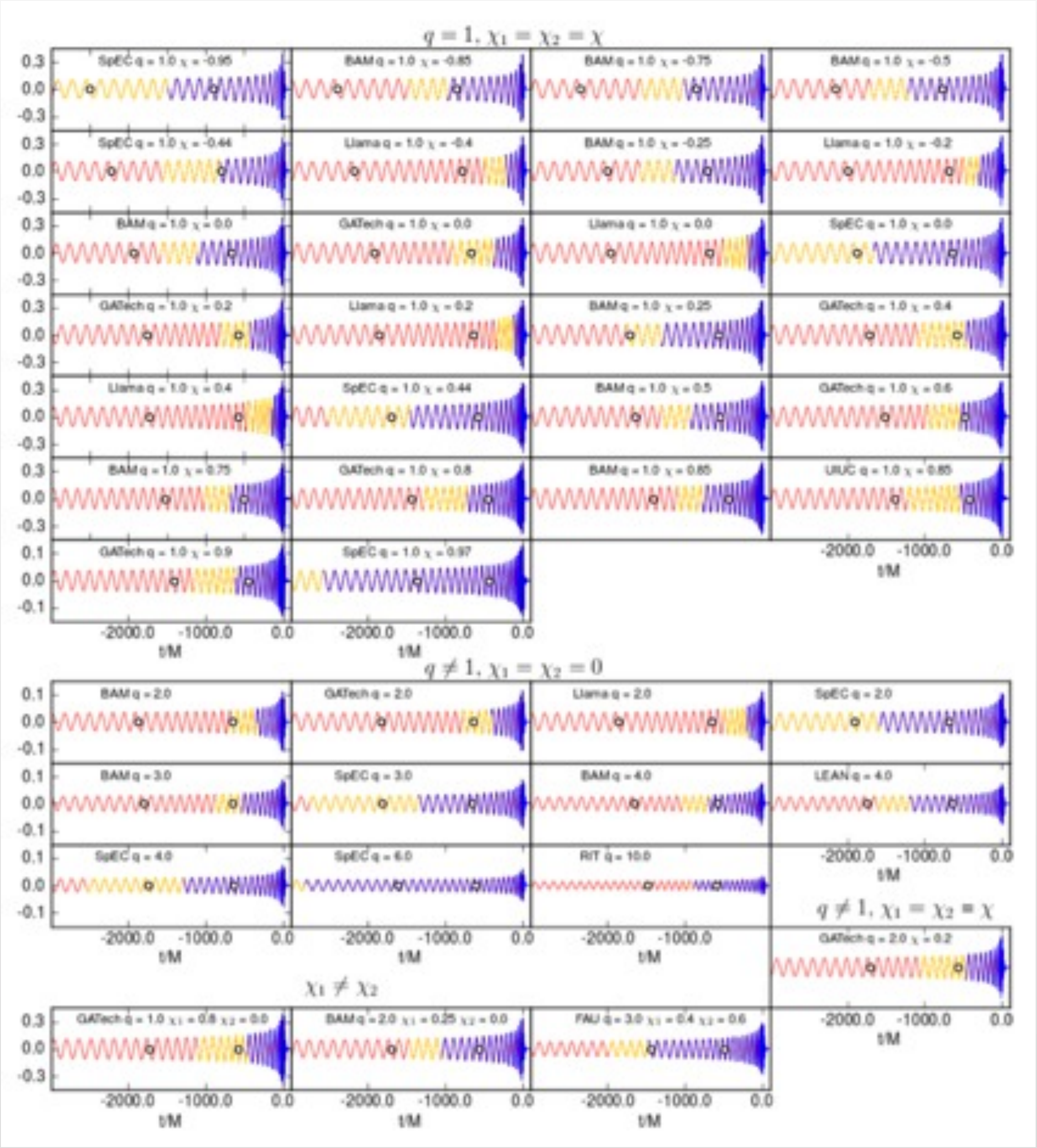


Lack of parameter space coverage

❖ BH-BH simulations are hard

- World-wide NINJA-2 collaboration computed 40 spin-alinged systems (no precession at all)

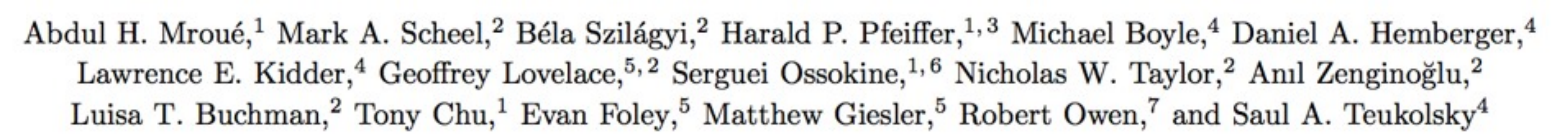




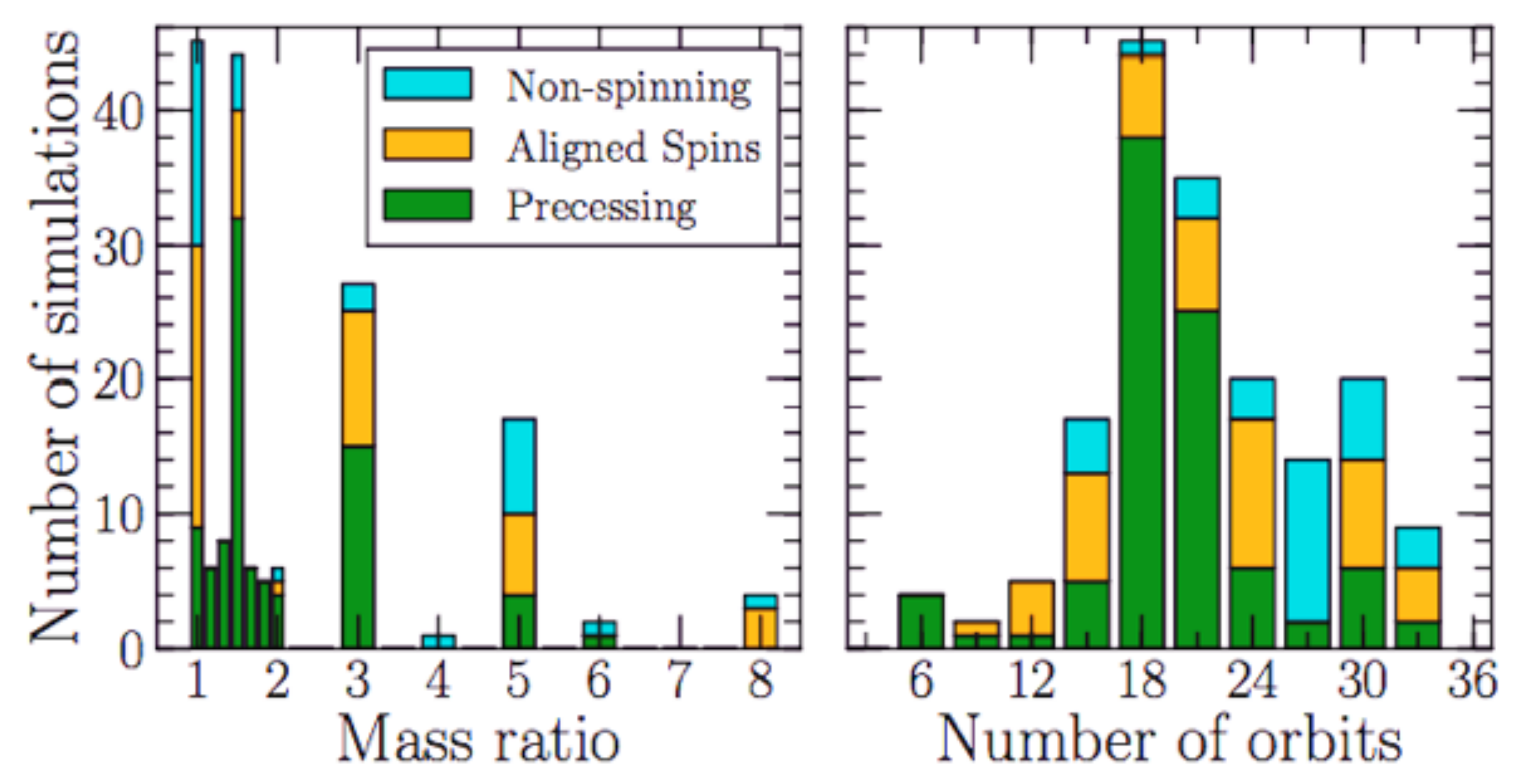
Ajith ea, 1211.5319

Pushing parameter space coverage

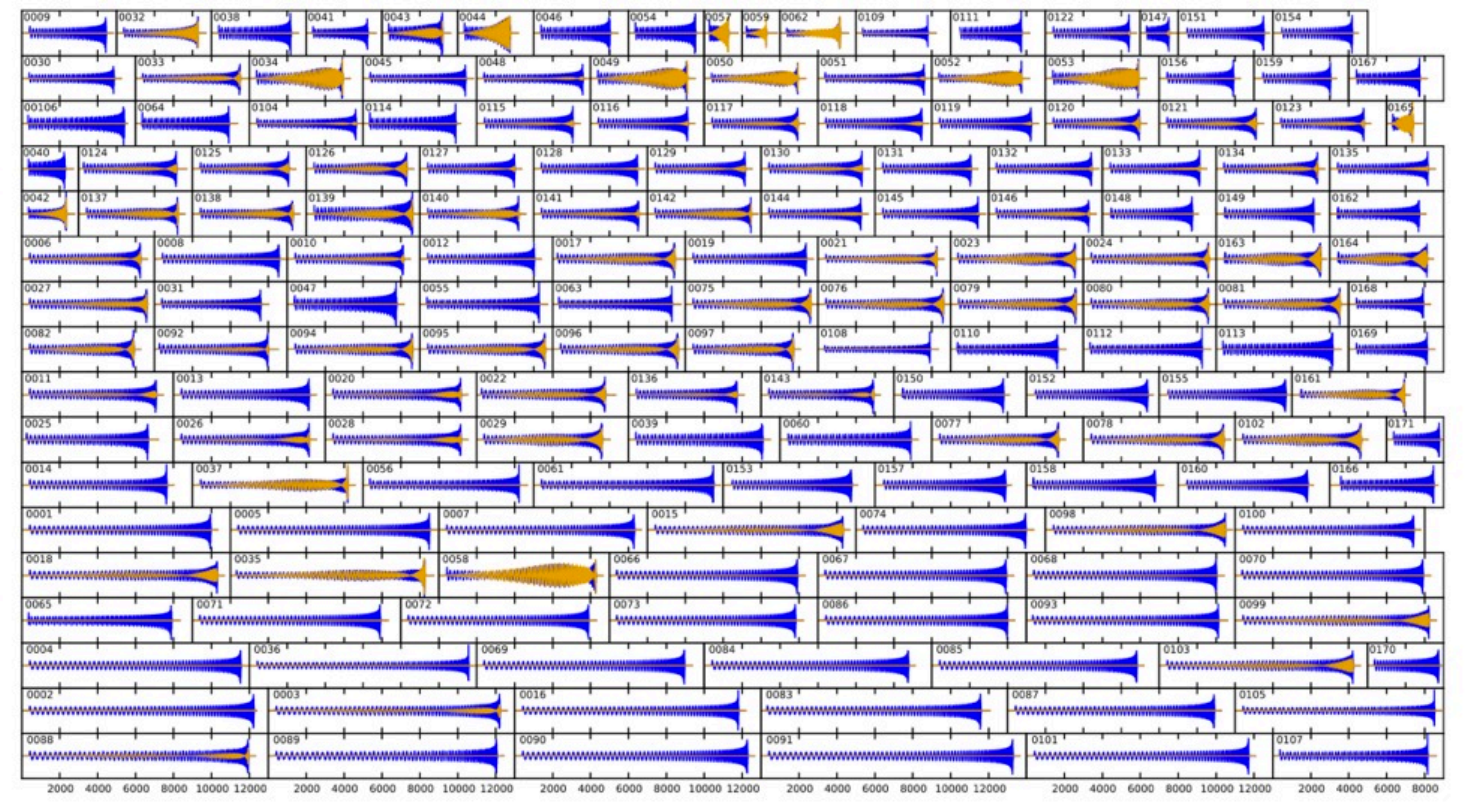
- ❖ 700 configurations quasi-circularized (Mroue, HP 1210.2958)
- ❖ 171 simulations completed
  - Mroue ea, arXiv:1304.6077



Abdul H. Mroué,[1] Mark A. Scheel,[2] Béla Szilágyi,[2] Harald P. Pfeiffer,[1,3] Michael Boyle,[4] Daniel A. Hemberger,[4]
Lawrence E. Kidder,[4] Geoffrey Lovelace,[5,2] Serguei Ossokine,[1,6] Nicholas W. Taylor,[2] Anıl Zenginoğlu,[2]
Luisa T. Buchman,[2] Tony Chu,[1] Evan Foley,[5] Matthew Giesler,[5] Robert Owen,[7] and Saul A. Teukolsky[4]



171 waveform catalog

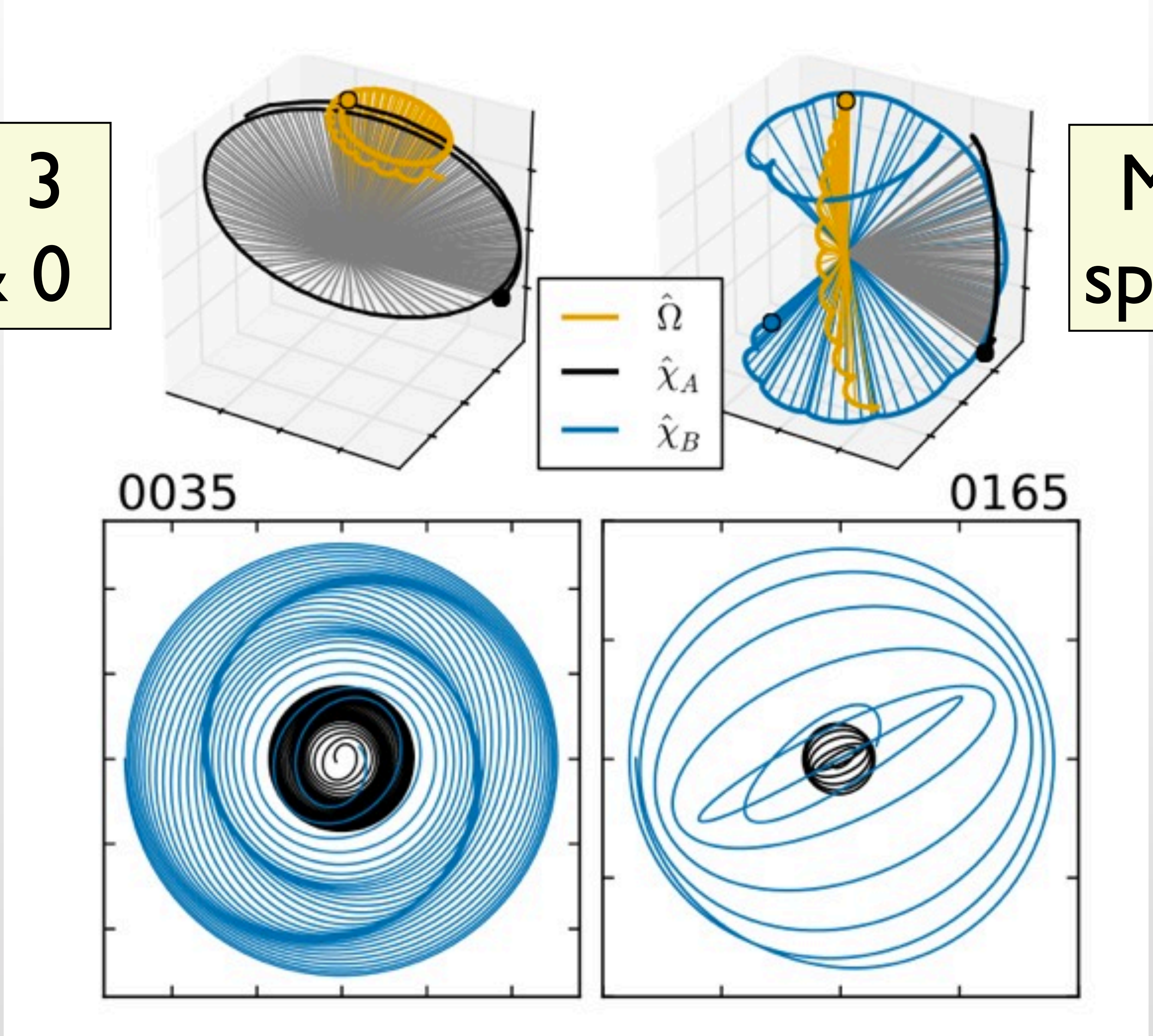


3 years, 50 Mio CPU-hours

Mroue ea, arXiv:1304.6077

Examples of precessing binaries

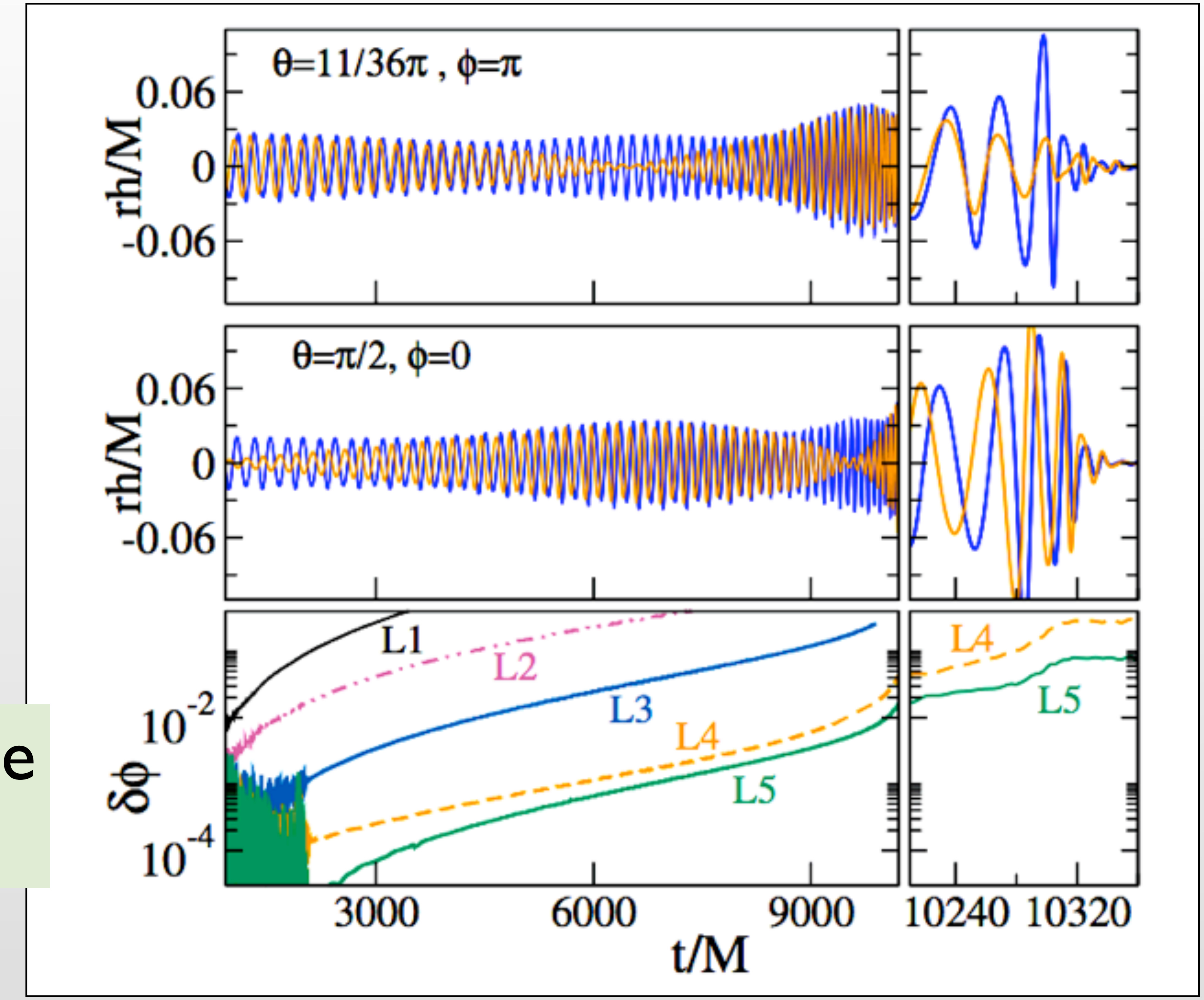
Mass-ratio 3
spins 0.5 & 0



Mass-ratio 6
spins 0.9 & 0.3

Mroue ea, arXiv:1304.6077

Orientation-dependence of waveform



Convergence test!

Mroue ea, arXiv:1304.6077

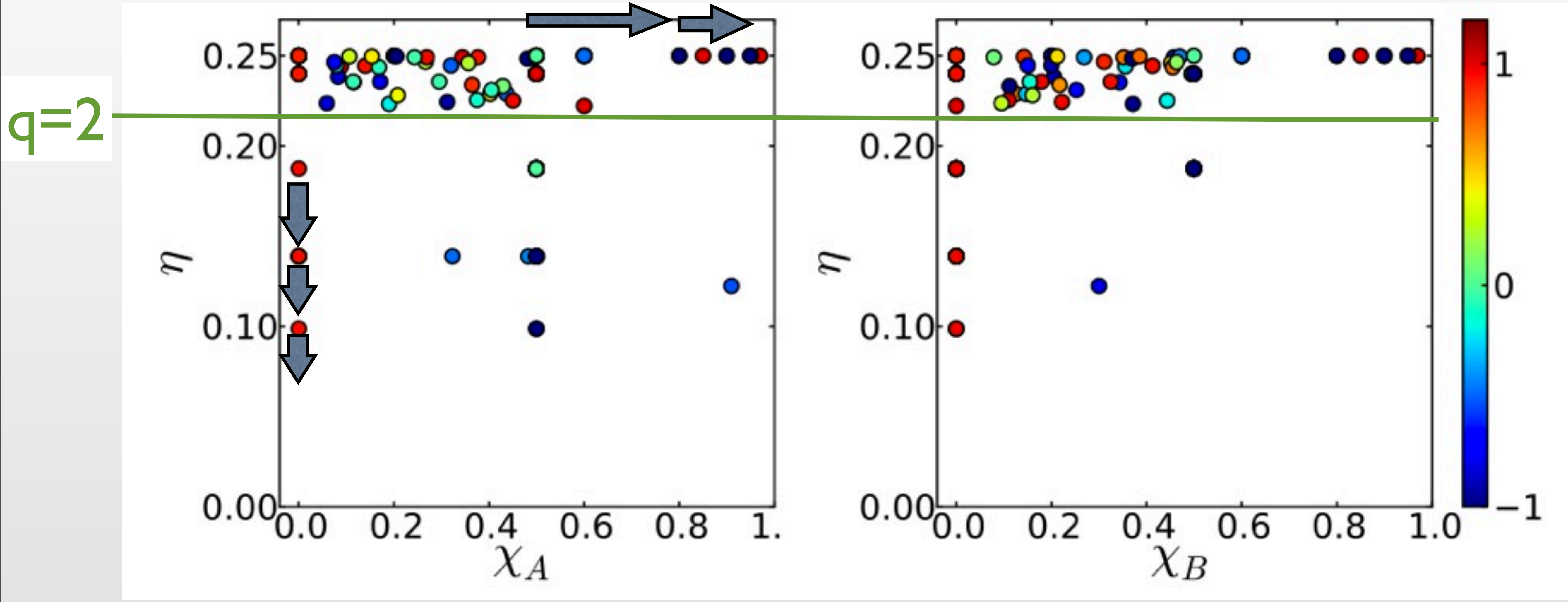
Alternative study: Georgia Tech

❖ Pekowsky ea 1304.3176, continuing sequence of papers

- 191 generic BBH waveforms
- precessing waveform = [non-precessing waveform] x [Rotation]
- IMRPhenomB fits to better than 95% for 200Msun<M<2500Msun
- At low masses, GW's can measure BH-BH properties
- At high masses, GW's can measure remnant properties

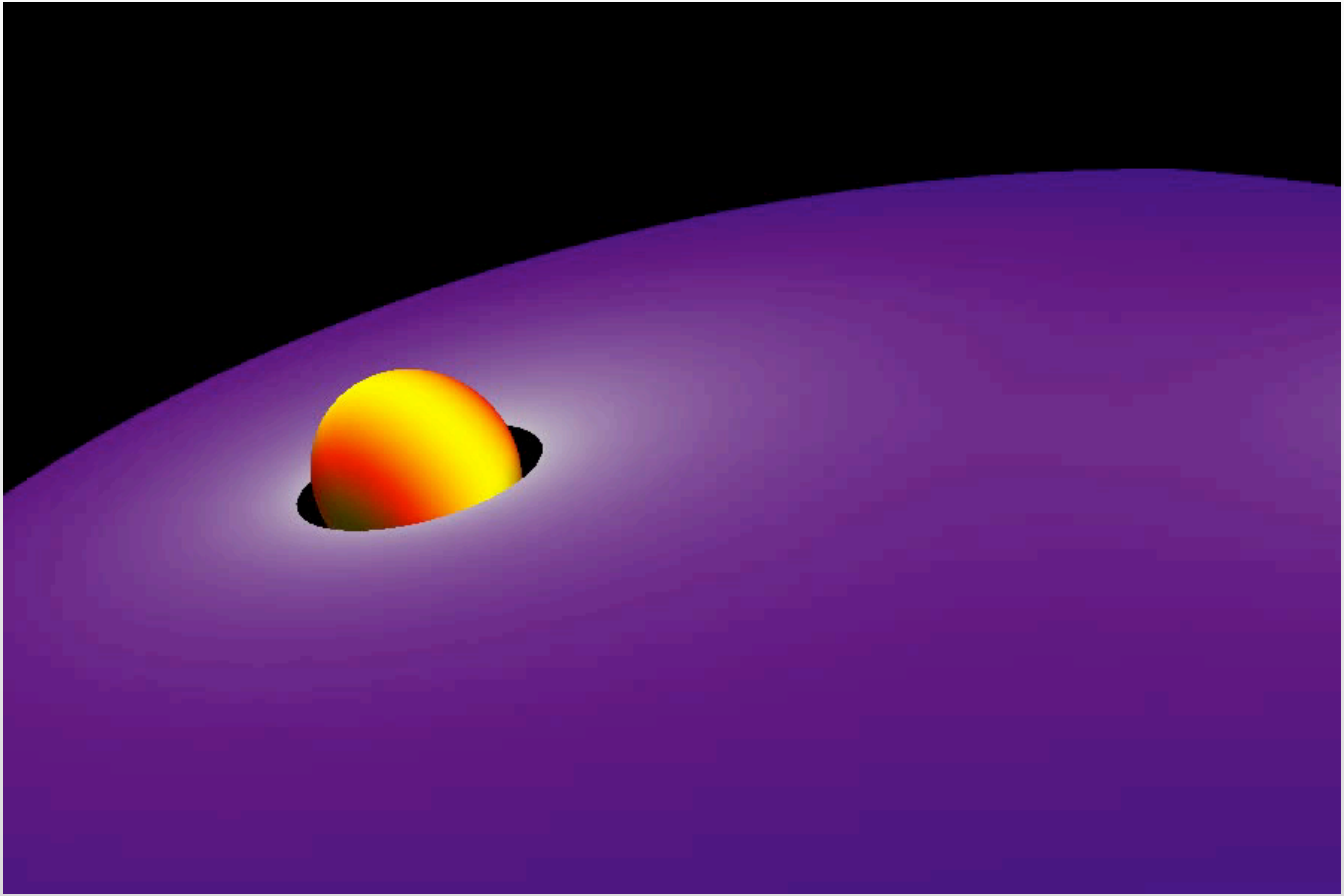
Expanding parameter space coverage

- ❖ Most spinning runs at q<2

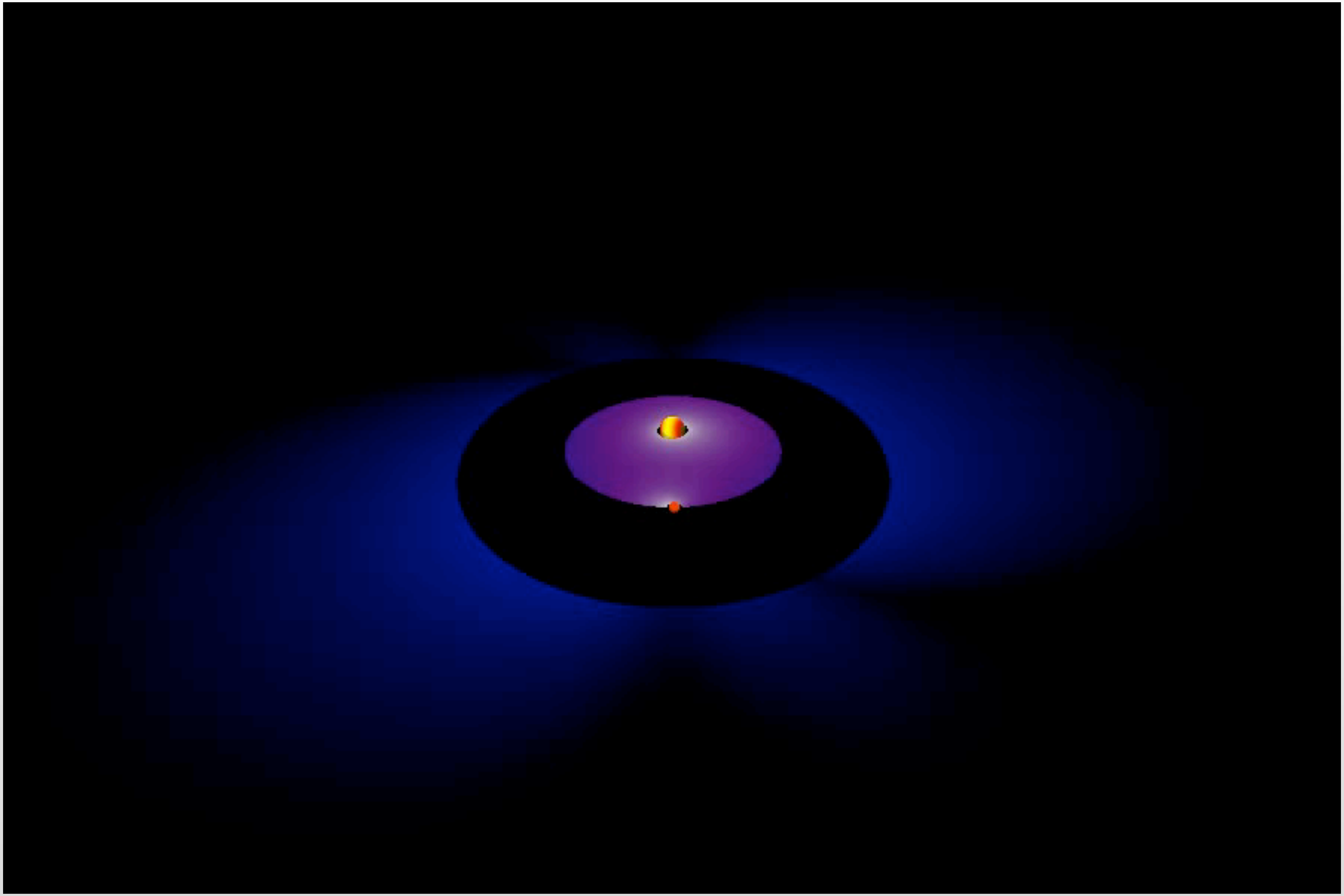


- ❖ So far, pushing parameters was *always* difficult
  - Each arrow 1-2years hard work

Precessing Movie



Precessing Movie



BH-BH the big picture

❖ BH-BH simulations quite mature

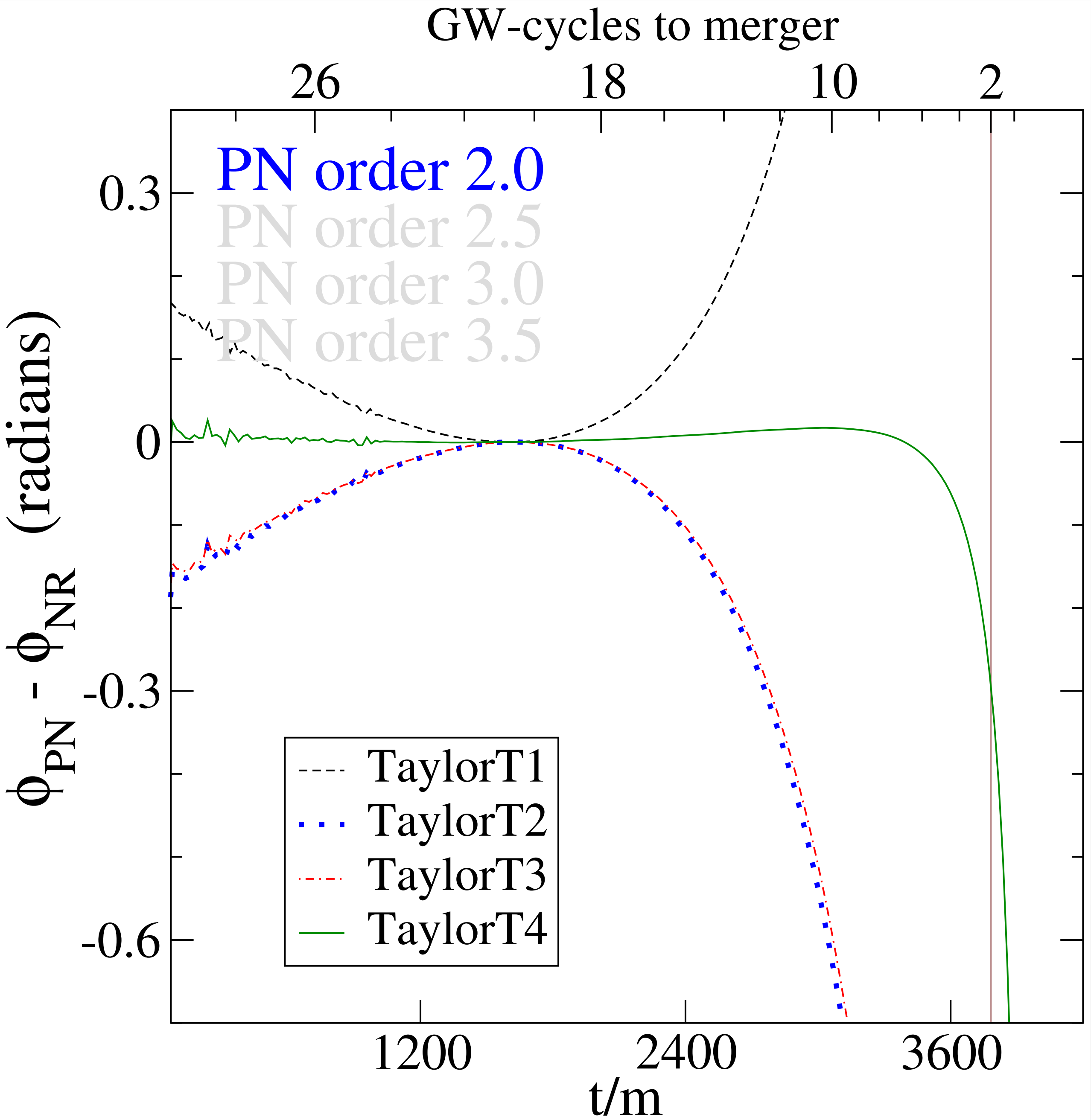
- Lot's of parameter space remains for exploration
- Boundaries of parameter space challenge current codes:
  - spins very close to maximal
  - mass-ratios beyond 10

❖ Present goal: Remove waveform modeling errors from GW data-analysis

- Explore all parameters w/ sufficient accuracy
- Check where Post-Newtonian is sufficient
- Construct waveform models
- Perform injection studies

post-Newtonian -- NR comparison

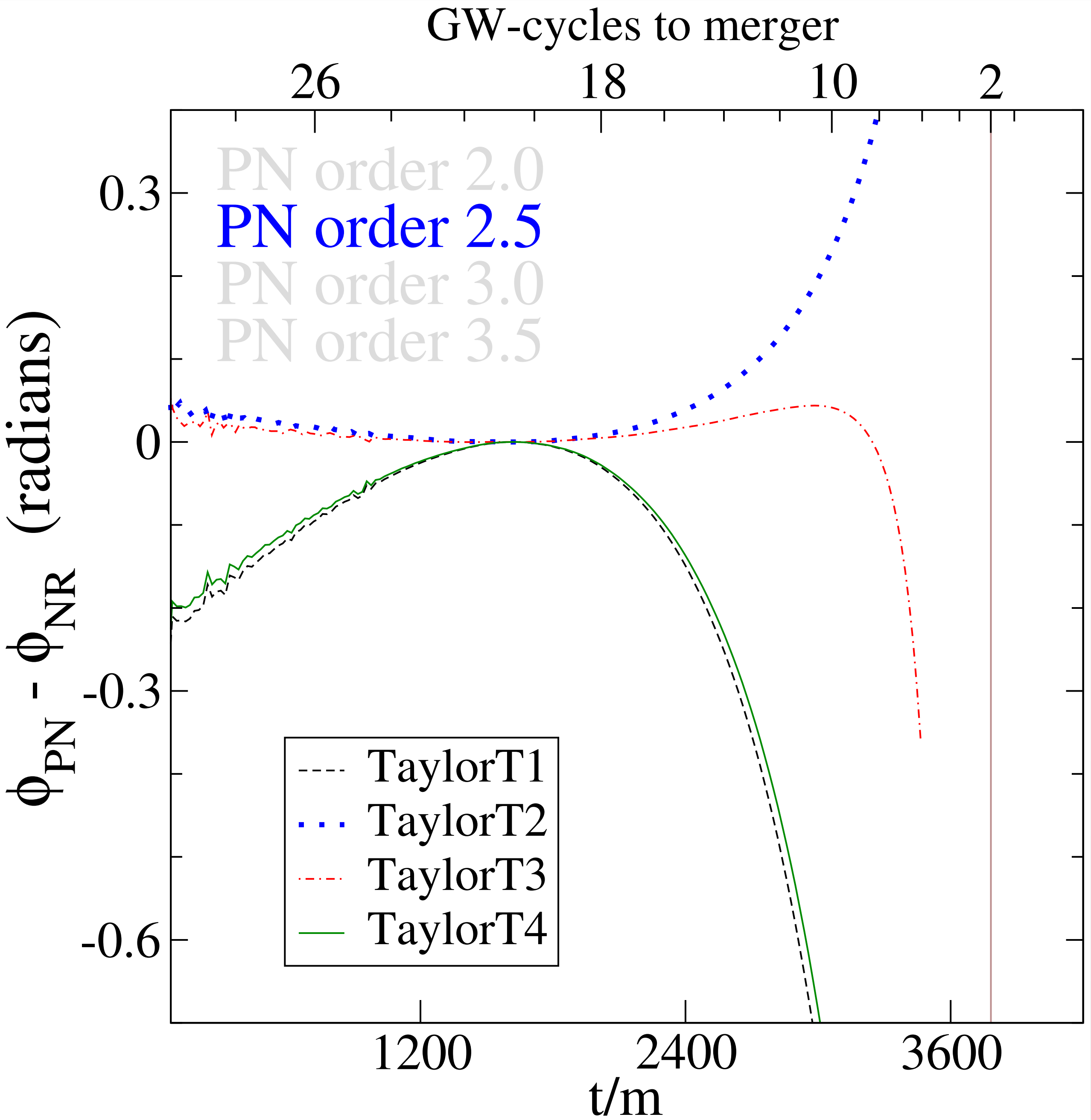
- ❖ NR & PN agree!



Boyle et al 2007

post-Newtonian -- NR comparison

- ❖ NR & PN agree!



Boyle et al 2007

post-Newtonian -- NR comparison

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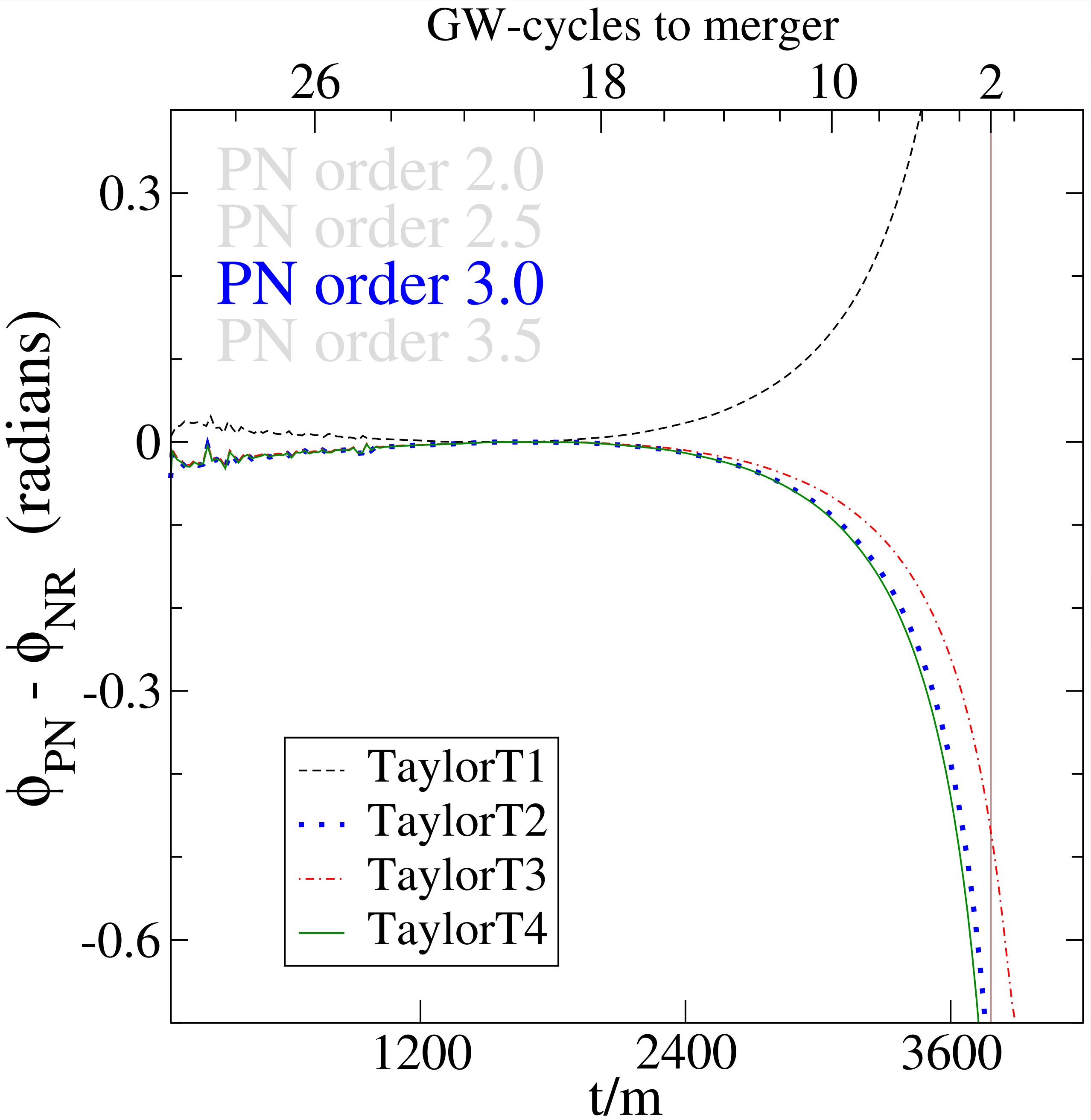
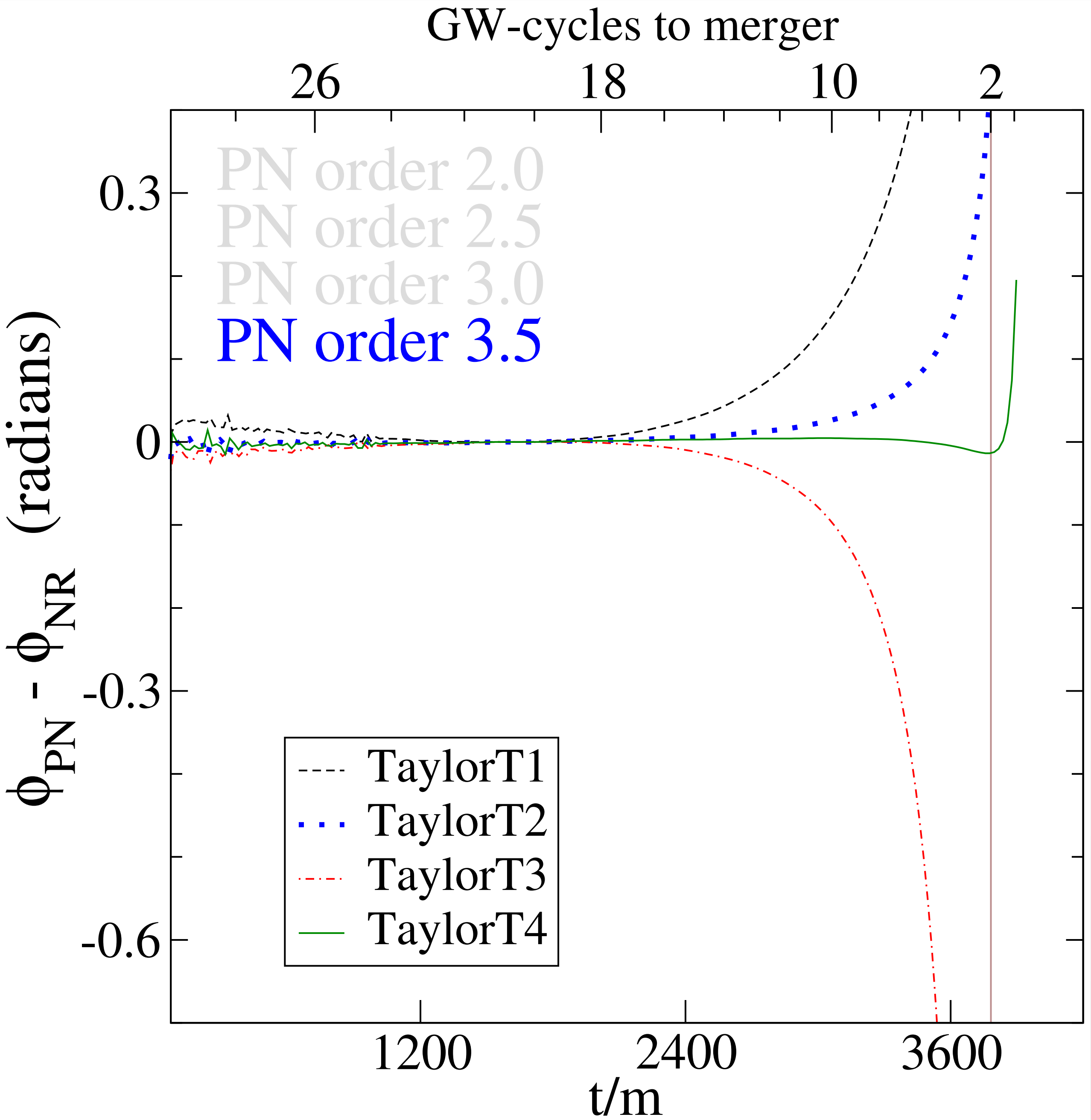


Boyle et al 2007

post-Newtonian -- NR comparison

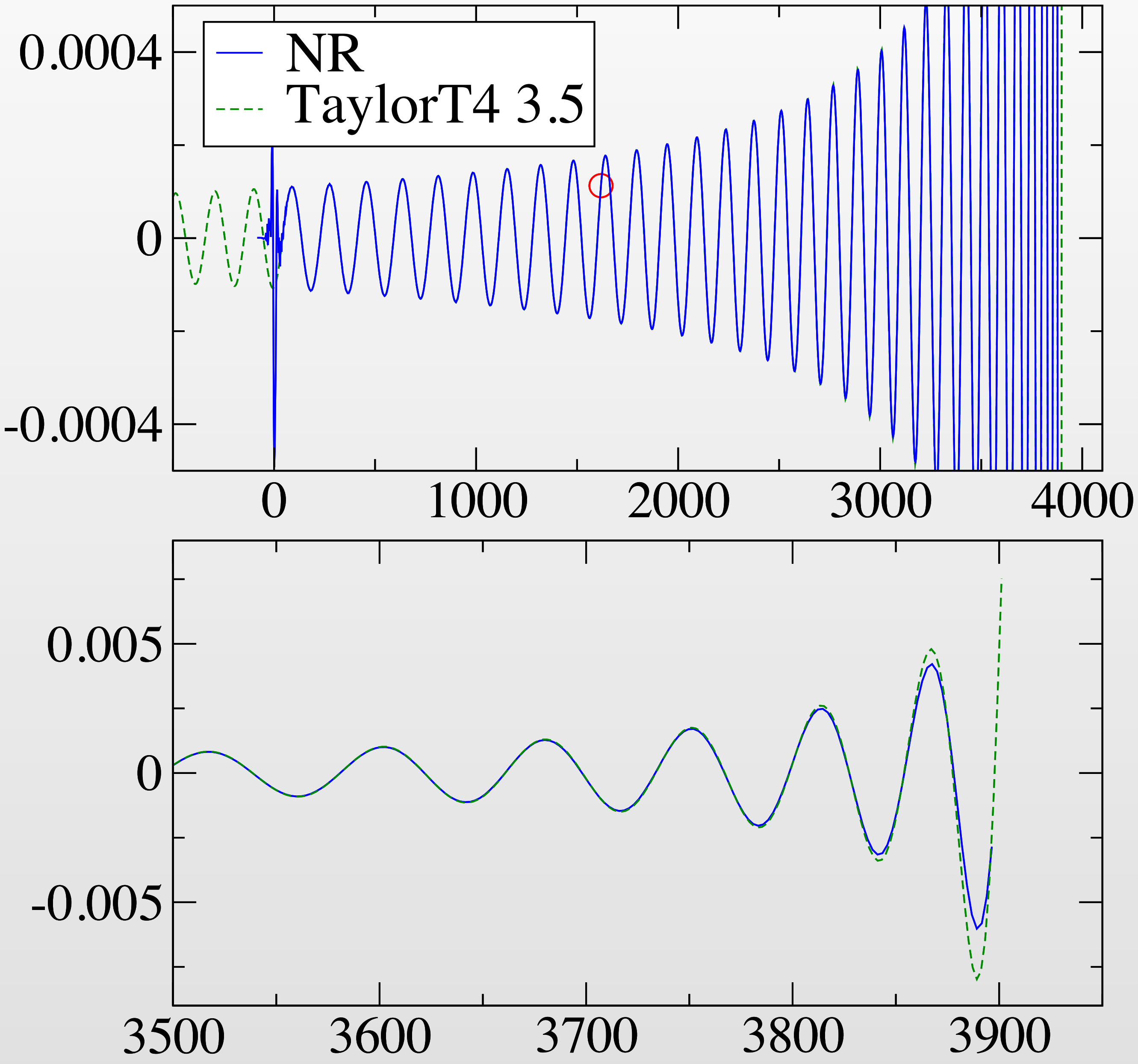
- NR & PN agree!



Boyle et al 2007

post-Newtonian -- NR comparison

- ❖ NR & PN agree!
- ❖ Or do they?
  - SOME versions of PN match very well
  - NO a priori knowledge which ones work (if any)

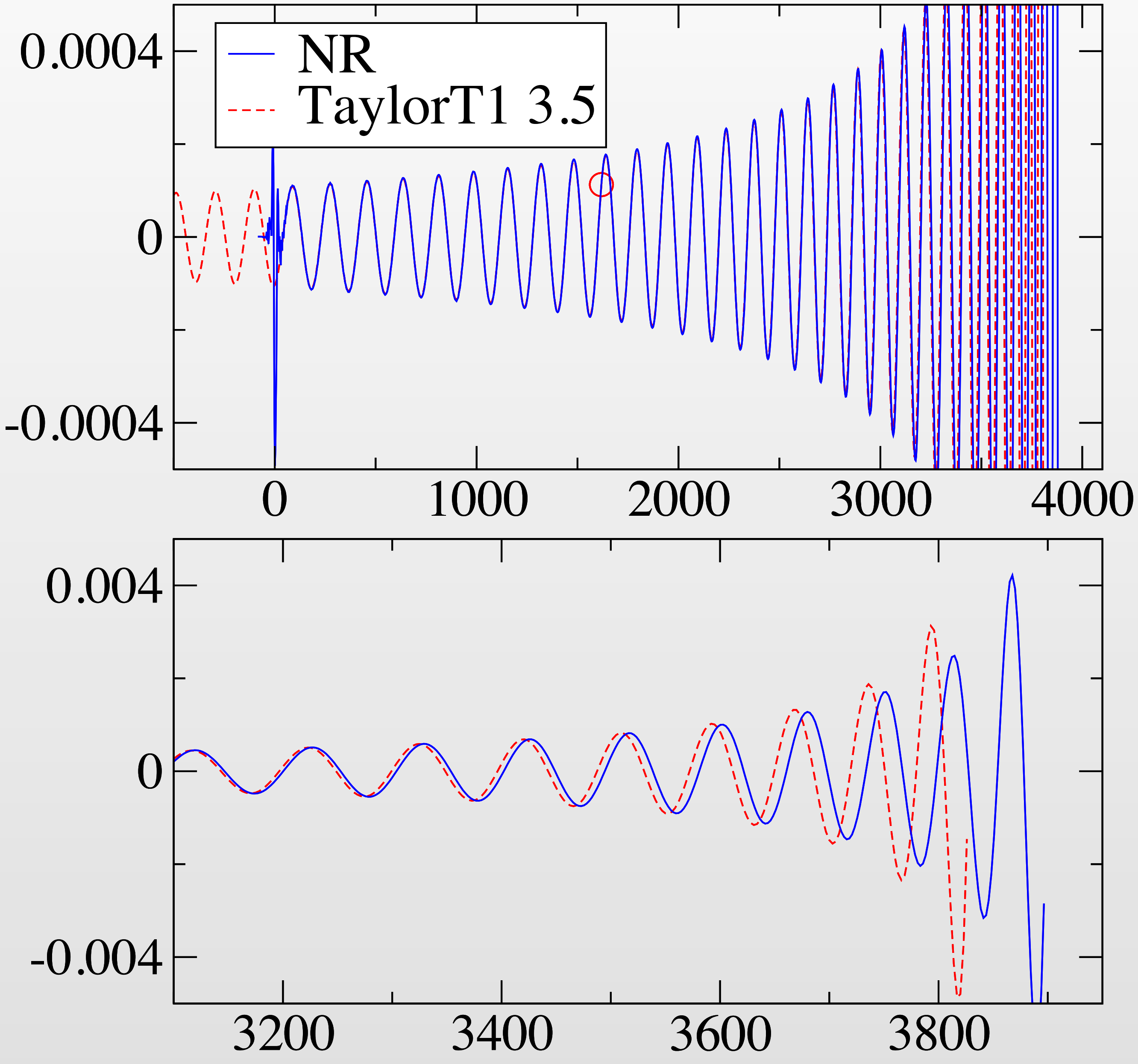


Boyle et al 2007

post-Newtonian -- NR comparison

CITA–ICAT

- NR & PN agree!
- Or do they?
  - SOME versions of PN match very well
  - NO a priori knowledge which ones work (if any)



Boyle et al 2007

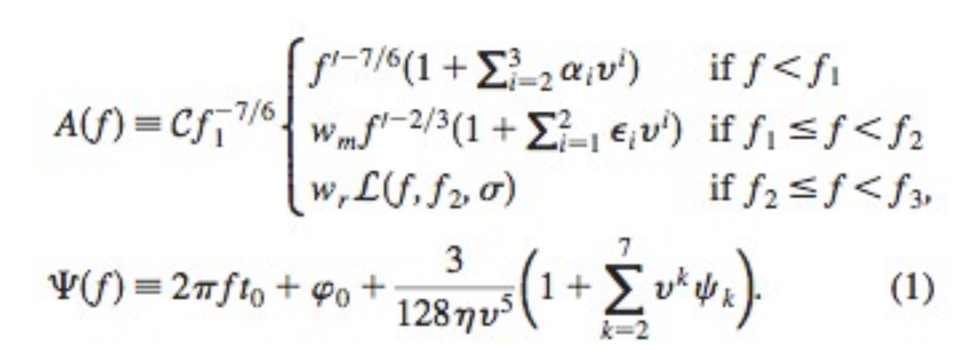
Phenomenological, aligned spins

❖ Unequal-mass, aligned spins (Ajith ea 2011) "IMRPhenomC"

- 2-dim waveform family (mass-ratio, effective spin)
- (2,2) mode calibrated against 24 sims (BAM, Ccatie, Llama)

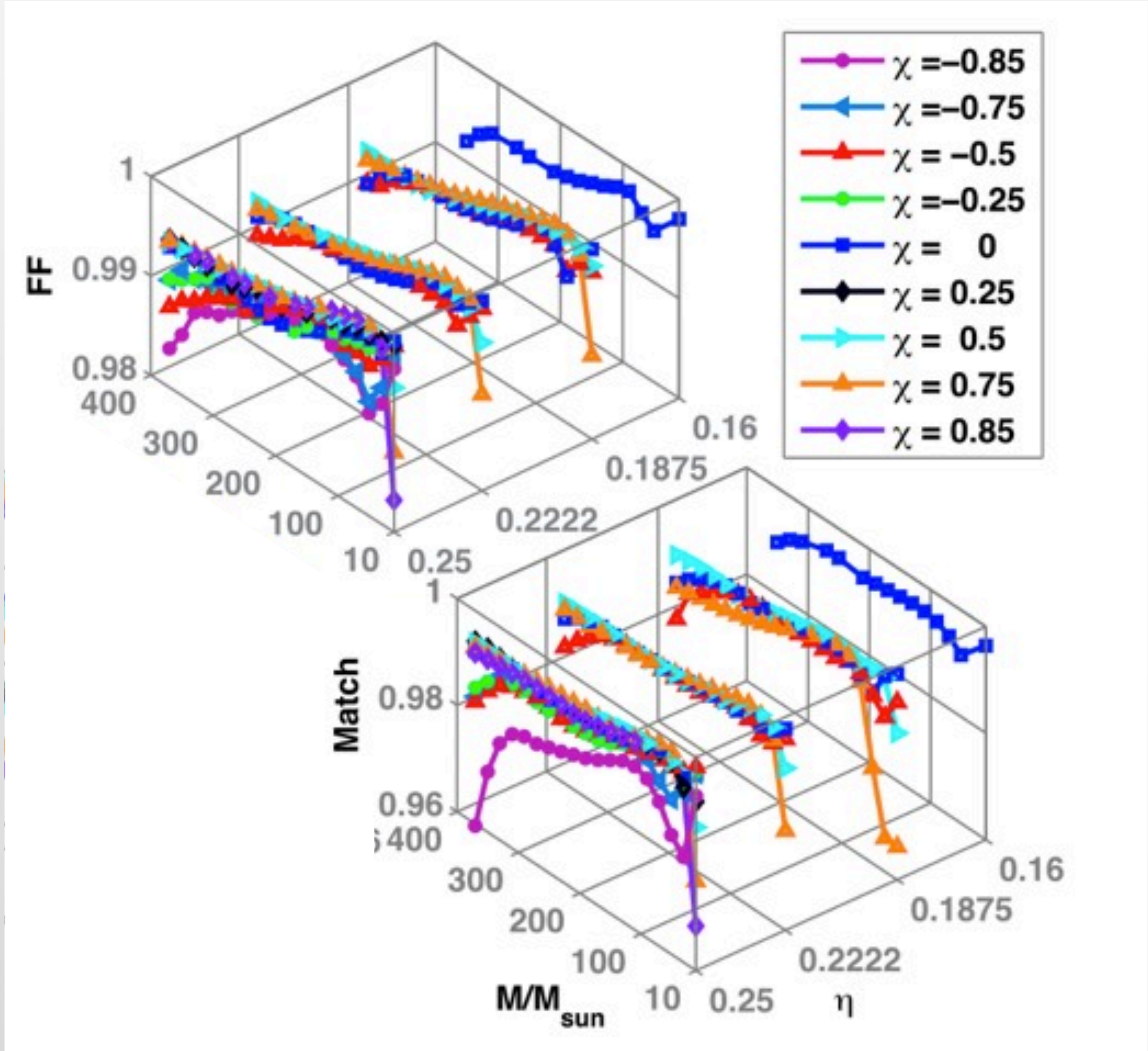
❖ Two stages:

1. construct TaylorT1+NR hybrids
2. fit model to hybrids



$$A(f) \equiv C f_1^{-7/6} \begin{cases} f'^{-7/6}(1+\sum_{i=2}^{3}\alpha_i \upsilon^i) & \text{if } f<f_1 \\ w_m f'^{-2/3}(1+\sum_{i=1}^{2}\epsilon_i \upsilon^i) & \text{if } f_1 \le f < f_2 \\ w_r \mathcal{L}(f, f_2, \sigma) & \text{if } f_2 \le f < f_3, \end{cases}$$

$$\Psi(f) \equiv 2\pi f t_0 + \varphi_0 + \frac{3}{128\eta\upsilon^5}\left(1+\sum_{k=2}^{7}\upsilon^k \psi_k\right). \qquad (1)$$



EOB + NR

- Effective one body
    - Buonanno, Damour 1999; many papers since
- Inspiral-Merger-Ringdown waveform model based on
    - Effective Hamiltonian to capture conservative dynamics

$$H=\mu\sqrt{p_r^2+A(r)\left[1+\frac{p_r^2}{r^2}+2(4-3\nu)\nu\frac{p_r^4}{r^2}\right]},\qquad A(r)=\sum_{k=0}^{4}\frac{a_k(\nu)}{r^k}+\frac{a_5(\nu)}{r^5}$$

-
    - Radiation reaction terms

$$\frac{dp_r}{dt}\;=\;-\frac{\partial H}{\partial p_r}+a^r_{\mathrm{RR}}\frac{\dot r}{r^2\Omega}\widehat{\mathcal{F}}_\phi$$

$$\frac{dp_\varphi}{dt}\;=\;=0-\frac{v_\Omega^3}{\nu V_\phi^6}F_4^4(V_\phi;\nu,v_{\mathrm{pole}}),\quad \text{using 4-PN term } \mathcal{F}_{8,\nu=0}+\nu A_8$$

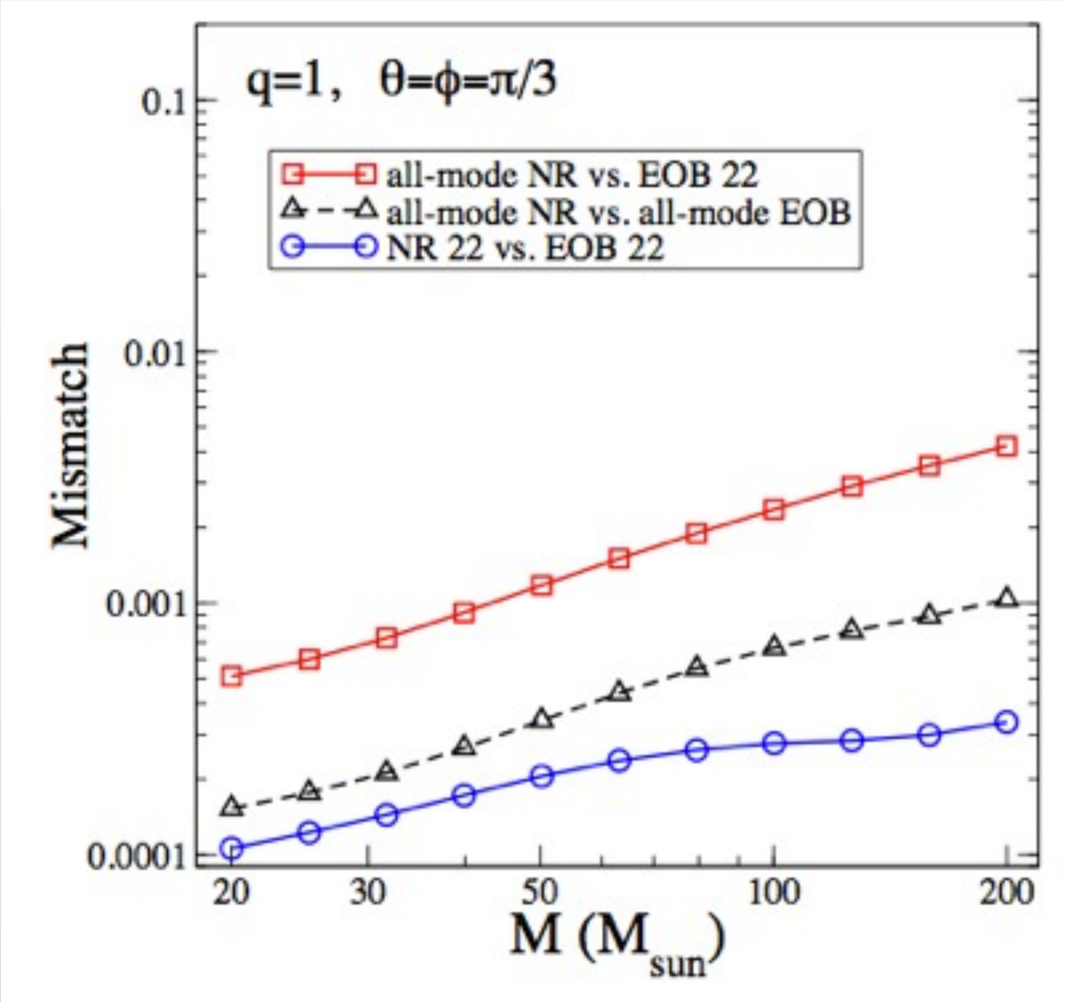
-
    - Attach ringdown modes

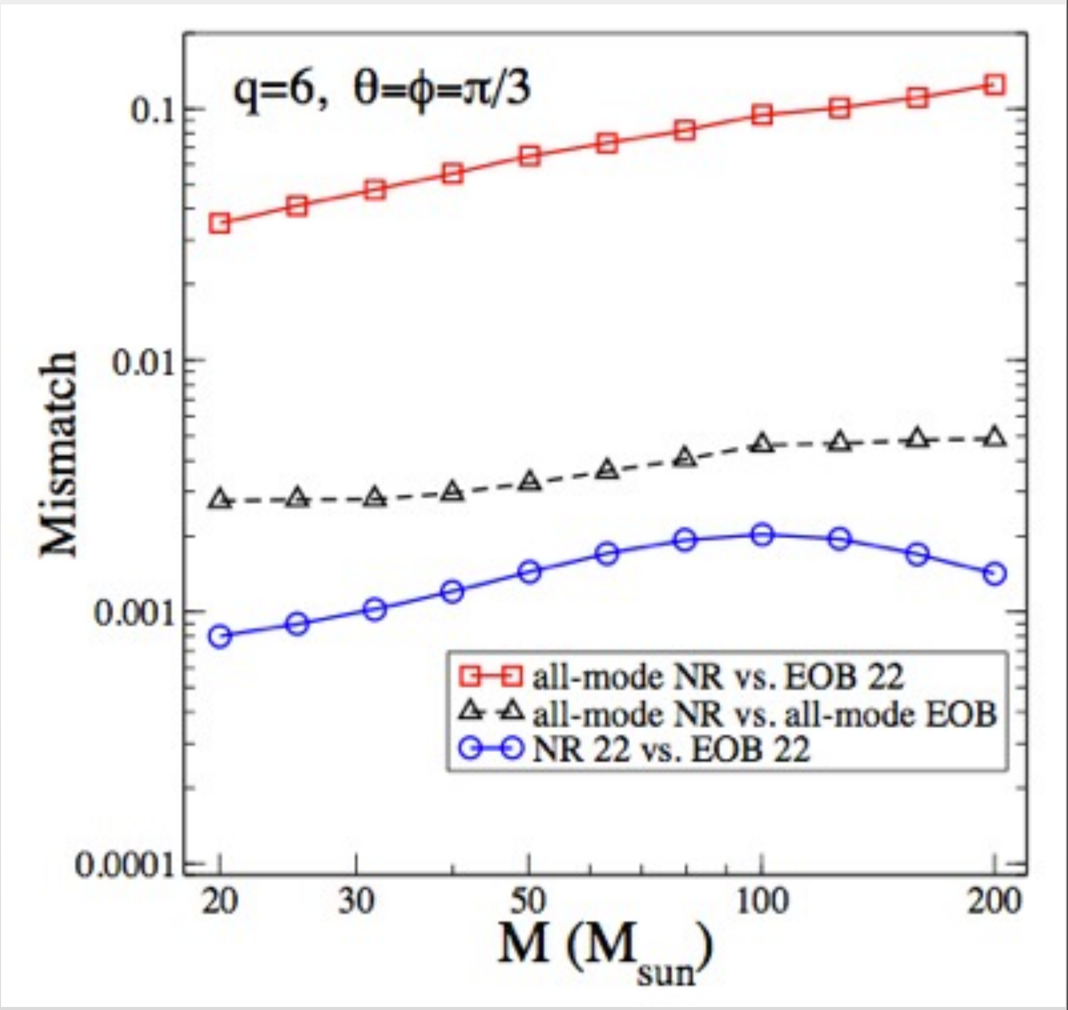
★ Fit parameters to NR simulations

EOB for non-spinning BH-BH

- ❖ Physical parameter mass-ratio q
- ❖ “EOBNRv2” Pan ea, 2011
  - supersedes EOBNRv1 (Buonnano ea 2007)
  - Five modes: (2,2), (2,1), (3,3), (4,4), (5,5)
  - calibrated against SpEC q=1,2,3,4,6.

Mismatch with NR waveforms used in fitting





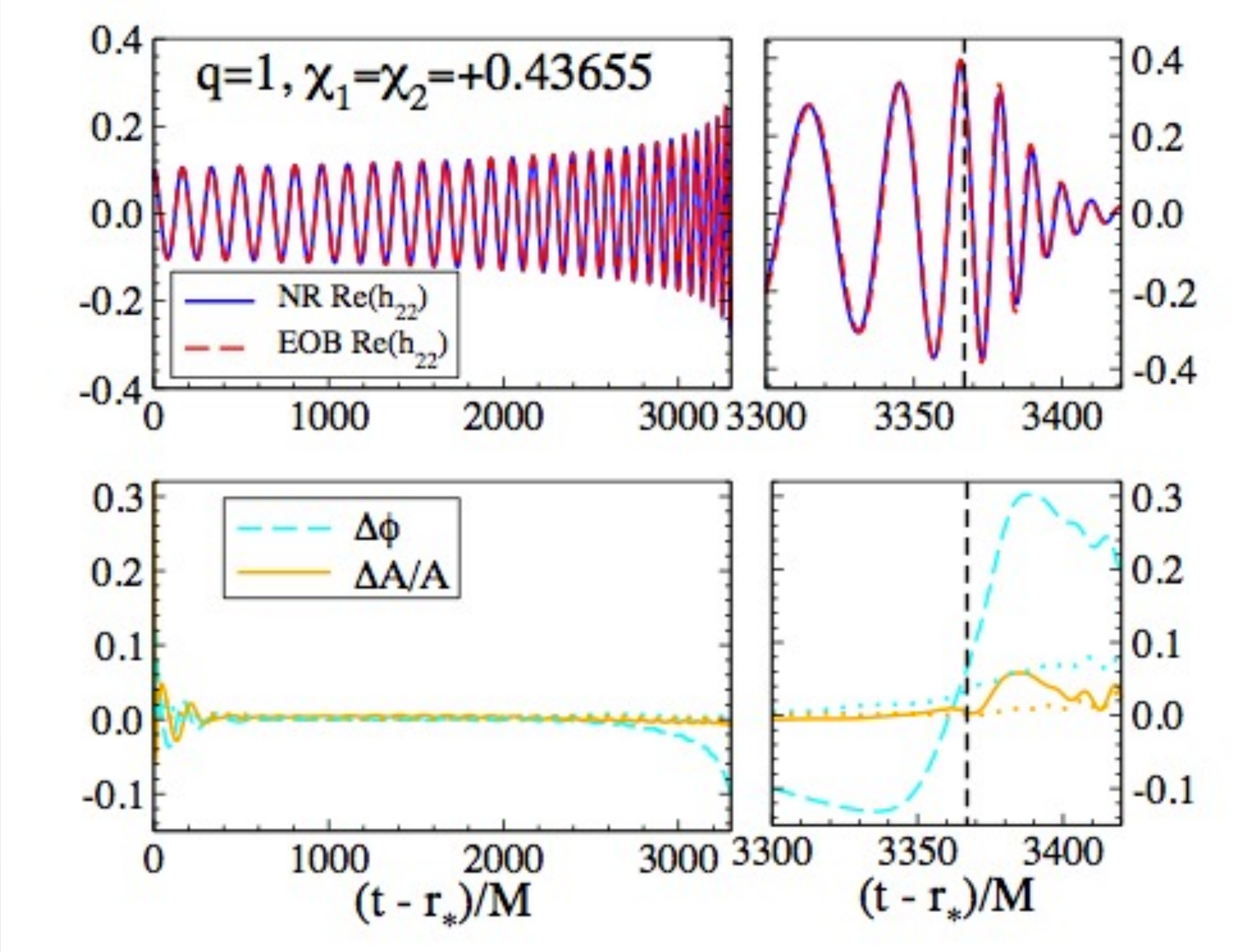
EOB for aligned spins

❖ EOB w/ aligned spins "SEOBNRv1"

- Taracchini ea 2012
- (2,2) mode calibrated against 7x SpEC & Teukolsky code
- Prototype-model: Intended for re-calibration with more NR sims

❖ Caveats:

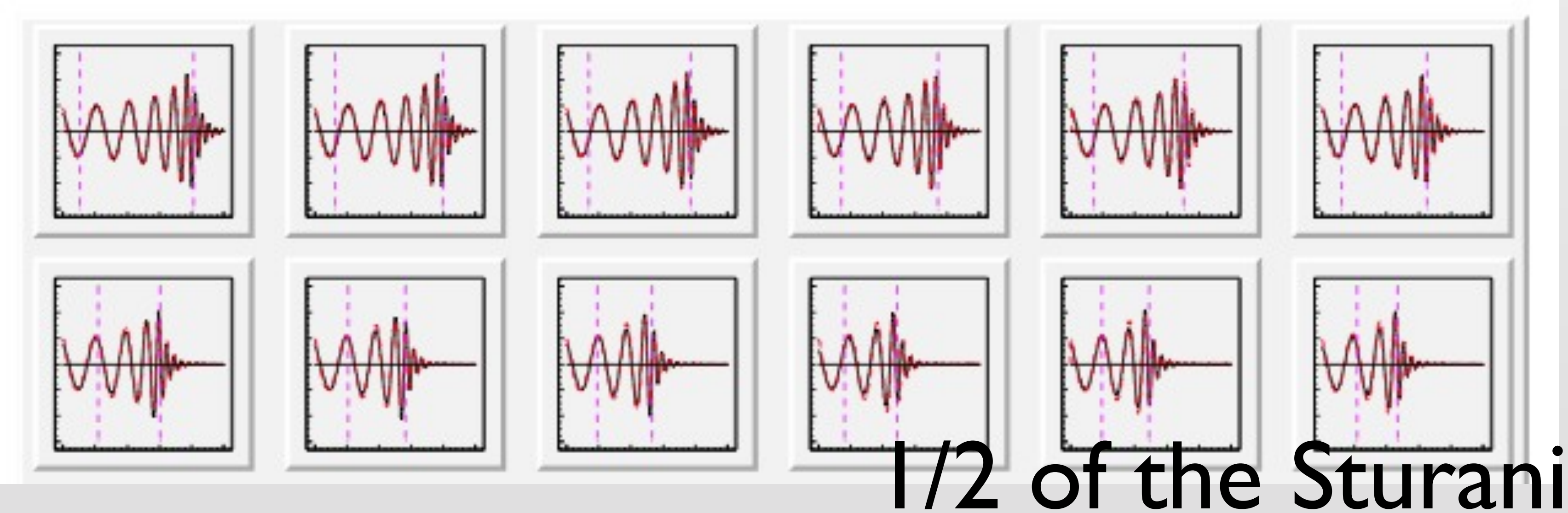
- Calibrated in tiny region of param space:
  (a) zero spin q=1,2,3,4,6
  (b) q=1, equal spin ±0.44
- Current EOB model fails for aligned spins >0.7



Taracchini ea 2012

Precessing BH-BH

- First generic spin model (Sturani ea 2010)
  - Based on 24 MayaKranc sims
  - TaylorT4 until very close to merger & phenomenological Ansatz

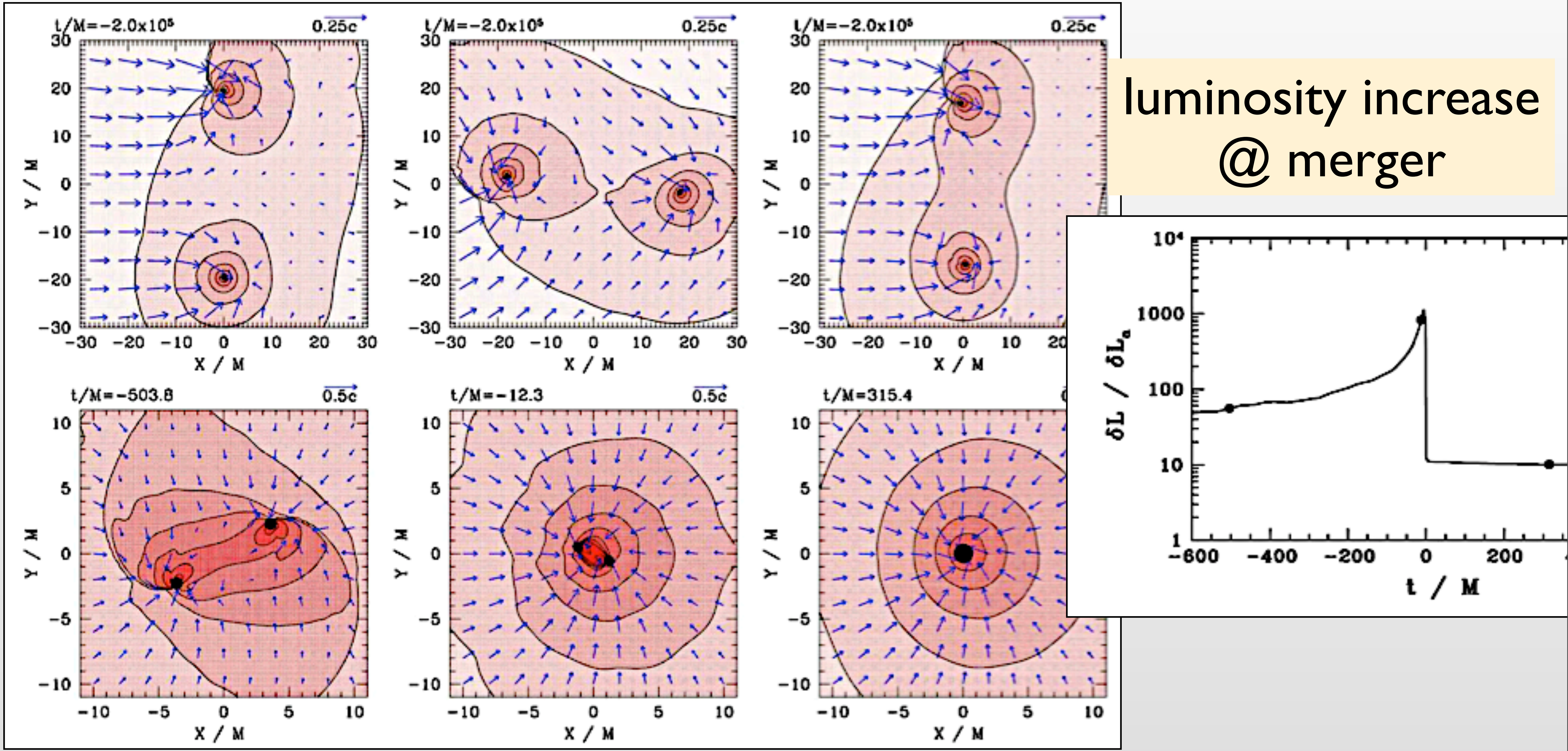


1/2 of the Sturani ea NR waveforms

BH-BH in non-vacuum environment

BH-BH in gaseous environment

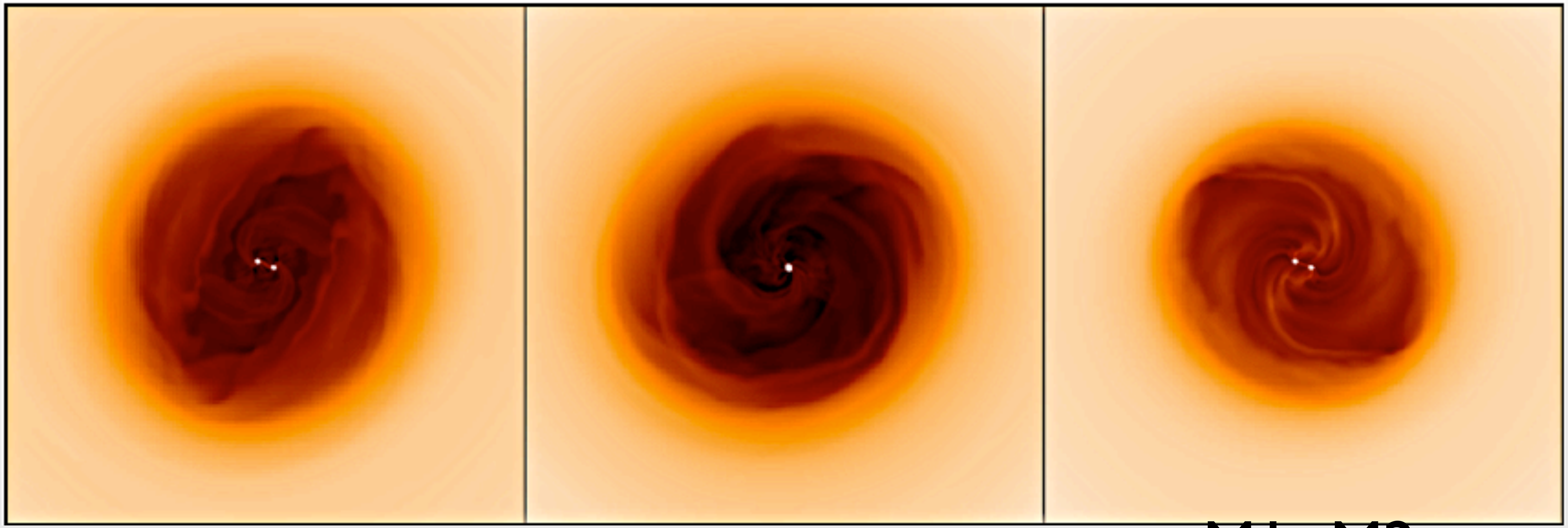
- ❖ center-of-mass **v=0.1c** "Bondi-Hoyle-Lyttleton BH-BH"



luminosity increase
@ merger

Farris, Liu, Shapiro 10

BH-BH in gaseous environment



M1=M2
aligned spins 0.6

M1/M2=2
random spins 0.6

M1=M2
aligned spins 0.6
retrograde disk

Bode, Bogdanovic, Haas, Healy, Laguna, Shoemaker 1101.4684

Black hole magnetosphere

- ❖ Tenuous plasma. Inertia dominated by magnetic field

$$\rho\vec E + \vec\jmath\times\vec B = \rho_{\rm plasma}\vec a_{\rm plasma} \equiv 0$$

  - Maxwell's equations close; matter disappears from eqns.

$$\begin{aligned}
\partial_t\vec E &= \nabla\times\vec B - \vec j,\\
\partial_t\vec B &= -\nabla\times\vec E,\\
\nabla\cdot\vec E &= \rho,\\
\nabla\cdot\vec B &= 0,
\end{aligned}$$

$$\vec j = \frac{\vec B}{B^2}\left[\vec B\cdot(\nabla\times\vec B) - \vec E\cdot(\nabla\times\vec E)\right] + \frac{\vec E\times\vec B}{B^2}\nabla\cdot\vec E.$$

- ❖ Appropriate for pulsar and BH magnetospheres

- ❖ cf. MHD: B-field attached to moving fluid

BH in force-free plasma

- ❖ Single BH's consistent with Blandford-Znajek

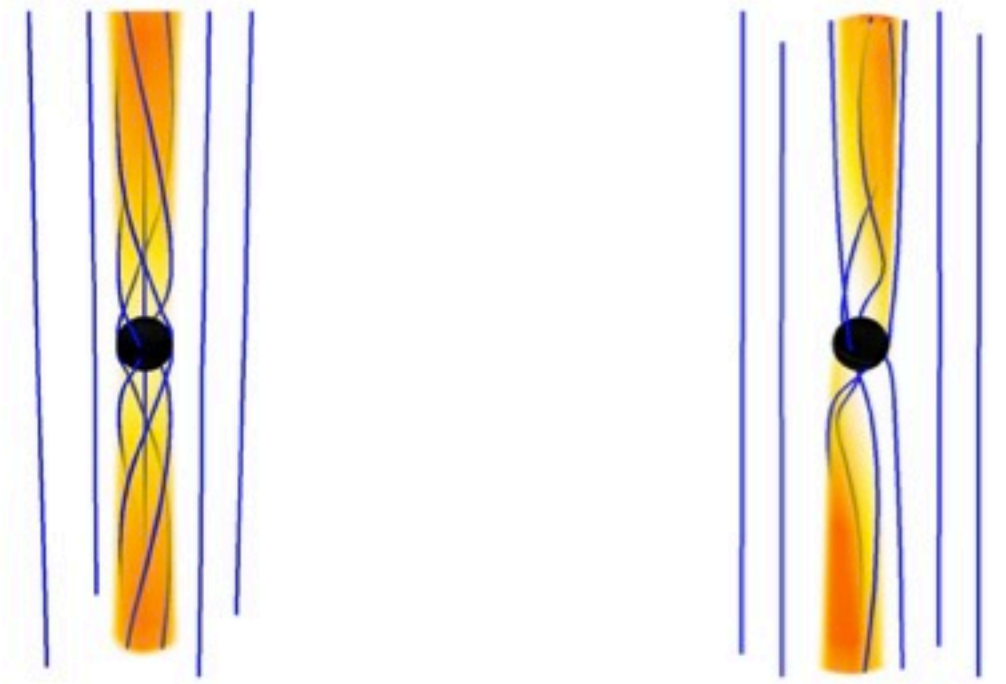
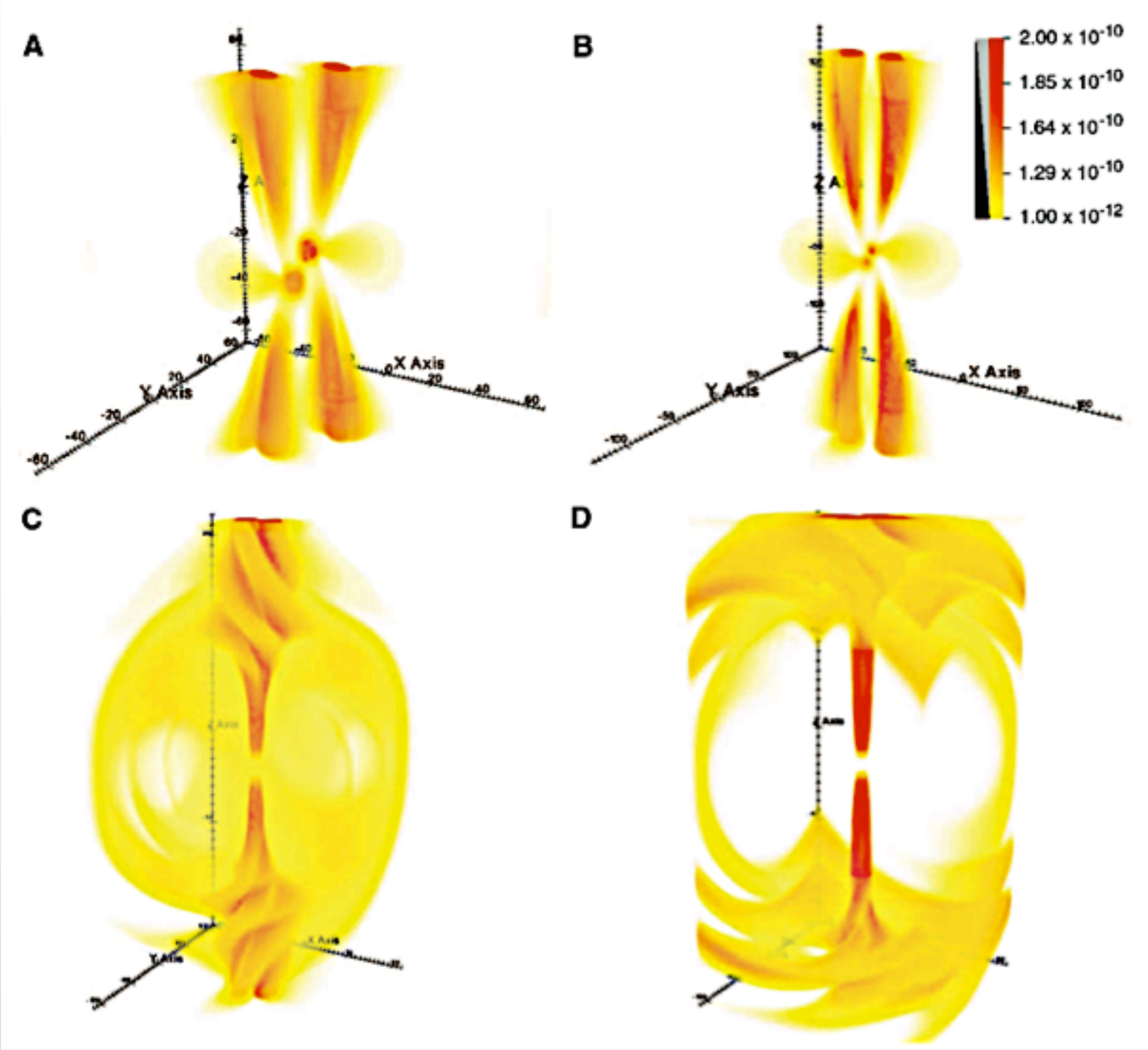
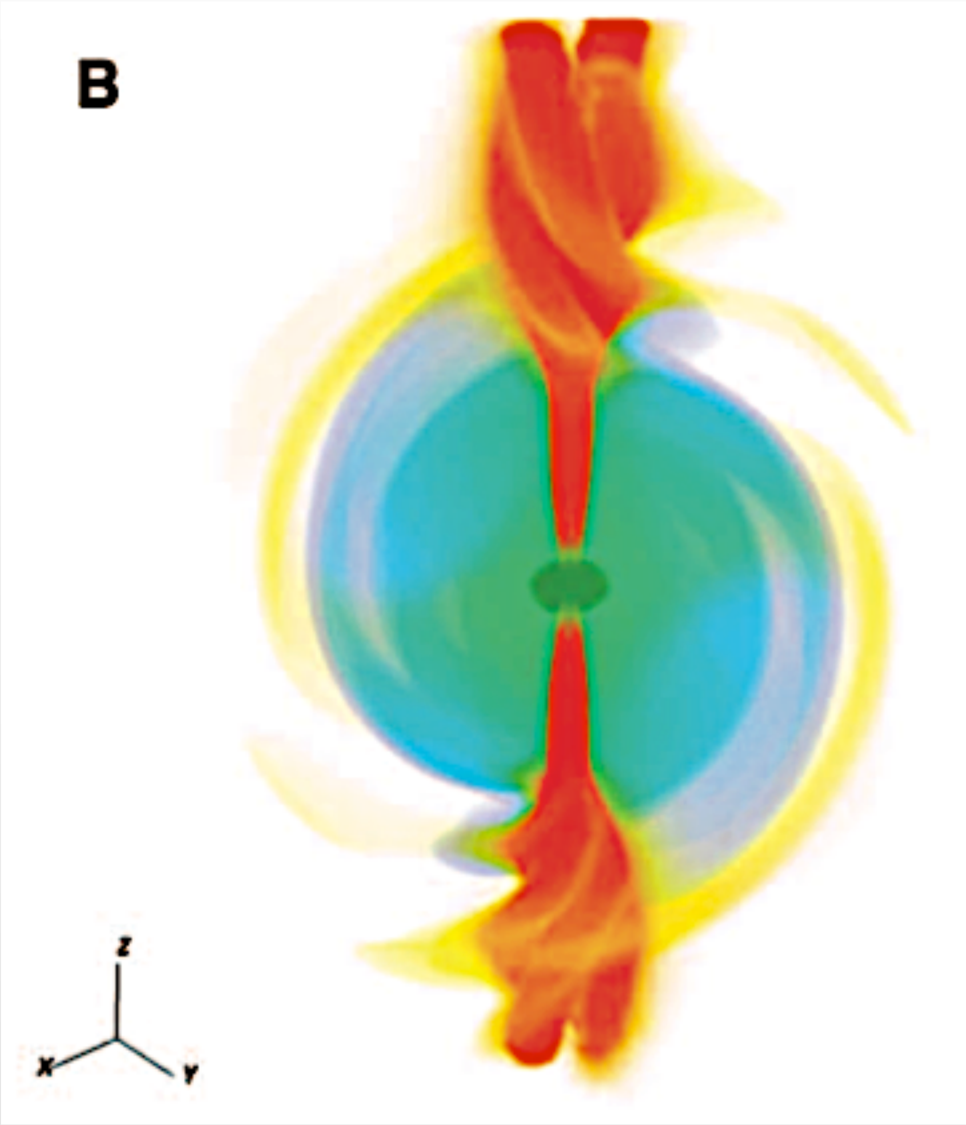


FIG. 7 (color online). Density of energy flux F_{EM} at $t = 100M$ for a single spinning black hole with $a = 0.99$, together with the magnetic field lines. *Left:* An aligned black hole ($\theta_o = 0$). *Right:* A misaligned black hole with $\theta_o = \pi/4$.

Palenzuela, Garrett, Lehner, Liebling 10

BH-BH in force-free plasma





Palenzuela, Lehner, Liebling 10

BH–NS & NS–NS

Much harder than BH-BH

* BH-BH: Completely solve the problem for GW detectors

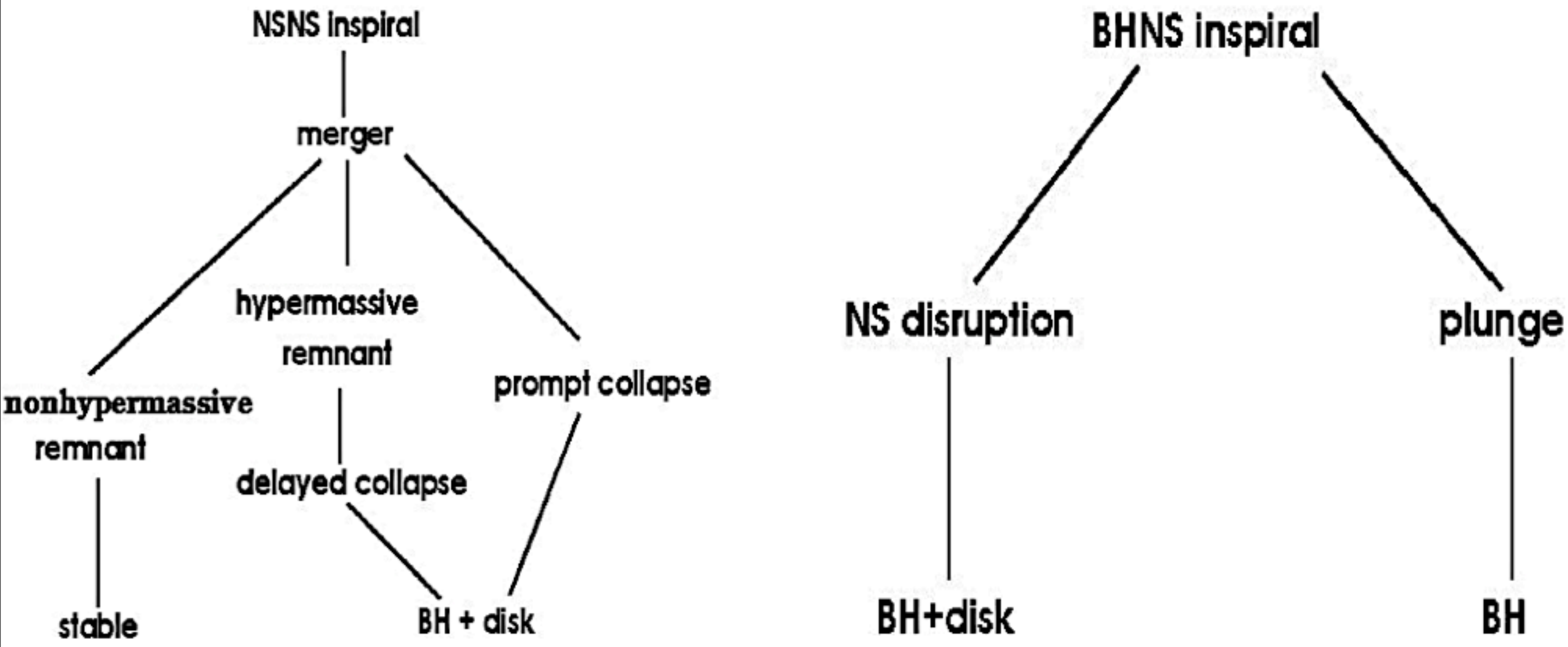
* BH-NS, NS-NS: investigate qualitative features

BH-NS & NS-NS: Goals

- ❖ Gravitational Waves
  - Long inspirals, accurate bulk evolution, accurate metric & GW extraction
- ❖ Test short GRB viability
  - Need massive disk, baryon poor region
  - Need MHD and/or neutrio radiation for energy extraction
- ❖ Ejecta effects (r-process elements, afterglow emission)
  - Need to track outflows with reasonable accuracy

Courtesy Matt Duez

Basic stages



NSNS inspiral

merger

nonhypermassive remnant

hypermassive remnant

prompt collapse

delayed collapse

stable

BH + disk

BHNS inspiral

NS disruption

plunge

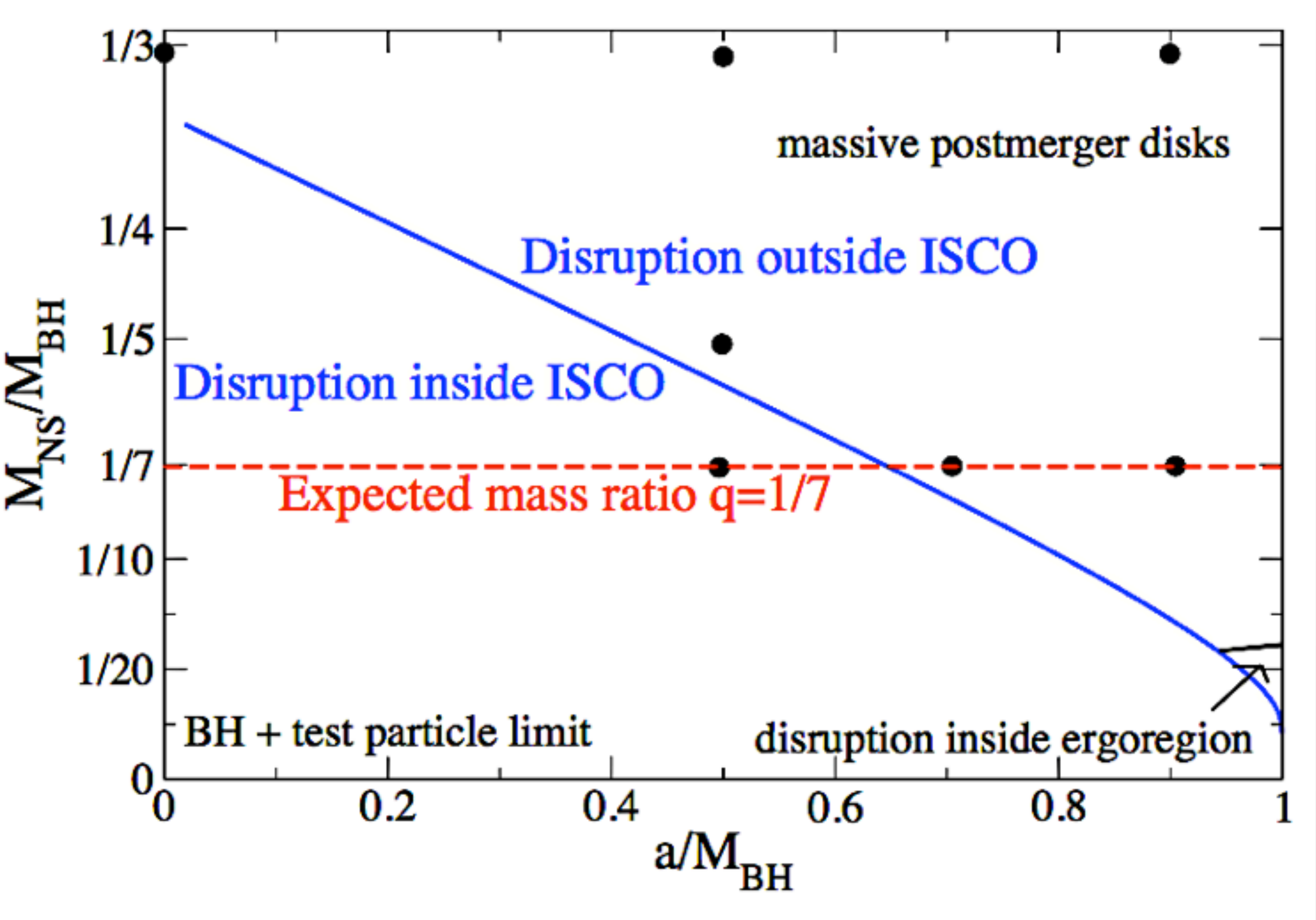
BH+disk

BH

Courtesy Matt Duez

BH-NS: disruption vs. direct plunge

- ❖ Toy-model w/ tidal effects (courtesy Matt Duez)



BH-NS

❖ Early fully GR simulations

- Shibata et al 06,07,08: Polytropic EOS
- Etienne et al 08,09: Polytropic EOS, aligned spin BH
- Duez et al 08: Polytropic EOS

❖ Current activity:

- Higher mass-ratios
- Effect of BH-spin (aligned, and non-aligned)
- Magnetic fields
- Neutrino cooling
- Realistic EOS

❖ Different groups chose different order of attack

BH-NS: some recent activities

- Kyotuko, Shipara, Taniguchi 10
  - Piecewise polytropic EOS's, zero spin, q=2,3
- Kiuchi et al
  - Study of compactness, mass-ratio, BH spin

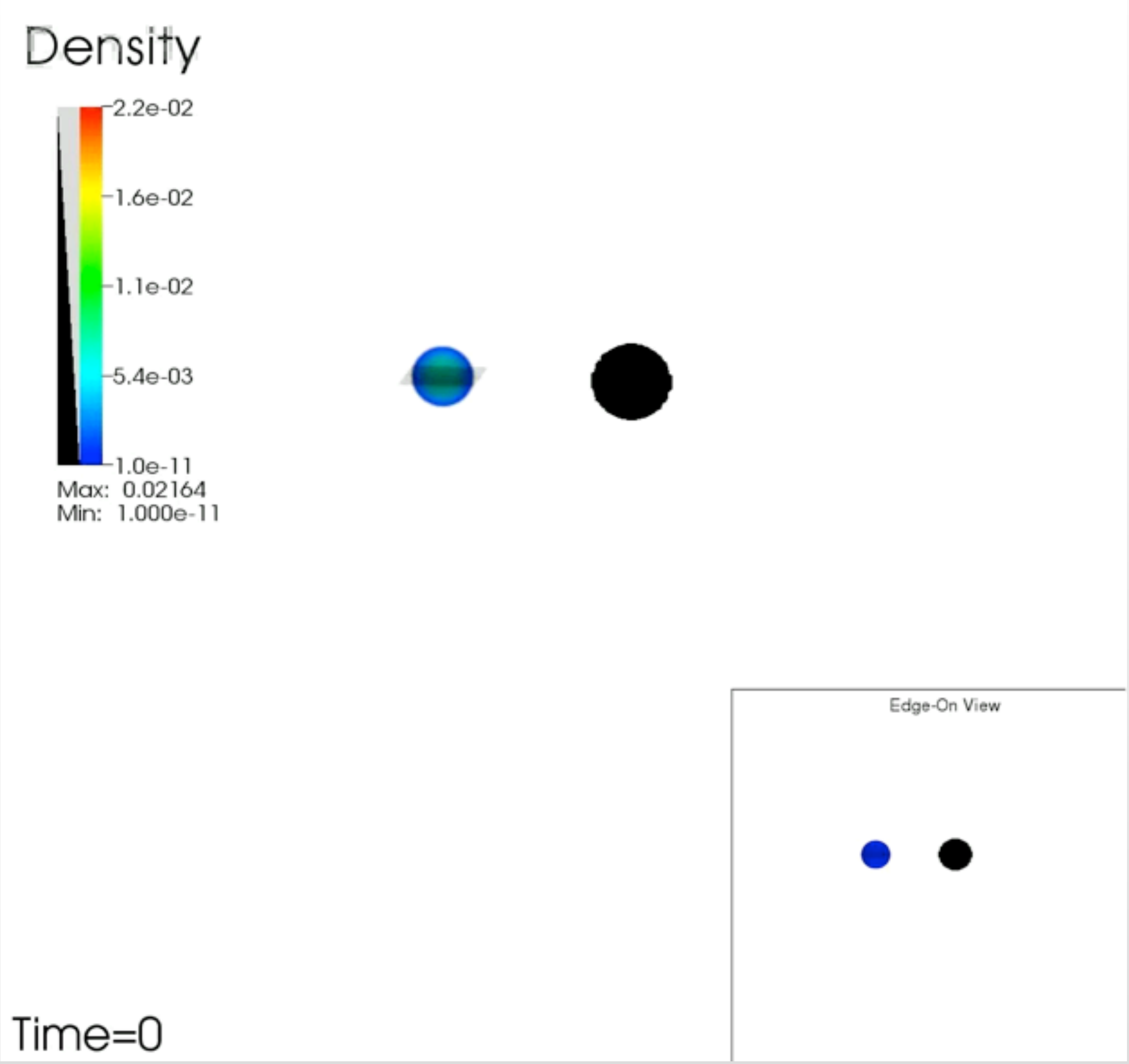
- Chawla et al, 10
  - BH spin 0.5, q=5, MHD

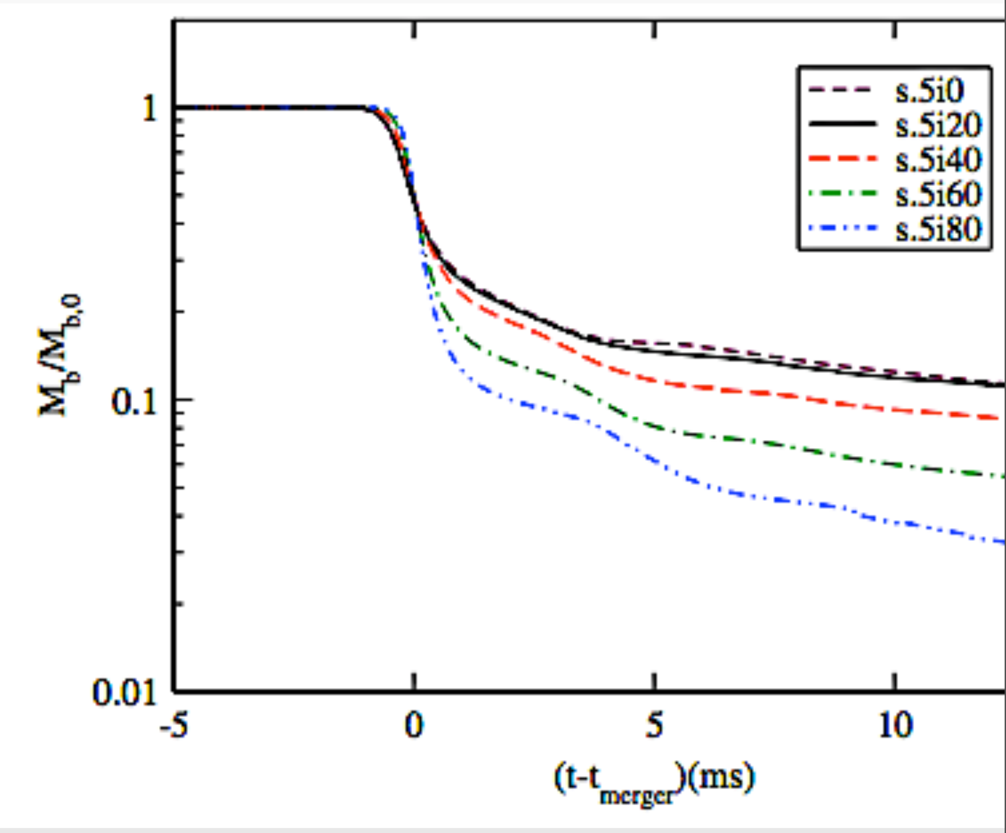
- Geleazzi et al
  - q=5, MHD

- Duez, et al 10
  - Polytropic & Shen EOS's, spin 0.5, q=3
- Foucart et al 11
  - BH spin 0 ... 0.9, vary angle, q=3

Precessing BH-NS

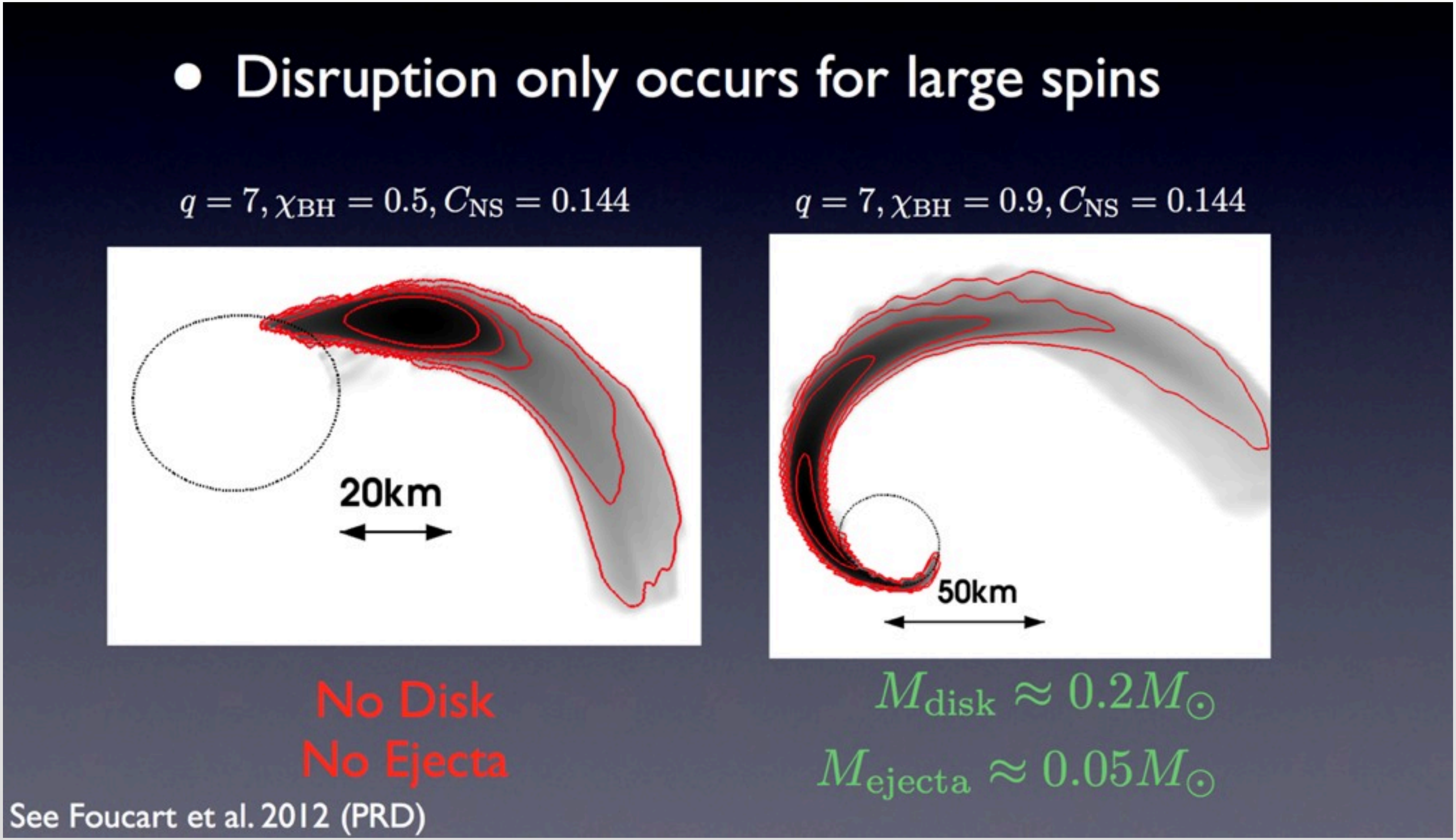
Foucart et al, 11





- ❖ Tidal tails precess
- ❖ Mdisk depends on spin orientation

High mass-ratio (q=7, as expected)



- Disruption only occurs for large spins

$q = 7, \chi_{\rm BH} = 0.5, C_{\rm NS} = 0.144$

$q = 7, \chi_{\rm BH} = 0.9, C_{\rm NS} = 0.144$

No Disk
No Ejecta

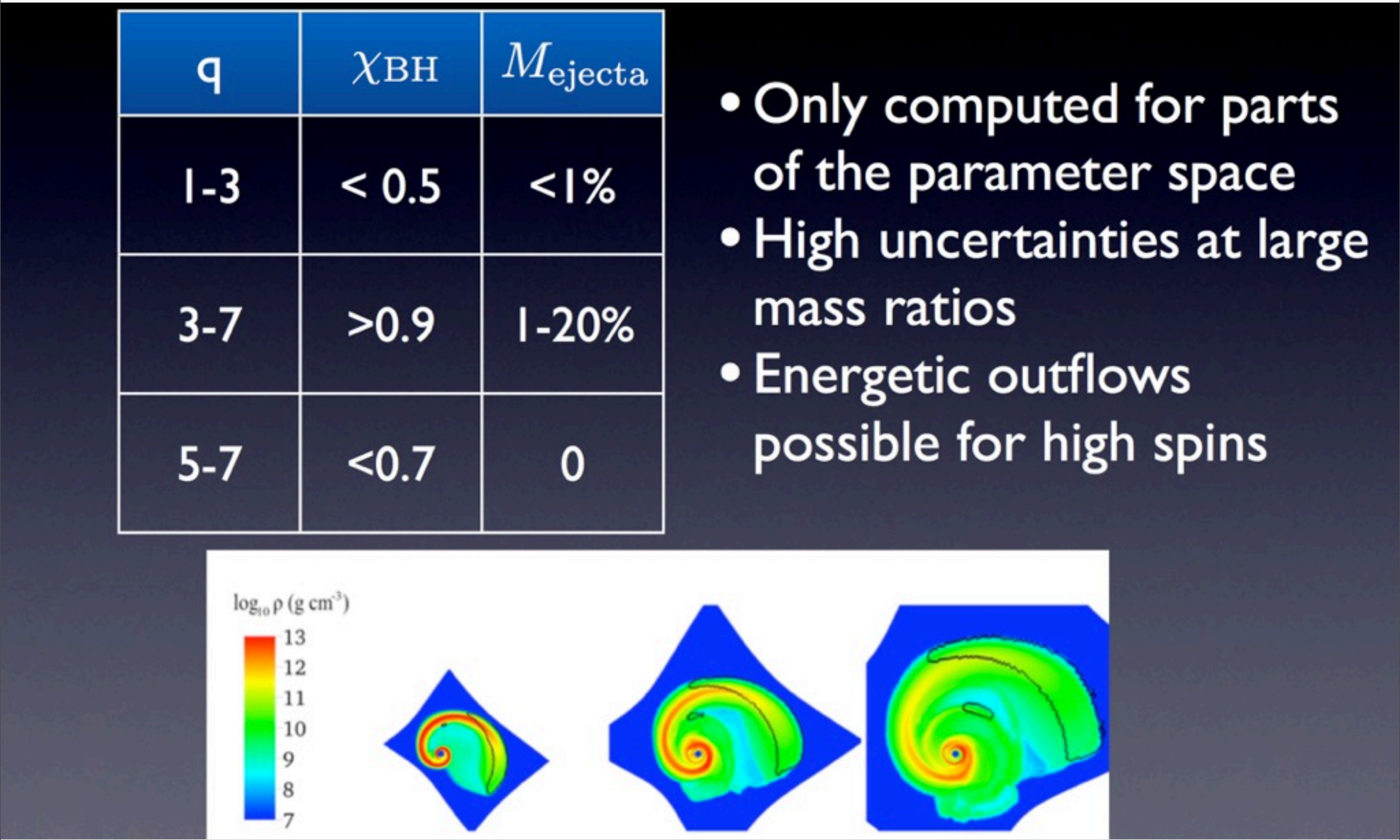
$M_{\rm disk} \approx 0.2 M_\odot$

$M_{\rm ejecta} \approx 0.05 M_\odot$

See Foucart et al. 2012 (PRD)

Slide courtesy Francois Foucart

Outflows: Important for counterparts



q	χ_{BH}	M_{ejecta}
1-3	< 0.5	<1%
3-7	>0.9	1-20%
5-7	<0.7	0

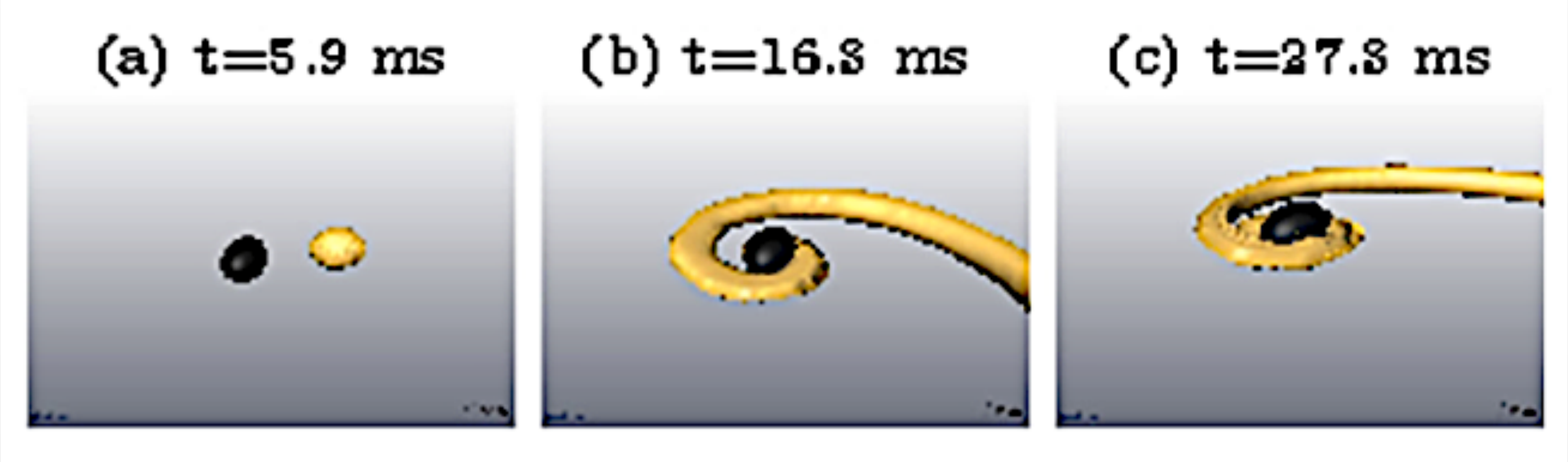
- Only computed for parts of the parameter space
- High uncertainties at large mass ratios
- Energetic outflows possible for high spins

Slide courtesy Francois Foucart

Magnetic BH-NS

Chawla et al 11

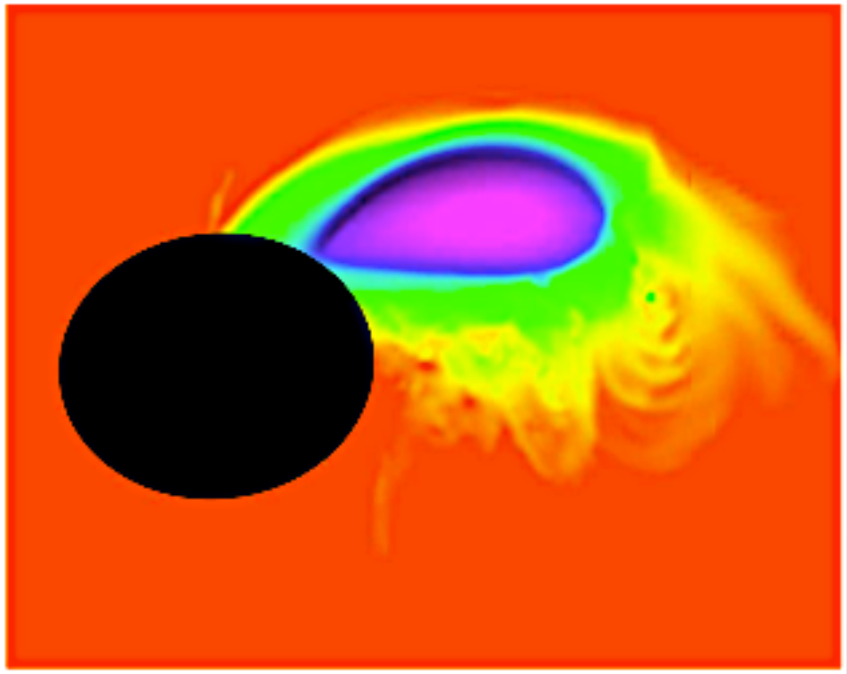
- q=5, spin 0.5, B=0 and B=10^{12}G
  - Essentially no unbound material
  - Magnetic field irrelevant



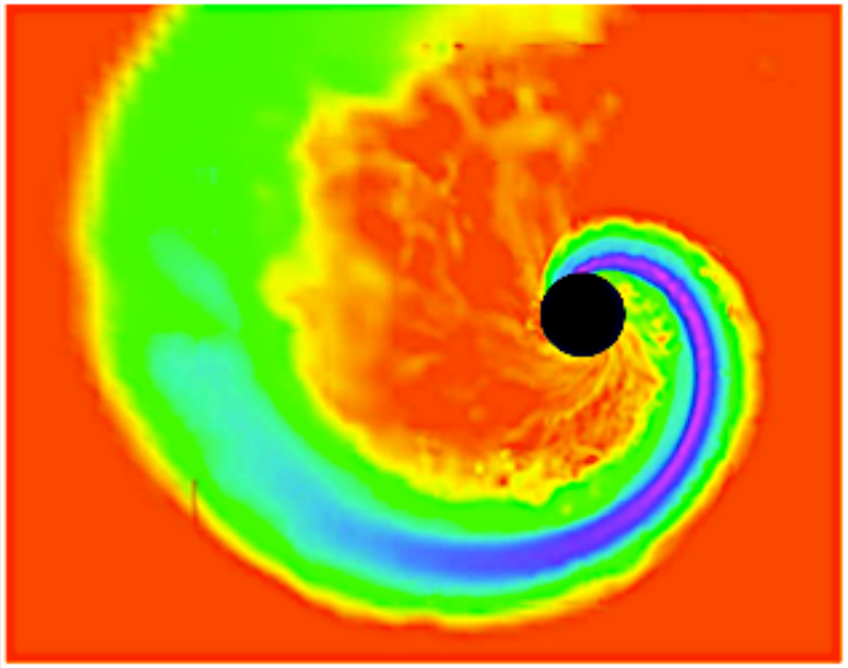
Hyperboloidal BH-NS encounters

Stephens, East, Pretorius 11

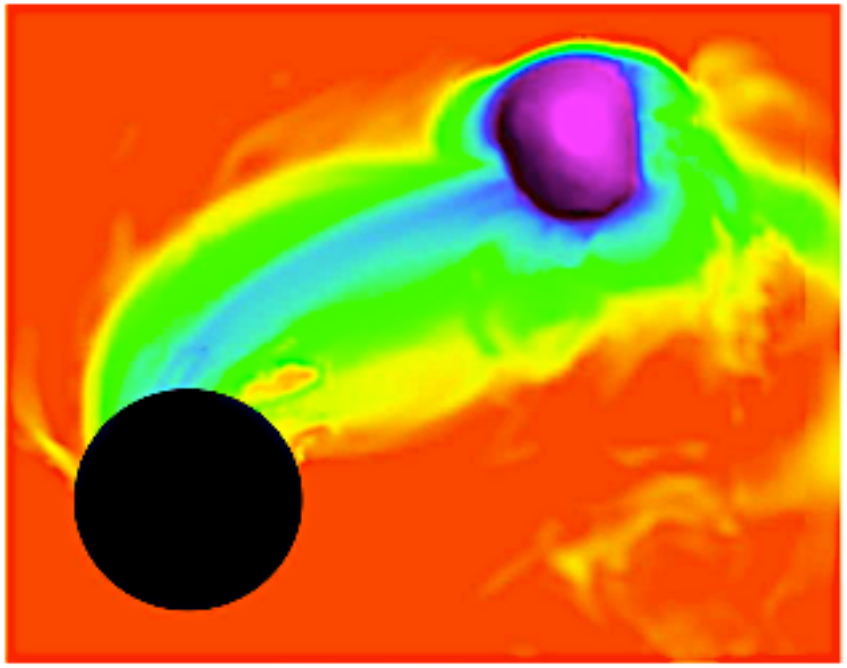
- ❖ Vary impact parameter p



Small p
direct coalescence



Larger p
Tidal disruption & tail



Yet larger p
periodic mass-transfer

- ❖ Large Mdisk even for a=0
- ❖ Unbounded ejecta, zoom-whirl behavior

Some recent NS-NS simulations

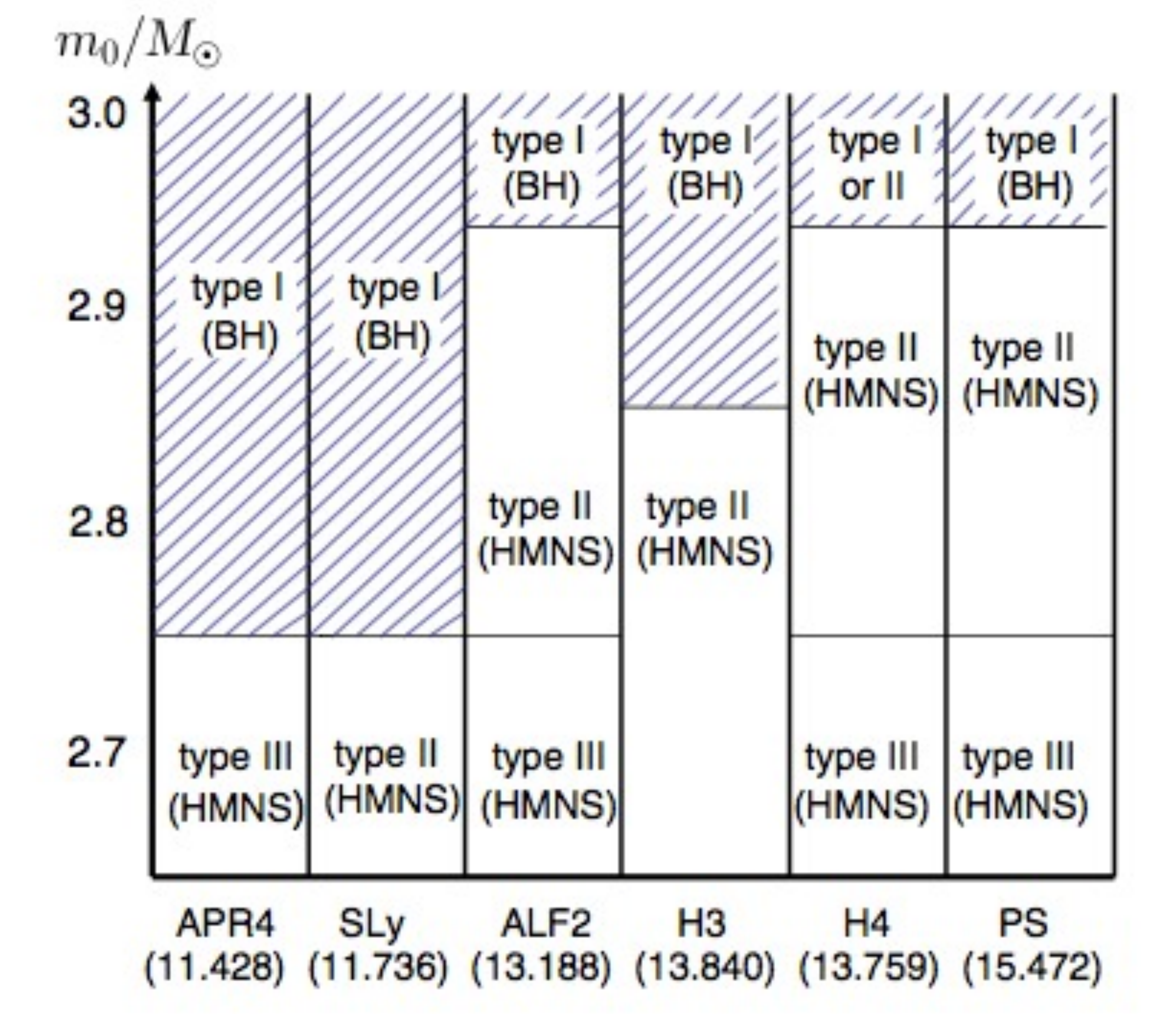
- Baiotti, Shibata, Yamamoto 10
  - Comparison SACRA vs. Whisky
- Baiotti et al 1103.3874
  - 20GW cycles & comparison to EOB
- Giacomazzo et al 11, Rezzolla et al 11
  - MHD, long run-time after merger

- Hotokezaka et al 11
  - 6 piecewise polytropic EOS
- Segikuchi et al 10
  - finite-temperature Shen EOS, neutrio cooling

Dependence on EOS

- Hotokezaka et al 11
  - 6 EOS's from Read et al 09
  - different masses for 16 simulations

- Different merger outcome
  - Type 1: Prompt BH formation
  - Type 2: short-lived HMNS
  - Type 3: long-lived HMNS

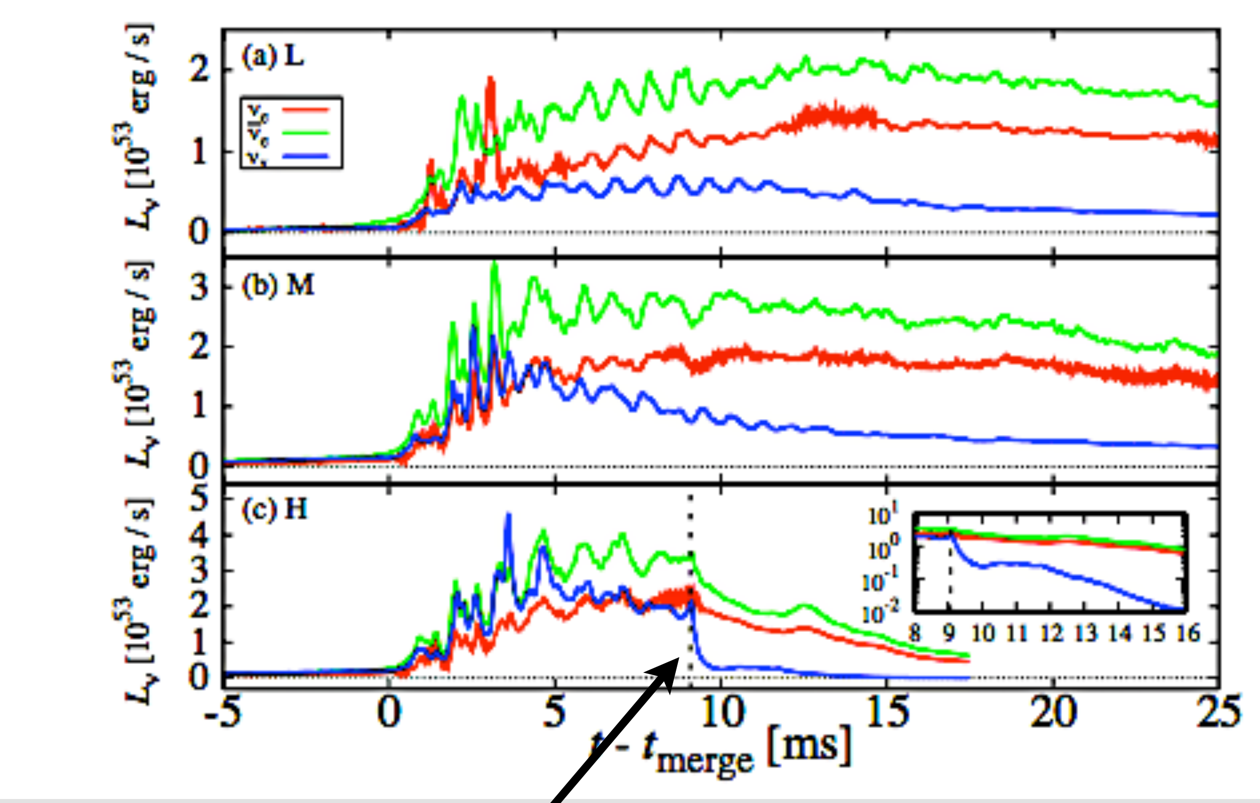


NS-NS w/ neutrino cooling

Sekiguchi ea
1105.2125

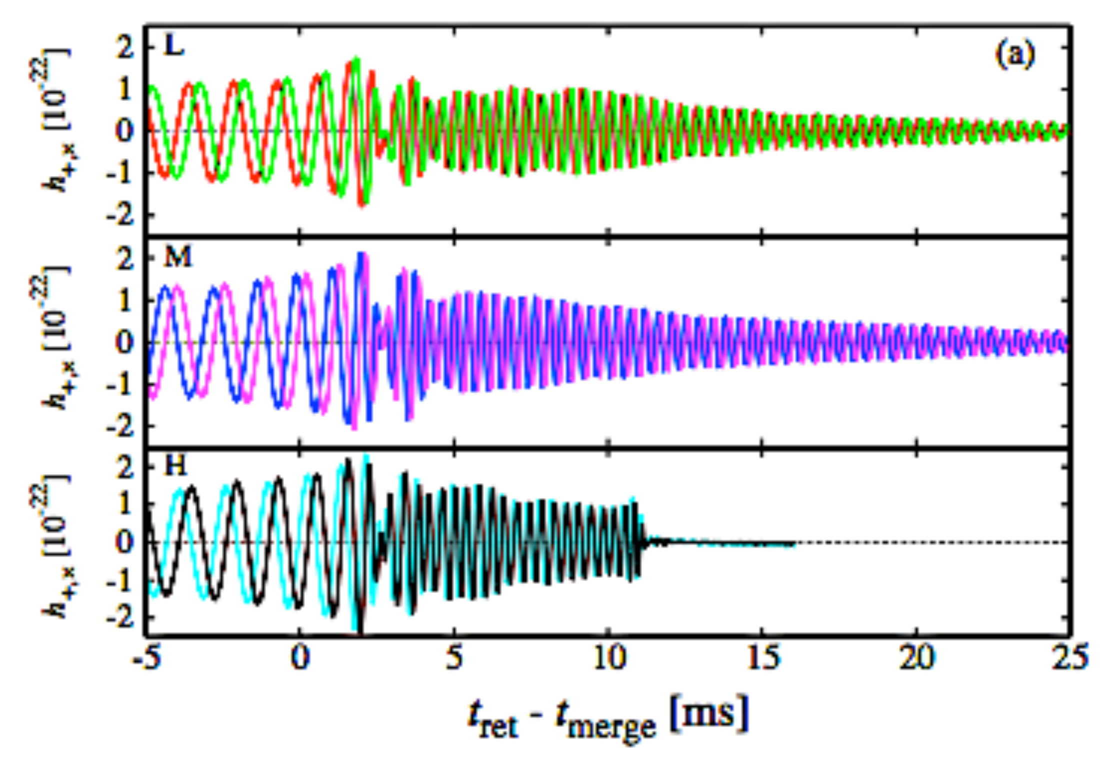
- Shen EoS, neutrino cooling
  - Equal mass NS-NS, M=1.45,1.5, 1.6Msun

Neutrino luminosity (three flavours)



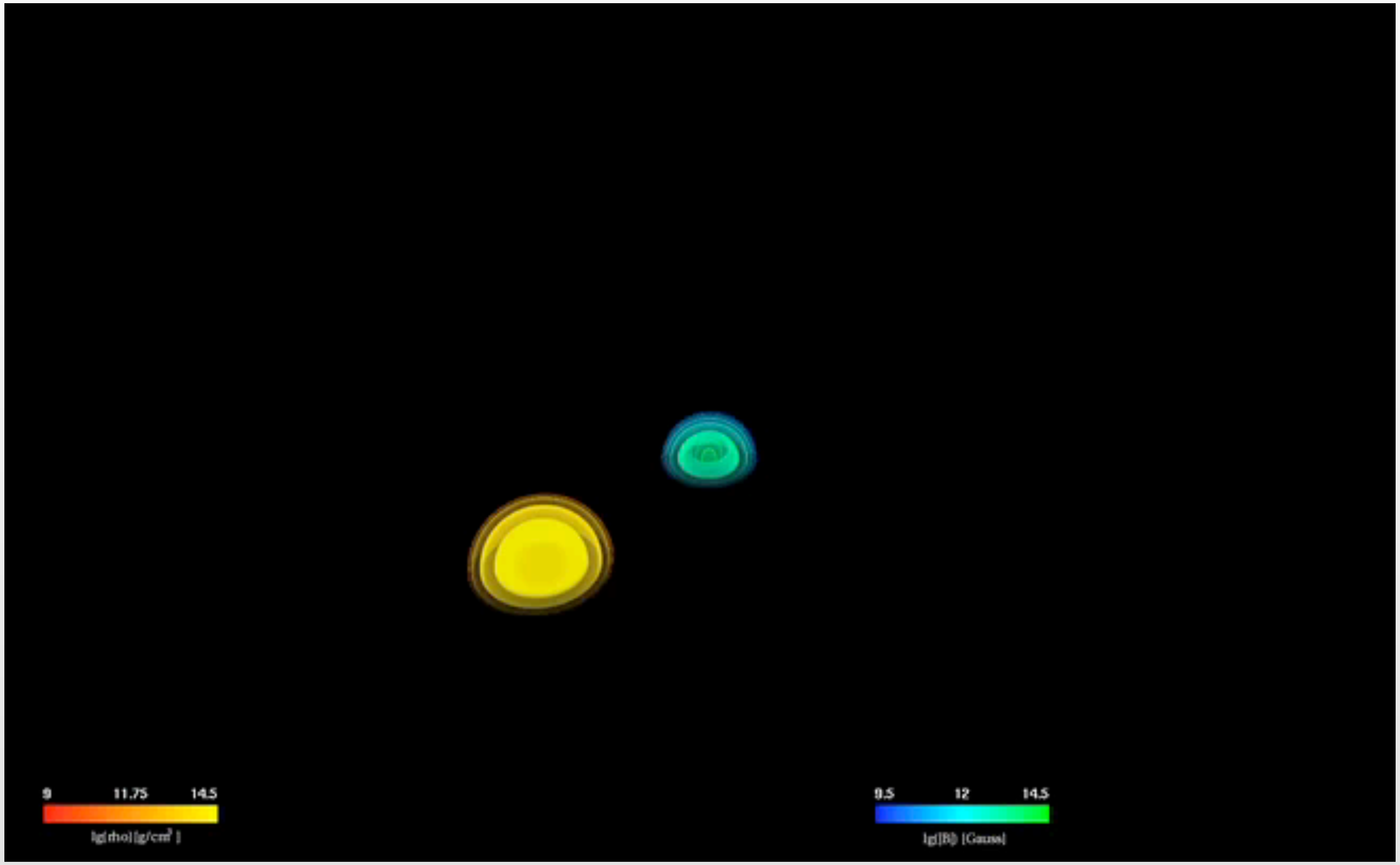
BH
formation

GW Waveforms



NS-NS simulation shows jets

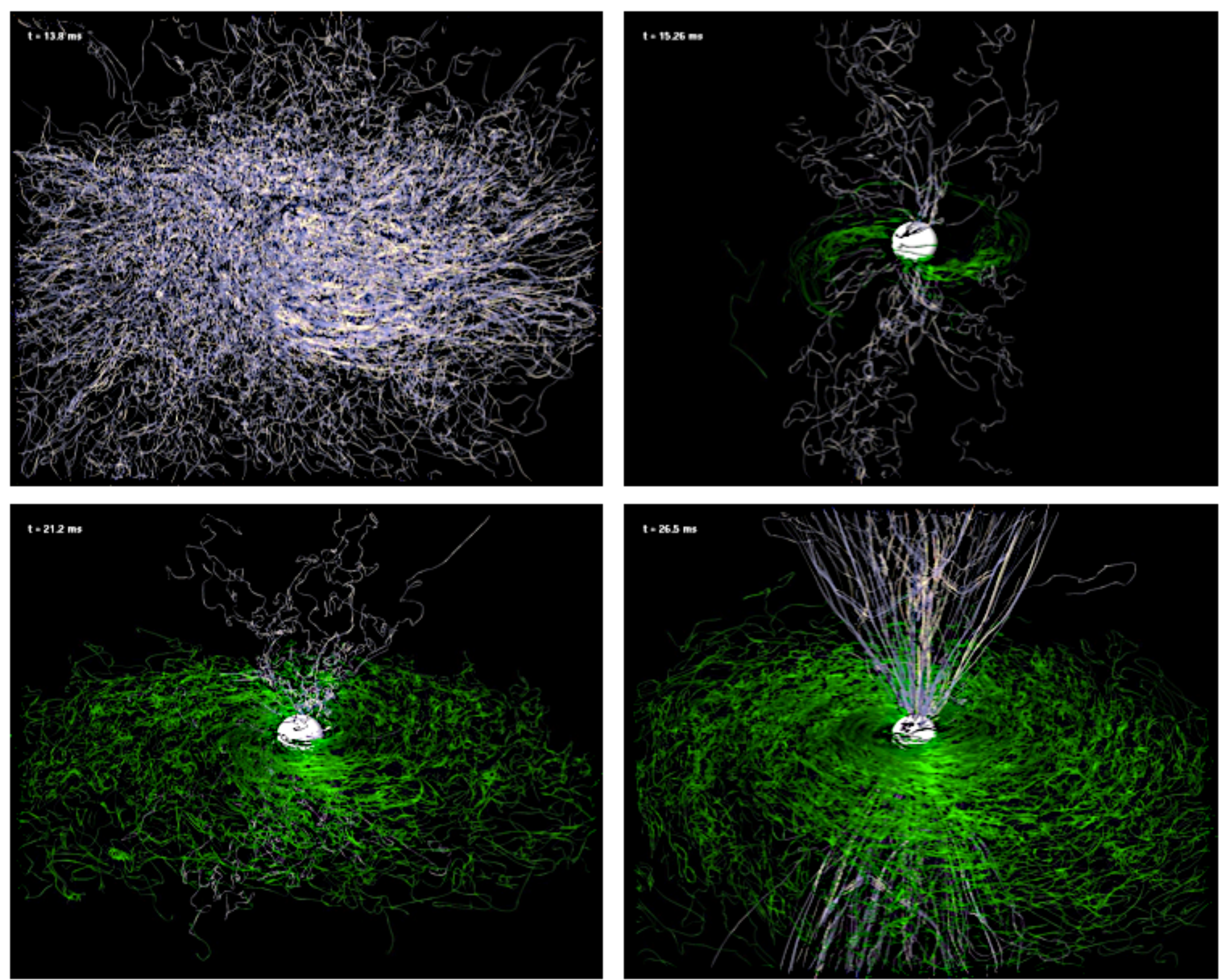
- ❖ GR+MHD+ideal fluid+long evolution (~30ms)



- ❖ B-fields amplified and ordered
- ❖ Baryon-poor funnel

Giacomazzo ea 10, Rezzolla ea 11

NS-NS simulation shows jets



Rezzolla ea 11

NS-NS in scalar-tensor theories

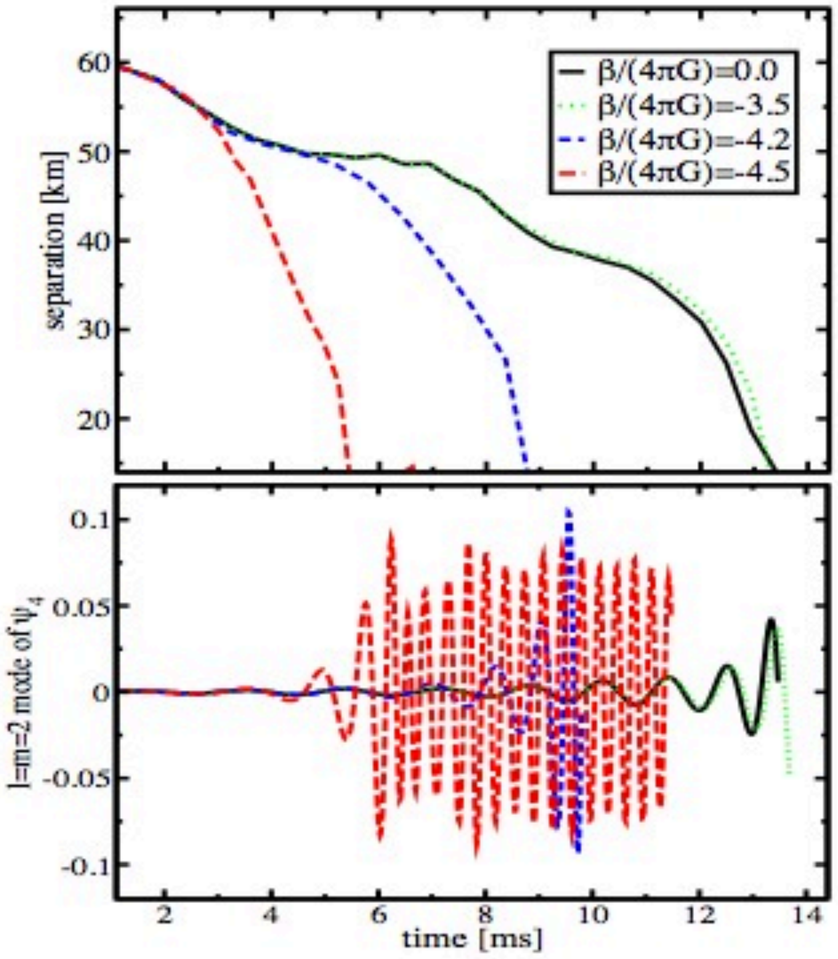


FIG. 1: The separation and the dominant mode of the ψ_4 scalar (encoding the effect of GWs) for a binary with gravitational masses $\{1.58, 1.67\}M_\odot$, and for different values of β.

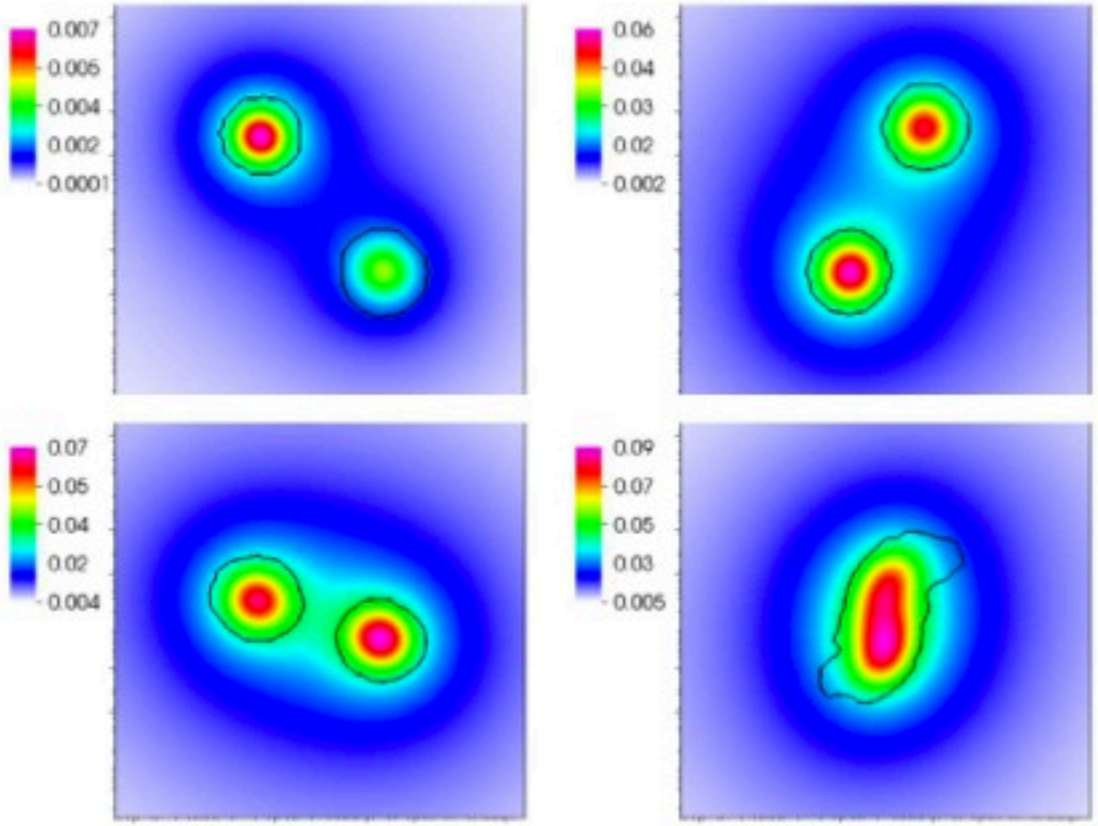
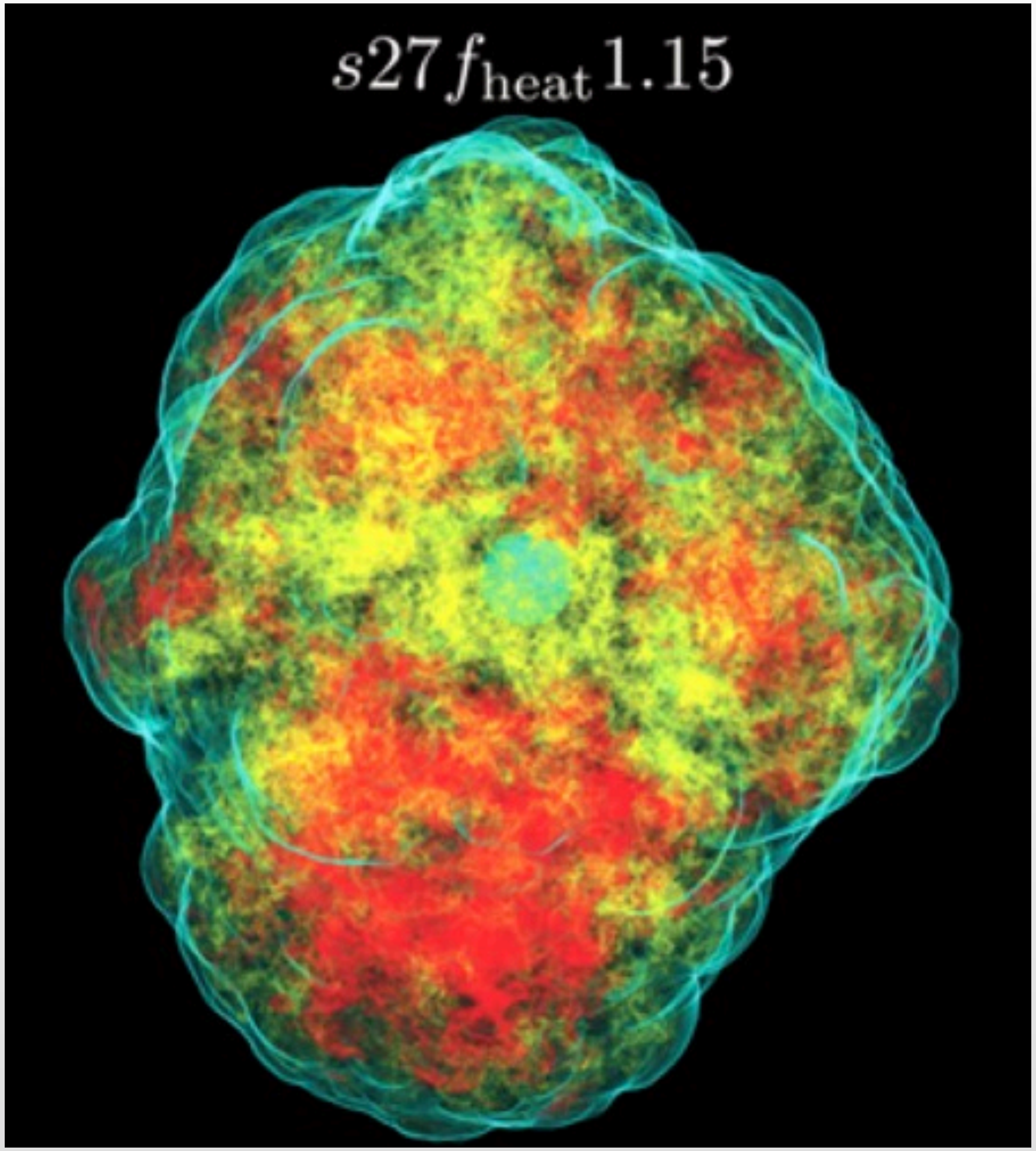


FIG. 2: The scalar field $\varphi G^{1/2}$ (color code) and the NS surfaces (solid black line) at $t = \{1.8, 3.1, 4.0, 5.3\}$ms for $\beta/(4\pi G) = -4.5$, and the binary of Fig. 1.

Barausse et al, arXiv/1212.5053

Beyond compact object binaries

- ❖ Supernova simulations
  - Require more micro-physics
  - Yet higher computational cost



Ott ea, 2012

Future challenges (BH-NS, NS-NS)

- Accuracy
  - Hydro converges <=3rd order (shocks 1st order)
  - Realistic EOS has *discontinuities* which are hard to model
- Neutrino transport
  - leakage schemes and beyond
- Magnetic fields
  - magnetospheres are *not* MHD
  - magneto-rotational instability excites small scales
- BH-NS parameter space

The future

Likely themes for the coming years

- BH-BH
  - precessing waveform models
  - targeted simulations, *in response* to LIGO detection
  - eccentric systems
  - alternative gravity; higher dimensional gravity
  - can GW observations distinguish between GR and alternative theories?

- Matter simulations
  - Tremendous amount of work needed for micro-physics
  - Parameter studies with
    - increasing amount of micro-physics
    - increasing parameter space coverage
  - Understand jets and their EM, nu signatures
  - Understand ejecta and their EM, nu signatures
  - What NS properties can GW detectors measure?