Bethe Ansatz in non-equilibrium statistical mechanics: techniques and applications

The aim of these lectures is to present some exact solutions in non-equilibrium statistical physics, by using the Bethe Ansatz, a method originally developed by Hans Bethe in 1931 to solve quantum mechanical problems. The Bethe Ansatz has become a seminal tool to derive exact results in theoretical physics, ranging from solid-state to high energy physics. We shall explain how it can be applied fruitfully to systems far from equilibrium, allowing us to calculate large deviation functions, that play a role similar to the thermodynamic free energy.

The outline of the lectures will be as follows:

1. General overview of equilibrium versus non-equilibrium statistical mechanics. The asymmetric exclusion process (ASEP) and the Matrix Representation Ansatz.

2. Coordinate Bethe Ansatz: Calculation of the gap of the ASEP.

3. Functional Bethe Ansatz: Exact results for large deviation functions.

4. ASEP and Vertex Models: Introduction to the Algebraic Bethe Ansatz.

5. Infinite systems and determinantal processes: current fluctuations and Tracy-Widom statistics.

6. Macroscopic Fluctuation Theory: a search for classical integrability?

Here are some general references, following the plan given above, (more precise references will be given during the lectures):

P. L. Krapivsky, S. Redner and E. Ben-Naim, 2010, *A Kinetic View of Statistical Physics* (Cambridge: Cambridge University Press).

R. J. Baxter, 1990, *Exactly solved models in statistical mechanics*, (Academic Press).

K. Mallick, *Some recent developments in non-equilibrium statistical mechanics*, PRAMANA **73**, 417 (2009).

B. Derrida, M. R. Evans, V. Hakim and V. Pasquier, *Exact solution of a 1D asymmetric exclusion model using a matrix formulation*, J. Phys. A: Math. Gen. **26**, 1493 (1993).

B. Derrida, *An exactly soluble non-equilibrium system: the asymmetric simple exclusion process*, Phys. Rep. **301**, 65-83 (1998).

B. Derrida, *Nonequilibrium steady states: fluctuations and large deviations of the density and of the current*, J. Stat. Mech: Theor. Exp P07023 (2007).

R. Blythe and M. R. Evans, *Nonequilibrium steady states of matrix product form: a solver's guide*, J. Phys. A: Math. Gen. **40**, R333 (2007).

T. Chou, K. Mallick and R. K. P. Zia, *Nonequilibrium statistical mechanics: from a paradigmatic model to biological transport*, Rep. Prog. Phys. **74**, 116601 (2011).

O. Golinelli and K. Mallick, *the asymmetric simple exclusion process: an integrable model for non-equilibrium statistical mechanics*, J. Phys. A: Math. Gen. **39**, 12679 (2006).

S. Prolhac, *Tree structures for the current fluctuation in the exclusion process*, J. Phys. A: Math. Gen. **43**, 105002 (2008).

T. Sasamoto, *Fluctuations of the one-dimensional asymmetric exclusion process using random matrix techniques*, J. Stat. Mech.: Theor. Exp. P07007 (2007).