Gravitational Waves from Inspiraling Compact Binaries

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State of the art

- What is the state of the art in the analytical modeling for all classes of compact binaries?
- Brief Update on Talk at "Science from the First GW detections", South Padre Island, May 2013. Slides 13-15 here.

Approaches

- Multipolar Post Minkowskian (MPM) formalism
- Hamiltonian (ADM)
- Direct Integration of Relaxed Einstein Eqns - DIRE
- Strong field point particle limit
- Effective field theory techniques
- Self-force approaches for EMRI's...
- Numerical Relativity-Analytical Relativity comparison
- Self force-PN comparisons....

Ingredients

Successful wave-generation formalisms are a cocktail of

- Post-Minkowskian (PM) methods
  Expansions in G - non-linearity expns,
- Post-Newtonian (PN) methods
  Expansions in $1/c$,
- Multipole (M) expansions
  Expansions in irreducible representations of the rotation group,
- Perturbations around Curved Backgrounds.

GW from ICB

1. Garden variety ICB would have radiated away their eccentricity and be moving in quasi-circular orbits during the late inspiral
2. Since matched filtering is sensitive to the phase it is more important to first control higher order phasing than higher order amplitudes - Newtonian Amplitude + Best available phasing: Restricted waveform
3. The inspiral can be treated in the adiabatic approximation as a sequence of circular orbits..This allows one to treat separately the radiation reaction effects and the conservative effects
4. One can go to higher PN orders in the inspiral or radiation reaction without getting technically bogged down in controlling the much more difficult higher order conservative PN terms
5. For compact objects the effects of finite size and quadrupole distortion induced by tidal interactions are of order 5PN. Hence, neutron stars and black holes can be modelled as point particles represented by Dirac δ-functions (+ Self-field regularization).

GW from ICB- Three Modules

Thus modelling ICB waveforms for inspiral involves three tasks

1. Motion: Given a Binary system, iterate Einstein's Eqns to discuss conservative motion of the system. Compute CM Energy E & AM J
2. Generation: Given the motion of the binary system on a fixed orbit, iterate EE to compute multipoles of the Grav field and hence the FZ flux of Energy and AM carried by GW. Compute $\mathcal{L}$ and $\mathcal{J}$
3. Radiation Reaction: Given the Conserved energy & AM and Radiated Flux of Energy and AM, *ASSUME* the Balance Eqns to Compute the effect of Radiation on the Orbit. Compute $F(t)$, $\phi(t)$, $r(t)$;

Adiabatic Phasing - Circular orbit, Leading order; $x \sim \frac{1}{c^2}$

$$M \equiv m_1 + m_2;\ \mu \equiv \frac{m_1 m_2}{M};\ \nu \equiv \mu M;\ x \equiv (\pi G M F/c^3)^{2/3};\ \frac{c^5}{G} \approx 3.63 \times 10^{52}\ \mathrm{W},$$

$$E(x) = -\frac{1}{2}\mu c^2 x;$$

$$\mathcal{L}(x) = \frac{32}{5}\ \frac{c^5}{G}\ \nu^2 x^5;$$

and heuristic Energy Balance equation

$$\frac{dE}{dt} = -\mathcal{L};\ \rightarrow \frac{dx}{dt} = -\frac{\mathcal{L}(x)}{E'(x)}$$

$$x(t) = \frac{1}{4}\tau^{-1/4};\quad \tau = \frac{c^3\nu}{5Gm}(t_c - t)$$

$$\phi = \int \omega dt = -\frac{5}{\nu}\int x^{2/3} d\tau;$$

$$\phi_{\mathrm{c}} - \phi(t) = \frac{1}{\nu}\tau^{5/8},$$

Basic Inputs - 3PN Energy and 3.5PN Energy Flux, $x \sim \frac{1}{c^2}$

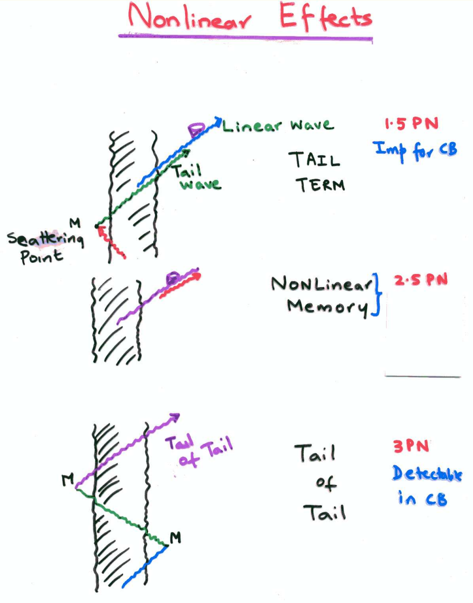
State of the art restricted GW phasing for ICB

$$
\begin{aligned}
E_3(x) &= -\frac{1}{2}\nu x \left[1 - \left(\frac{3}{4} + \frac{1}{12}\nu\right) x - \left(\frac{27}{8} - \frac{19}{8}\nu + \frac{1}{24}\nu^2\right) x^2 \right. \\
&\quad \left. - \left\{\frac{675}{64} - \left(\frac{34445}{576} - \frac{205}{96}\pi^2\right)\nu + \frac{155}{96}\nu^2 + \frac{35}{5184}\nu^3\right\} x^3 \right], \\
\mathcal{L}_{3.5}(x) &= \frac{32c^5}{5G} x^5 \nu^2 \left\{1 + \left(-\frac{1247}{336} - \frac{35}{12}\nu\right) x + 4\pi x^{3/2} \right. \\
&+ \left(-\frac{44711}{9072} + \frac{9271}{504}\nu + \frac{65}{18}\nu^2\right) x^2 + \left(-\frac{8191}{672} - \frac{535}{24}\nu\right) \pi x^{5/2} \\
&+ \left(\frac{6643739519}{69854400} + \frac{16\pi^2}{3} - \frac{1712}{105} C - \frac{856}{105} \ln(16x) \right. \\
&+ \left. \left[\frac{41\pi^2}{48} - \frac{134543}{7776}\right] \nu - \frac{94403}{3024}\nu^2 - \frac{775}{324}\nu^3 \right) x^3 \\
&+ \left. \left(-\frac{16285}{504} + \frac{176419}{1512}\nu + \frac{19897}{378}\nu^2\right) \pi x^{7/2} + \mathcal{O}(x^4) \right\}
\end{aligned}
$$

Higher order Phasing is equivalent to inclusion of higher order Gravitational Radiation Reaction (GRR).

3.5PN (v^7/c^7) beyond leading RR at 2.5PN (v^5/c^5)

3PN GW Flux includes..



Post Newtonian Approximants - Different Families

- PNA computes orbital phase $\phi(t)$ of a CB as perturbative expn in a small parameter, $v = (\pi M F)^{1/3}$ (characteristic velocity in the binary), or $x = v^2$, although other variants exist.
- In the adiabatic approximation and for restricted WF (GW phase twice orbital phase) phasing specified by a pair of differential eqns

$$\begin{aligned} \frac{d\phi}{dt} - \frac{v^3}{M} &= 0, \\ \frac{dv}{dt} + \frac{\mathcal{F}(v)}{M E'(v)} &= 0, \end{aligned}$$

$\mathcal{F}(v)$: GW Flux; $E(v)$: Binding energy; Prime: deriv wrt v

- Different PN families arise because one can choose to treat the ratio $\mathcal{F}(v)/E'(v)$ differently starting from the same PN order inputs.

Post Newtonian Approximants - Different Families

- TaylorT1: Retain PN expansions of the luminosity $\mathcal{F}(v)$ and $E'(v)$ as they appear and solve DE numerically
- TaylorT4: Expand rational polynomial $\mathcal{F}(v)/E'(v)$ in v to consistent PN order and solve DE
- TaylorT2: Follow the earlier expansion and integrate to obtain a pair of parametric equations $\phi(v)$ and $t(v)$
- TaylorT3: Invert the above $t(v)$ to get $v(t)$ and Write phasing as explicit function of time $\phi(v(t)) = \phi(t)$
- TaylorEt: Write the series in terms of Energy variable E, suitably adimensionalized i.e. $\zeta = -2E/\nu$.
- TaylorF2: Fourier reprn computed using stationary phase approxmn

Beyond the non-spinning case

- Computation of Spin-Orbit and Spin Spin terms in EOM and GW Flux - Waveform Phase and Amplitude
- Interplay of analytical PN and (very accurate numerical ?) self-force facilitates computation of unknown PN terms and binding energy for circular orbits at all PN orders and linear in symmetric mass ratio..
- Tidal effects - 7PN conservative dynamics and EOM, 6PN energy flux, 6PN Waveform phase, 6PN waveform amplitude
- Test particle limit - 22PN energy flux for Schwarzschild and 4PN for the Kerr case
- Black hole horizon absorbed flux - 6PN (beyond leading) for Schwarzschild, 6.5PN for Kerr. For comparable masses 4PN for the non-spinning and spinning case.
- 2.5PN EOM in Scalar Tensor gravity; Phasing in other theories
- RR from balance equations

$$\vec{S}_i = G m_i^2 \xi_i \hat{\vec{S}}_i$$

$$\kappa_i = \hat{\vec{S}}_i . \vec{l}$$

$$\text{Maximal Spin} \quad \rightarrow \xi_i = 1$$

$$-1 \leq \kappa_i \leq 1$$

$$\vec{S}_1^{\mathrm{c}} = \left(1 + \frac{G m_2}{c^2 r}\right) \vec{S}_1 - \frac{m_2^2}{2 c^2 m^2} S_{1\lambda} r^2 \omega^2 \vec{\lambda}$$

$$\vec{S}_2^{\mathrm{c}} = \left(1 + \frac{G m_1}{c^2 r}\right) \vec{S}_2 - \frac{m_1^2}{2 c^2 m^2} S_{2\lambda} r^2 \omega^2 \vec{\lambda}$$

Spin

Table: Post-Newtonian contributions to the number of GW cycles (**??**) accumulated from $\omega_{\rm min} = \pi \times 10\,{\rm Hz}$ to $\omega_{\rm max} = \omega_{\rm ISCO} = 1/(6^{3/2}\, m)$ for binaries detectable by LIGO and Virgo. For comparison, we add the contributions of spin-spin terms at 2PN order (we denote $\xi^{\rm c} = \hat{\mathbf{S}}^{\rm c}_1 \cdot \hat{\mathbf{S}}^{\rm c}_2$) and non-spin terms at 3PN and 3.5PN orders.

	$(10 + 1.4) M_\odot$	$(10 + 10) M_\odot$	$(1.4 + 1.4) M_\odot$
N	3577	601	16034
1	$+213$	$+59.3$	$+441$
1.5	$-181 + 114\, \kappa_1^{\rm c}\, \chi_1^{\rm c} + 11.8\, \kappa_2^{\rm c}\, \chi_2^{\rm c}$	$-51.4 + 16.0\, \kappa_1^{\rm c}\, \chi_1^{\rm c} + 16.0\, \kappa_2^{\rm c}\, \chi_2^{\rm c}$	$-211 + 65.7\, \kappa_1^{\rm c}\, \chi_1^{\rm c} + 65.7\, \kappa_2^{\rm c}$
2	$+9.8 - 4.4\, \kappa_1^{\rm c}\, \kappa_2^{\rm c}\, \chi_1^{\rm c}\, \chi_2^{\rm c} + 1.5\, \xi^{\rm c}\, \chi_1^{\rm c}\, \chi_2^{\rm c}$	$+4.1 - 3.3\, \kappa_1^{\rm c}\, \kappa_2^{\rm c}\, \chi_1^{\rm c}\, \chi_2^{\rm c} + 1.1\, \xi^{\rm c}\, \chi_1^{\rm c}\, \chi_2^{\rm c}$	$+9.9 - 8.0\, \kappa_1^{\rm c}\, \kappa_2^{\rm c}\, \chi_1^{\rm c}\, \chi_2^{\rm c} + 2.$
2.5	$-20 + 33.9\, \kappa_1^{\rm c}\, \chi_1^{\rm c} + 2.9\, \kappa_2^{\rm c}\, \chi_2^{\rm c}$	$-7.1 + 5.7\, \kappa_1^{\rm c}\, \chi_1^{\rm c} + 5.7\, \kappa_2^{\rm c}\, \chi_2^{\rm c}$	$-11.7 + 9.3\, \kappa_1^{\rm c}\, \chi_1^{\rm c} + 9.3\, \kappa_2^{\rm c}$
3	$+2.3$	$+2.2$	$+2.6$
3.5	-1.8	-0.8	-0.9

Table: Spin-orbit contributions to the number of gravitational-wave cycles $\mathcal{N}_{\mathrm{GW}} = (\phi_{\mathrm{max}} - \phi_{\mathrm{min}})/\pi$ accumulated from $\omega_{\mathrm{min}} = \pi \times 10\,\mathrm{Hz}$ to $\omega_{\mathrm{max}} = \omega_{\mathrm{ISCO}} = c^3/(6^{3/2} G m)$ for binaries detectable by ground-based detectors LIGO and VIRGO. For each compact object we define the magnitude χ_a and the orientation κ_a of the spin by $\mathbf{S}_a \equiv G\, m_a^2\, \chi_a\, \hat{\mathbf{S}}_a$ and $\kappa_a \equiv \hat{\mathbf{S}}_a \cdot \boldsymbol{\ell}$. For comparison, we give all the non-spin contributions up to 3.5PN order; however we neglect all the spin-spin terms.

	$1.4M_\odot + 1.4M_\odot$	$10M_\odot + 1.4M_\odot$	$10M_\odot + 10M_\odot$
Newtonian	15952.6	3558.9	598.8
1PN	439.5	212.4	59.1
1.5PN	$-210.3 + 65.6\kappa_1\chi_1 + 65.6\kappa_2\chi_2$	$-180.9 + 114.0\kappa_1\chi_1 + 11.7\kappa_2\chi_2$	$-51.2 + 16.0\kappa_1\chi_1 + 16.0\kappa_2\chi_2$
2PN	9.9	9.8	4.0
2.5PN	$-11.7 + 9.3\kappa_1\chi_1 + 9.3\kappa_2\chi_2$	$-20.0 + 33.8\kappa_1\chi_1 + 2.9\kappa_2\chi_2$	$-7.1 + 5.7\kappa_1\chi_1 + 5.7\kappa_2\chi_2$
3PN	$2.6 - 3.2\kappa_1\chi_1 - 3.2\kappa_2\chi_2$	$2.3 - 13.2\kappa_1\chi_1 - 1.3\kappa_2\chi_2$	$2.2 - 2.6\kappa_1\chi_1 - 2.6\kappa_2\chi_2$
3.5PN	$-0.9 + 1.9\kappa_1\chi_1 + 1.9\kappa_2\chi_2$	$-1.8 + 11.1\kappa_1\chi_1 + 0.8\kappa_2\chi_2$	$-0.8 + 1.7\kappa_1\chi_1 + 1.7\kappa_2\chi_2$

Quasi-eccentric case

- Phasing needs Conserved Energy E and AM J and FZ flux of Energy and AM $\mathcal{L}$ and $\mathcal{J}$.
- To average over the orbit needs Generalized quasi-Keplerian representation
- Secular evolution of orbital elements generalizing Peters-Mathews
- Beyond secular evolution by method of variation of constants

State of the art

Non-Spinning (Quasi-circular)

- 3PN Conservative EOM, 3.5PN RR EOM [1]
- 3.5PN Energy Flux, 3.5PN Phasing (Implicit 6PN EOM (partial))[2]
- 3PN GW Polarizations and Modes [3]
- 3.5PN h_{22} [4]
- 2.5PN LMF and Recoil, [5]
- 4PN EOM (Incomplete)[6]

Non-Spinning (Quasi-eccentric)

- 3PN Conservative EOM, 3.5PN RR EOM [1]
- 3PN Energy Flux, 3PN AM Flux, 3PN quasi-Keplerian representation, 3PN secular evolution of orbital elements [7]
- 3PN Modes for instantaneous terms [8]
- 3PN modes for hereditary terms for small eccentricity [8]
- Post-adiabatic corrections to evolution of orbital elements [9]
- Fourier domain in SPA (e expanded) [10]
- Frequency Domain - 2PN [11]

State of the art

Spinning

- SO 3PN [12]
- SO 3.5PN terms in Evolution of Spins [13]
- EOM SS S1S1 3PN [14]
- EOM S1S2 4PN [15]
- GW Flux, Phasing 3.5PN [16]

Eccentric Spinning

- QK reprn; Waveform LO SO, S1S2, S1S1 [17]

Test particle limit (Quasi-circular)

- Schwarzschild case: 22PN Energy Flux, 22PN Approximants [18]

State of the art

EOB

- EOB for non-spinning, spinning and tidal cases [19]

Improved EOB

- Schwarzschild case: 22PN ρ_{lm} [18, 20]
- Kerr case: 4PN ρ_{lm} [21]

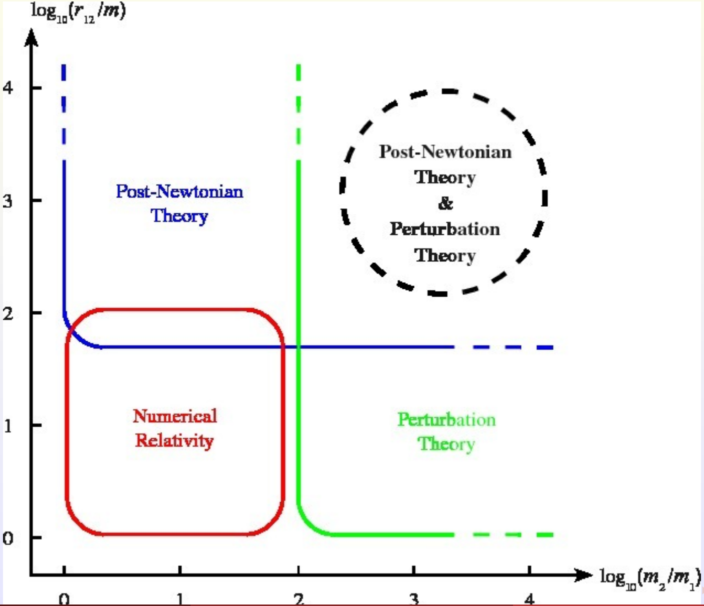
RR from Balance Eqns for Energy and AM

- Non-Spinning 3.5PN, 4.5PN [22]
- Spinning SO 3.5PN [23]

Other theories of gravity

- Quadratic Modified Gravity [24]
- Dynamical Chern-Simons Gravity [25]
- Scalar-Tensor gravity - 2.5PN EOM [26]

Overview of comparisons - Blanchet, Detweiler, Tiec, Whiting



Multipolar Post Minkowskian (MPM) formalism

- MPM: Currently the most successful since it can deal with *all* aspects: the Conservative EOM, Radiation field at infinity, Non-linear efffects related to Tails. Has evolved over the last two decades into a consistent algorithmic approach to analytical GW computations..
  Blanchet Liv Rev Rel 9:4 2006; Gravitational Waves - M. Maggiore

Beyond PNA

- PNA cannot model merger and ringdown..Break down of adiabatic approx $\dot{F}_{\rm orb}/F^2_{\rm orb} \ll 1$, Monotonicity of freq evoln (??)..
- Suggestion to use *Resummation methods* to extend numerical validity of PN expansions (at least) up to the LSO e.g Padé approximants → *Effective-one-Body*

Effective-One-Body (EOB)

- Effective-One-Body (EOB) approach - New resummation, to extend validity of suitably resummed PN results beyond the LSO, and up to the merger
- At Newtonian approx, the Hamiltonian $H_0(\mathbf{q}, \mathbf{p})$ can be thought of as describing a 'test particle' of mass μ orbiting around an 'external mass' GM. ($M \equiv m_1 + m_2$ and $\mu = m_1\, m_2/M$);
- EOB approach is *general relativistic generalization* of this. Consists in looking for an 'external spacetime geometry' $g_{\mu\nu}^{\rm ext}(x^\lambda; GM)$ s.t 'geodesic' dynamics of 'test particle' of mass μ within $g_{\mu\nu}^{\rm ext}(x^\lambda, GM)$ is *equivalent* (when expanded in powers of $1/c^2$) to original, relative PN-expanded dynamics.

Effective-One-Body (EOB)

- *Four* essential elements of the EOB approach are:
- (i) *Hamiltonian* H_{real} describing *conservative* part of relative dynamics of 2 BH
- (ii) *Radiation-reaction force* $\mathcal{F}_\varphi$ describing loss of (mechanical) angular momentum, and energy, of binary system;
- (iii) Definition of various multipolar components of *"inspiral-plus-plunge" (metric) waveform* $h_{\ell m}^{\mathrm{insplunge}}$;
- (iv) Attachment of subsequent *"Ringdown waveform"* $h_{\ell m}^{\mathrm{ringdown}}$ around certain (EOB-determined) "merger time" t_m.
- Estimated complete GW signal emitted by inspiralling, plunging, merging and ringing binary black holes

Improved Effective-One-Body (EOB)

- Novel improvement concerns radiation reaction force $\mathcal{F}_\varphi$. Uses "improved resummation" of PN multipolar waveforms.

$$\mathcal{F}_\varphi \equiv -\frac{1}{8\pi\,\Omega}\sum_{\ell=2}^{\ell_{\rm max}}\sum_{m=1}^{\ell}(m\,\Omega)^2\,|R\,h_{\ell m}^{(\epsilon)}|^2\,.$$

(Ω: EOB orbital frequency; $\ell_{\rm max}=8$)

- Approach is *Multiplicative*. Any relativistic quantity is decomposed as *Product* of various contributions
- Choice of factors based on physical intuition of the main physical effects entering the final waveform. Resum separately each factor.
- One important factor Resums leading logarithms and a *l*th root a growing *l* dependence in the modes

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