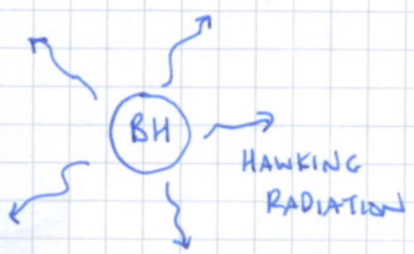
QUANTUM INFORMATION & BLACK HOLES

GOAL: A BLACK HOLE CENTRIC TOUR THROUGH TOOLS OF QI

BH INFO PARADOX →
- MEASURES OF STATE DISTINGUISHABILITY
- QUANTIFYING ENTANGLEMENT
- ENTROPY & INFORMATION
- QUANTUM ERROR CORRECTION (THERMALIZATION)

BH INFO PARADOX



BASIC RELATIONSHIPS:

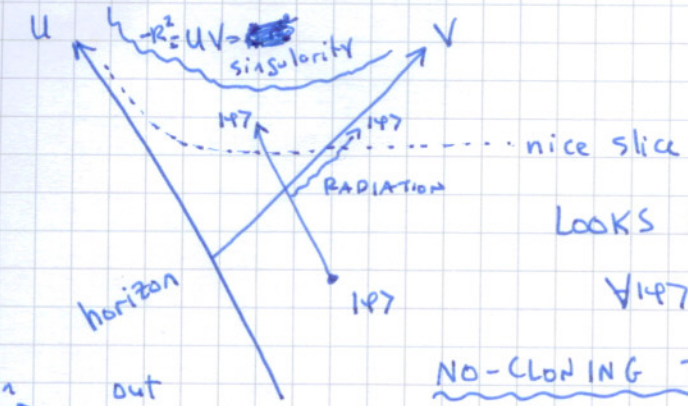
$M \sim r_S \qquad A \sim r_S^2 \qquad S \sim A$

$\hbar = c = G = 1.$

INFO ESCAPE?

NO $\Rightarrow$ PHYSICS ISN'T UNITARY

YES $\Rightarrow$ APPARENT VIOLATIONS OF UNITARITY: CLONING



RADIAL LIGHT RAYS

LOOKS LIKE A PHYSICAL TRANSFORMATION

$\forall |\varphi\rangle \qquad |\varphi\rangle \mapsto |\varphi\rangle|\varphi\rangle$

NO-CLONING TH'M: NO SUCH EVOLUTION IS POSSIBLE.

PROOF: COMPARE INNER PRODUCTS BEFORE & AFTER

WANT $\quad U|\varphi\rangle|0\rangle|0\rangle = |\varphi\rangle|\varphi\rangle|\zeta_\varphi\rangle \qquad \forall |\varphi\rangle$

BEFORE: $\left| \left(\langle\varphi_1| \langle 0| \langle 0| \right) \left(|\varphi_2\rangle|0\rangle|0\rangle \right) \right| = |\langle\varphi_1|\varphi_2\rangle|$

AFTER: $\left| \left(\langle\varphi_1| \langle\varphi_1| \langle\zeta_{\varphi_1}| \right) \left(|\varphi_2\rangle|\varphi_2\rangle|\zeta_{\varphi_2}\rangle \right) \right|$

$= |\langle\varphi_1|\varphi_2\rangle|^2 \, |\langle\zeta_{\varphi_1}|\zeta_{\varphi_2}\rangle|$

$\leq |\langle\varphi_1|\varphi_2\rangle|^2$

Require $|\langle\varphi_1|\varphi_2\rangle| = |\langle\varphi_1|\varphi_2\rangle|^2$ TO SUCCEED. $\square$.

BH COMPLEMENTARITY PRINCIPLE : YES, BUT...

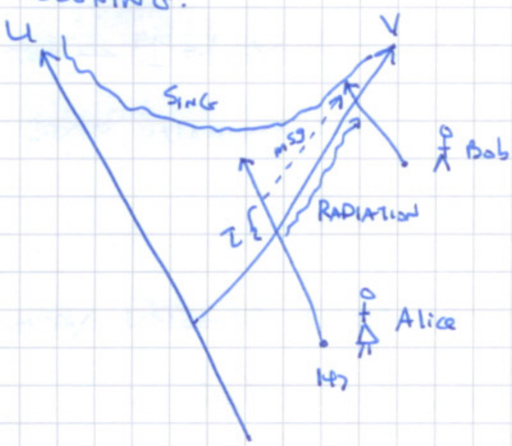
EVIDENCE FROM STRING THEORY (AdS/CFT) THAT BH EVAPORATION IS UNITARY. HOW TO MAKE SENSE OF APPARENT CLONING AND RESOLVE THE PARADOX?

PROPOSAL: THE TWO COPIES OF $|\psi\rangle$ ARE NOT INDEPENDENT.

INFALLING & OUTSIDE OBSERVER'S DESCRIPTIONS ARE COMPLEMENTARY, ANALOGOUS TO POSITION AND MOMENTUM.

$\boxed{\Rightarrow \text{CLONING SHOULD BE UNVERIFIABLE.}}$

SO CONSIDER THOUGHT EXPERIMENTS THAT TRY TO VERIFY CLONING.



ALICE & BOB NEED TO COMPARE THEIR COPIES OF $|\psi\rangle$.

$$\tau \sim r_s \exp\left(-\Delta t / r_s\right)$$

SCHWARZSCHILD TIME DELAY BETWEEN ALICE & BOB.

Alice PROPER TIME AVAILABLE TO SEND MSG

LONGER BOB STAYS OUTSIDE COLLECTING HAWKING RADIATION, LESS TIME ALICE HAS TO SEND MSG.

SPS ALICE MSG HAS REASONABLE ENERGY. TO AVOID DISTORTING BH,

$$\tau > \frac{1}{M} \Rightarrow \frac{1}{M} \lesssim r_s \exp\left(-\Delta t / r_s\right)$$

$$\Delta t \lesssim r_s \log\left(M r_s\right)$$

$$\Delta t \lesssim r_s \log(r_s^2) \sim r_s \log r_s .$$

CALL t_{info} AMT OF TIME INFO REMAINS INSIDE BH UNAVAILABLE TO BOB. FOR CONSISTENCY OF BH COMPLEMENTARITY, NEED EXPERIMENT TO FAIL:

$$\boxed{t_{info} > r_s \log r_s}$$

BH COMPLEMENTARITY CONSISTENCY CONDITION.

SUSSKIND, THORLACIUS, UGLUM, PRD 48(8):3743, 1993.
IDEA FOR EXPT: PAGE, PRESKILL, SUSSKIND 1993

DON PAGE, AVERAGE ENTROPY OF A SUBSYSTEM
PHYS. REV. LETT. 71(9):1291, 1993

INFO RETENTION: PAGE ANSATZ

GET QUALITATIVE UNDERSTANDING BY STUDYING GENERIC STATES.

SPS $|\varphi\rangle \in \mathcal{H}_{BH} \otimes \mathcal{H}_{rad} \cong \mathbb{C}^d$ $\qquad \mathcal{H}_{BH} \cong \mathbb{C}^b \quad \mathcal{H}_{rad} \cong \mathbb{C}^r$

↑ SELECTED AT RANDOM

QUESTION: DOES $\varphi_{rad} \equiv \mathrm{tr}_{BH} |\varphi\rangle\langle\varphi|$ DEPEND ON $|\varphi\rangle$?

IF NOT, NO INFO RELEASED FROM BH.

CONSIDER ρ DENSITY OPERATOR ON $\mathcal{H}_{rad}$.

DEFINE PURITY $P(\rho) = \mathrm{tr}\,\rho^2$.

IF $\rho = |\omega\rangle\langle\omega|$ THEN $P(\rho) = \mathrm{tr}\,|\omega\rangle\langle\omega|^2 = \mathrm{tr}\,|\omega\rangle\langle\omega| = 1$ ← PURE

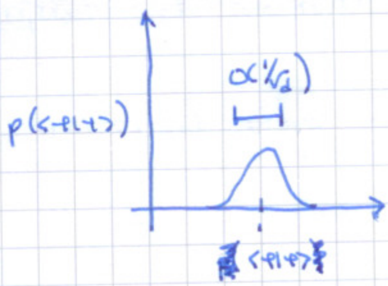
IF $\rho = \begin{pmatrix} \frac{1}{r} & & 0 \\ & \frac{1}{r} & \\ 0 & & \ddots & \\ & & & \frac{1}{r} \end{pmatrix}$ THEN $P(\rho) = r \cdot \frac{1}{r^2} = \frac{1}{r}$ ← MAXIMALLY MIXED

ESTIMATE $\mathbb{E}P(\rho)$ BY CHOOSING $|\varphi\rangle$ TO BE A GAUSSIAN VECTOR

$|\varphi\rangle = \sum_{jk} g_{jk} |j\rangle|k\rangle$ WHERE g_{jk} GAUSSIAN $N_{\mathbb{C}}(0, \frac{1}{d})$

$\mathbb{E}\, g_{jk} = 0 \quad \mathbb{E}|g_{jk}|^2 = \frac{1}{d} \quad \mathbb{E}|g_{jk}|^4 = \frac{2}{d^2}$

THEN $\mathbb{E}[\langle\varphi|\varphi\rangle] = \mathbb{E}\sum_{jk} |g_{jk}|^2 = 1.$



$\mathbb{V}[\langle\varphi|\varphi\rangle] = \frac{1}{d}.$

$$\begin{aligned}
\mathbb{V}[\langle\varphi|\varphi\rangle] &= \mathbb{E}\Big[(\langle\varphi|\varphi\rangle - 1)(\langle\varphi|\varphi\rangle - 1)\Big] \\
&= \mathbb{E}\Big[\big(\sum_{jk} |g_{jk}|^2 - 1\big)\big(\sum_{mn} |g_{mn}|^2 - 1\big)\Big] \\
&= \sum_{jkmn} \mathbb{E}\big[|g_{jk}|^2 |g_{mn}|^2\big] - 1 \\
&= \sum_{(j,k) \neq (m,n)} \frac{1}{d} \cdot \frac{1}{d} + \sum_{jk} \mathbb{E}|g_{jk}|^4 - 1 \\
&= \frac{d^2 - d}{d^2} + d\,\frac{2}{d^2} - 1 \\
&= \frac{1}{d}
\end{aligned}$$

PURITY ESTIMATE

$$\varphi_{rad} = \mathrm{tr}_{BH} |\varphi\rangle\langle\varphi| = \mathrm{tr}_{BH} \sum_{jkj'k'} g_{jk}\,\bar{g}_{j'k'}\; |j\rangle\langle j'|_{BH} \otimes |k\rangle\langle k'|_{rad}$$

$$= \sum_{jkk'} g_{jk}\,\bar{g}_{jk'}\, |k\rangle\langle k'|$$

$$P(\varphi_{rad}) = \mathrm{tr}\, \varphi_{rad}^2 = \mathrm{tr}\left[\sum_{jkk'} g_{jk}\,\bar{g}_{jk'}\, |k\rangle\langle k'| \overset{\delta_{k'm}\,\delta_{km'}}{\sum_{lmm'}} g_{lm}\, g_{lm'}\, |m\rangle\langle m'| \right]$$

$$= \sum_{jklm} g_{jk}\,\bar{g}_{jm}\, g_{lm}\, \bar{g}_{lk}$$

$$\mathbb{E}\, P(\varphi_{rad}) = \sum_{jklm} \mathbb{E}\left[g_{jk}\,\bar{g}_{jm}\, g_{lm}\, \bar{g}_{lk} \right]$$

$$= \sum_{jk} \mathbb{E}\left[|g_{jk}|^4 \right] + \sum_{\substack{j\neq l \\ k}} \mathbb{E}\left[|g_{jk}|^2 |g_{lk}|^2 \right] + \sum_{\substack{k\neq m \\ j}} \mathbb{E}\left[|g_{jk}|^2 |g_{jm}|^2 \right]$$

$$= br \cdot \frac{\cancel{4}2}{(br)^2} + (b^2-b)r \cdot \frac{1}{(br)^2} + (r^2-r)b \cdot \frac{1}{(br)^2}$$

$$= \cancel{\tfrac{2}{br}} \cancel{+} \frac{1}{r} + \frac{1}{b}$$

NOTE: IF $b \gg r$ THEN $\mathbb{E}\, P(\varphi_{rad}) \triangleq \frac{1}{r}$ SO φ_{rad} (ALMOST) MAXIMALLY MIXED.

EXPRESS THIS IN BITS:

DEFINE RENYI ENTROPY $S_\alpha(\rho) = \frac{1}{1-\alpha} \log \mathrm{tr}\rho^\alpha$

$\lim_{\alpha\to 1} S_\alpha(\rho) = S(\rho) = -\mathrm{tr}\rho\log\rho$ VON NEUMANN ENTROPY.

PROPERTIES: (i) $0 \leq S_\alpha(\rho) \leq \log \mathrm{rank}\,\rho$

(ii) $\alpha \geq \beta \Rightarrow S_\alpha(\rho) \leq S_\beta(\rho)$

SO $S(\rho) \geq S_2(\rho) = -\log P(\rho)$.

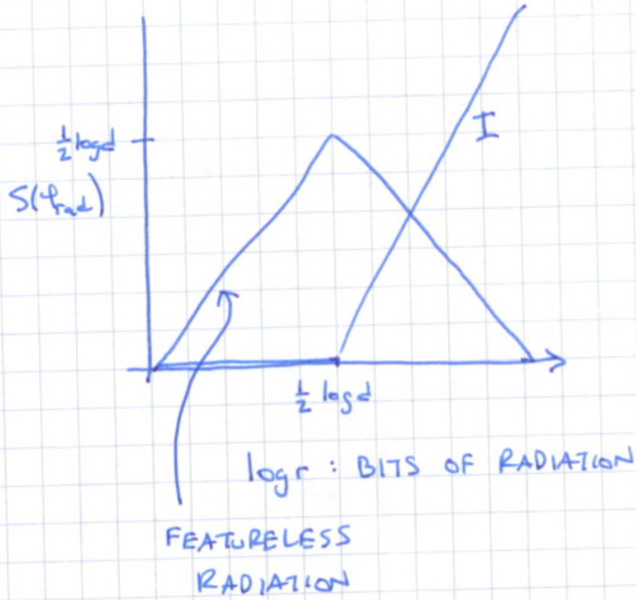
SO $\mathbb{E}\, S(\rho) \geq \mathbb{E}\, S_2(\rho) = \mathbb{E}\left[-\log P(\rho)\right]$

$\geq -\log \mathbb{E}\left[P(\rho)\right]$

AND $\mathbb{E}\, S(\varphi_{rad}) \geq -\log\left(\frac{1}{r} + \frac{1}{b} \cancel{+ \tfrac{1}{br}} \right)$

IF $r \ll b$: $\quad -\log\left(\frac{1}{r}\left(1+\frac{r}{b}+\frac{2}{b}\right)\right) \cong \log r - \frac{r\cancel{+2}}{b}$

$b \ll r$: $\quad \cong \log b - \frac{b\cancel{+2}}{r}$



GENERALIZATION TO THERMAL SYSTEMS:
POPESCU, SHORT, WINTER.
ENTANGLEMENT AND THE FOUNDATIONS OF STAT. MECH., NATURE PHYSICS, 2006

"INFO" IN RADIATION:

$$I = S\left(\frac{I_{rad}}{r}\right) - S(\varphi_{rad})$$

$$= \log r - S(\varphi_{rad})$$

$$= D\left(\varphi_{rad} \,\Big\|\, \frac{I_{rad}}{r}\right)$$

FREE QUANTUM INFO BOOK
M. WILDE arXiv: 1106.1445

$D(\rho\|\sigma)$: Sec 10.5, 11.8

WHERE $D(\rho\|\sigma) = \mathrm{tr}\left[\rho\log\rho - \rho\log\sigma\right]$ IS RELATIVE ENTROPY

CONCLUSION: NO INFO UNTIL BH AREA HAS SHRUNK BY ABOUT $\frac{1}{2}$.

$$\boxed{t_{info} \sim \text{BH LIFETIME} \sim r_s^3.}$$

$r_s^3 \gg r_s \log r_s$

$\Rightarrow$ BH COMPLEMENTARITY SAFE

COMPARE TO CLASSICAL

Z_1, Z_2 TWO R.V.'S UNIFORM INDEP. S.T. $Z_j = (X_j, Y_j)$

$x_j \in \{1, 2, \ldots, b\}$ $\quad Y_j \in \{1, 2, \ldots, r\}$.

$$\boxed{P_r\left[Y_1 \neq Y_2\right] = \frac{r-1}{r}}$$ ← DISTINGUISHABLE WHP.

GENERALIZATION TO THERMAL SYSTEMS

$$\underset{\substack{\uparrow \\ \text{CONSTRAINT SUBSPACE}}}{\mathcal{H}_c} \subseteq \underset{\substack{\uparrow \\ \text{SYSTEM}}}{\mathcal{H}_S} \otimes \underset{\substack{\uparrow \\ \text{ENVIRONMENT}}}{\mathcal{H}_E}$$

USUAL STAT MECH ASSUMPTION: STATE IS $\Omega = \frac{\Pi_c}{d_c}$

PROJECTOR ONTO $\mathcal{H}_c$.

DIMENSION

INSTEAD, SPS $|\varphi\rangle \in \mathcal{H}_c$ CHOSEN @ RANDOM.

WILL φ_S BE CLOSE TO Ω_S?

HOW TO DEFINE CLOSE?

TRACE DISTANCE $T(\rho, \sigma) = \|\rho - \sigma\|_1$ for $\|X\|_1 = \operatorname{tr}\sqrt{X^\dagger X} = \max_{\|\Lambda\|_\infty \le 1} \operatorname{tr} \Lambda X$

PROPERTIES:

(i) $T(\rho, \sigma) = 2(1 - 2p_{err})$ WHERE p_{err} IS MIN PROB OF ERROR OVER ALL MEASUREMENTS DESIGNED TO DISTINGUISH ρ FROM σ. [QI BOOK SEC 9.1.4].

(ii) $\frac{1}{2} T(\rho, \sigma)^2 \le D(\rho \| \sigma)$ (QUANTUM PINSKER INEQ). [QI BOOK TH'M 11.9.2].

THEN $\Pr\left\{ T(\varphi_S, \Omega_S) > \epsilon \right\} < \exp\left(-\text{const} \cdot d_R \cdot \delta^2\right)$

WHERE $\epsilon = \delta + \frac{1}{2}\sqrt{\frac{d_S}{d_E^{eff}}}$, $d_E^{eff} = \frac{1}{\operatorname{tr} \Omega_E^2}$.

> QI BOOK IS
>
> M. WILDE. FROM CLASSICAL TO QUANTUM SHANNON THEORY. PUBLISHED BY CUP. AVAILABLE AS arXiv:1106.1445

SIMPLEST EXAMPLE: n SPIN-1/2 IN MAGNETIC FIELD VERY WEAK INTERACTION

$\mathcal{H}_c$: SPAN OF ENERGY EIGENSTATES OF FIXED ENERGY.

S: k SPINS $\quad d_S = 2^k$

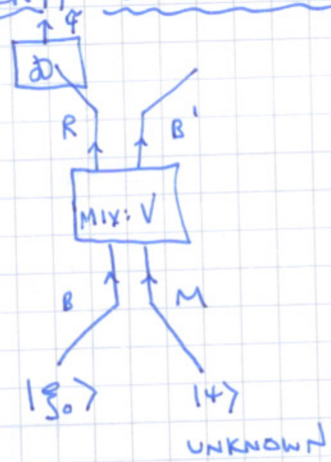
$$d_E^{eff} \simeq 2^{(n-k)H(p)}$$

WHERE H IS ENTROPY, p IS FRACTION OF FLIPPED SPINS

VERSION FOR MFD OF FIXED ENERGY EXPECTATION:
MUELLER, GROSS, EISERT. COMM. MATH. PHYS. 303:785, 2011.

DECODING RADIATION

1) (VERY) SIMPLE QUANTUM MODEL



B: BH $(n-k$ QUBITS$)$ $(d_B = 2^{n-k})$

M: MSG (k QUBITS)

R: RADIATION (r QUBITS)

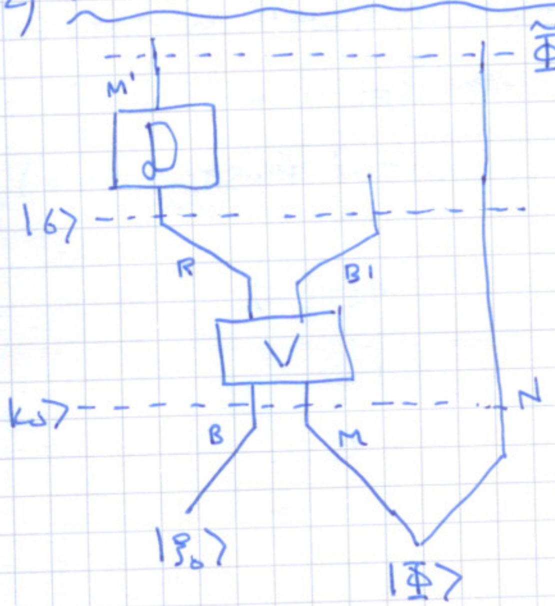
B': POST-EVAP BH. $(n-r$ QUBITS$)$

$\mathcal{D}$ SHOULD BE CONSISTENT WITH QM

WANT $\int \langle\psi|\tilde{\psi}|\psi\rangle\, d\psi > 1-\epsilon$

OR $\int D(\psi,\tilde{\psi})\, d\psi < 1-\delta(\epsilon)$. etc.

2) EQUIV SETUP: SEND ENTANGLEMENT



N: REFERENCE SYSTEM (k QUBITS)

$$|\Phi\rangle = \frac{1}{\sqrt{2^k}} \sum_{j=1}^{2^k} |j\rangle_M |j\rangle_N$$

WANT $\langle\Phi|\tilde{\Phi}|\Phi\rangle > 1-\epsilon$.

THEN $(2) \Rightarrow (1)$ (QI BOOK 9.5.3)

HOW TO TELL IF THIS IS POSSIBLE?

ASIDE ON BIPARTITE ENTANGLEMENT

CONSIDER $\rho_X = \sum_i \lambda_i |e_i\rangle\langle e_i|$, $\langle e_i|e_j\rangle = \delta_{ij}$

$$|\alpha\rangle_{XY} = \sum_i \sqrt{\lambda_i}\, |e_i\rangle_X |f_i\rangle_Y , \quad \langle f_i|f_j\rangle = \delta_{ij} \qquad (*)$$

$$|\beta\rangle_{XY} = \sum_i \sqrt{\lambda_i}\, |e_i\rangle_X |g_i\rangle_Y \qquad \langle g_i|g_j\rangle = \delta_{ij}$$

THEN $\rho_X = \alpha_X = \beta_X$: α, β BOTH PURIFY ρ_X

CLEARLY, $\exists$ UNITARY U ON Y s.t. $U|f_i\rangle = |g_i\rangle$

SO $(I_X \otimes U)|\alpha\rangle_{XY} = |\beta\rangle_{XY}$.

CLAIM: SUCH A UNITARY U EXISTS FOR ANY STATES

$|\alpha\rangle_{XY}, |\beta\rangle_{XY}$ s.t. $\alpha_X = \beta_X$.

PROOF: SUFFICES TO SHOW THAT $|\alpha\rangle$ CAN ALWAYS BE WRITTEN IN DIAGONAL FORM (*). "SCHMIDT DECOMP"

WHY? $|\alpha\rangle_{XY} = \sum_{jk} a_{jk} |j\rangle_X |k\rangle_Y$

SET $A = [a_{ij}]$. BY SVD, A CAN BE WRITTEN

$$A = U D W$$

U, W → UNITARY

D → DIAGONAL, NON-NEG.

So $|\alpha\rangle_{XY} = \sum_{jk\ell} U_{j\ell} D_{\ell\ell} W_{\ell k} |j\rangle_X |k\rangle_Y$.

$$= \sum_{\ell} D_{\ell\ell} |e_\ell\rangle_X |f_\ell\rangle_Y. \quad \square.$$

CRUCIAL EXAMPLE

WRITE $R \cong M' \otimes O$ AND SPS $\sigma_{B'N} = \sigma_{B'} \otimes \Phi_N$

ie POST-EVAPORATION STATE OF BH IS INDEPENDENT OF REFERENCE! CORRELATIONS DESTROYED.

COMPARE $|\sigma\rangle_{M'NOB'}$ TO HYPOTHETICAL $|\alpha\rangle = |\Phi\rangle_{M'N} |\zeta\rangle_{OB'}$

WITH $\zeta_{B'} = \sigma_{B'}$.

THEN $\sigma_{B'N} = \alpha_{B'N}$ SO $\exists$ UNITARY $U_{M'O}$ st

$$(I_{B'N} \otimes U_{M'O}) |\sigma\rangle_{M'NOB'} = |\Phi\rangle_{M'N} |\zeta\rangle_{OB'}$$

↳ PROVIDES DECODING $\mathcal{D}$!

DECORRELATING B' FROM N NECESSARY & SUFFICIENT FOR EXISTENCE OF DECODING

APPROXIMATE VERSION:

$$T(\sigma_{B'N}, \sigma_{B'} \otimes \Phi_N) < \delta \iff \exists \mathcal{D} \quad \langle \Phi | \tilde{\Phi} | \Phi \rangle \geq 1 - \epsilon.$$

[See e.g. quant-ph/0702005 Thm II].

NEXT DAY:

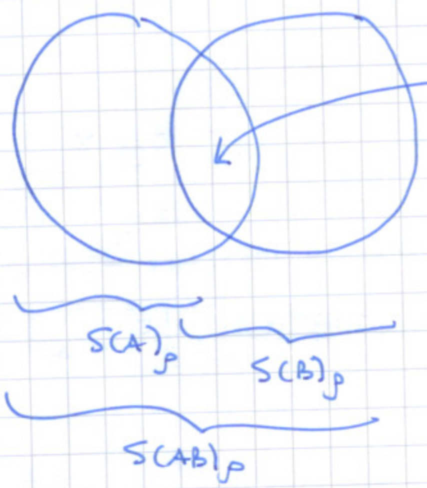
WILL SHOW $\int \| \sigma_{NB'} - \Phi_N \otimes \sigma_{B'} \|_1^2 \, dV \le 2^{n+k-2r}$

SO NEED $\frac{n+k}{2} \ll r$ TO RECOVER INFO.

CONFIRMS PAGE BUT GOES FURTHER: DECODING IS POSSIBLE SOON AFTER HALF BITS RADIATED FOR SMALL k. (QUANTUM ERROR CORRECTION!)

DEVELOP SOME INTUITION USING ENTROPIES

ρ_{AB} $\qquad S(A)_\rho = S(\rho_A)$ etc.



$I(A:B)_\rho = S(A)_\rho + S(B)_\rho - S(AB)_\rho$.

PROPERTIES: (i) $I(A:B)_\rho \ge 0$, EQUALITY IFF $\rho_{AB} = \rho_A \otimes \rho_B$.

(ii) IF $|\Phi\rangle = \frac{1}{\sqrt{d_A}} \sum_{j=1}^{d_A} |j\rangle_A |j\rangle_B$ THEN

$$I(A:B)_\Phi = \log d_A + \log d_A - 0 = 2 \log d_A$$

IN GENERAL $I(A:B)_\rho \le 2 \log d_A$.

(iii) $I(A;BC)_\rho \ge I(A;B)_\rho$ (STRONG SUBADDITIVITY)

(iv) $I(A:BC)_\rho \le I(A:B)_\rho + 2 \log d_C$.

APPLY TO EVAPORATION: (HERE $\log = \log_2$)

$$2k = I(N:M)_\omega = I(N:MB)_\omega$$
$$= I(N:RB')_\omega \quad \text{(V UNITARY)}$$
$$\le I(N:B')_\sigma + 2 \log d_R$$

$\cong 2\log d_R$ IF DECODING IS POSSIBLE ($\sigma_{NB'} \cong \sigma_N \otimes \sigma_{B'}$)

$= 2r.$

SO ALL WE HAVE LEARNED IS $r \geq k$.

EVAPORATED QUANTA $\geq$ # MSG QUANTA.

HMMM.... CAN ANOTHER SETUP SATURATE THIS BOUND?

LECTURE 3

RECALL SETUP (2) ON PG. 7.

WILL PROVE GENERAL BOUND

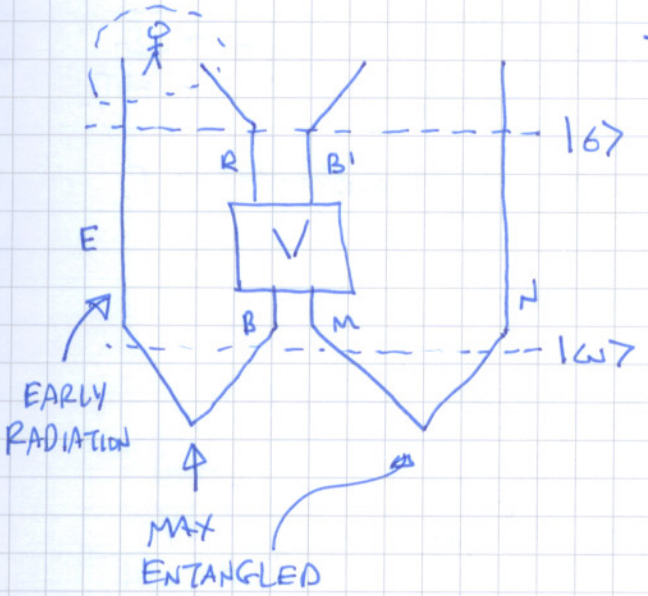
$$\boxed{\int \| \sigma_{NB'} - \Phi_N \otimes \zeta_{B'} \|_1^2 \, dV \quad \text{for } \zeta_{B'} = \frac{I_{B'}}{d_{B'}} \qquad \leq \frac{d_N d_{BM}}{d_R^2} \operatorname{tr}\left[(\omega_{BMN})^2 \right]}$$

looks like $\exp(I(N:BM) - 2r)$

$$= \frac{2^k 2^n}{2^{2r}} \cdot 1 = 2^{k+n-2r} \quad \leftarrow \text{FOR SETUP (2).}$$

DECODING WHEN $2r > k+n$.

OLD BLACK HOLE : [HAYDEN, PRESKILL. JHEP 2007].



THEN

$$\int \| \sigma_{B'N} - \Phi_N \otimes \zeta_{B'} \|_1^2 \, dV$$

$$\leq \frac{d_N d_{BM}}{d_R^2} \operatorname{tr}\left[(\omega_{BMN}^2) \right]$$

Schmidt

$$= \frac{2^k 2^n}{2^{2r}} \operatorname{tr}\left[\omega_E^2 \right]$$

$$= \frac{2^k 2^n}{2^{2r}} 2^{-[n-k]}$$

$$= 2^{2(k-r)}$$

$\boxed{\text{RECOVER INFO AS SOON AS } k > r. \ !}$ YIKES!

ELTS OF CALCULATION

$$\|X\|_1 = \sum_{j=1}^{d} |\lambda_j| \quad \text{IF } \lambda_j \text{ EIG OF HERMITIAN } X.$$

$$= \sum_{j=1}^{d} |\lambda_j \cdot 1|$$

$$\leq \sqrt{d} \sqrt{\sum_{j=1}^{d} \lambda_j^2} \quad \text{CAUCHY-SCHWARZ}$$

$$= \sqrt{d}\, \|X\|_2 .$$

SO ESTIMATE EASIER

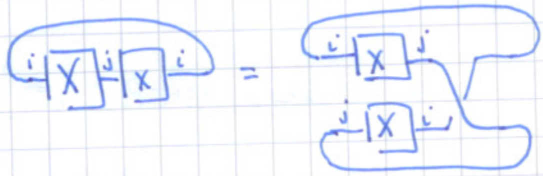
$$\int \| \sigma_{B'N} - \Phi_N \otimes \tau_{B'} \|_2^2 \, dV$$

$$= \int \mathrm{tr}\left[\sigma_{B'N}^2 \right] dV - \frac{1}{d_N d_{B'}}$$

FIRST TERM IS QUARTIC IN MATRIX ENTRIES OF V.

TWO TRICKS:

(i) $\mathrm{tr}[X^2] = \mathrm{tr}[X \otimes X \, F]$ WHERE $F|\varphi\rangle|\xi\rangle = |\xi\rangle|\varphi\rangle$



(ii) $$\int (V \otimes V)\, Y \,(V^\dagger \otimes V^\dagger)\, dV = \mathrm{tr}[Y \Pi_+] \frac{\Pi_+}{\mathrm{rk}\,\Pi_+} + \mathrm{tr}[Y \Pi_-] \frac{\Pi_-}{\mathrm{rk}\,\Pi_-}$$

WHERE $\Pi_\pm$ PROJECTORS ONTO ± 1 EIGENSPACES OF F.

$$\int \mathrm{tr}\left[\sigma_{B'N}(V)^2 \right] dV$$

$$= \int \mathrm{tr}\left[\sigma_{B_1'N_1} \otimes \sigma_{B_2'N_2} \; F_{B_1'B_2'} \otimes F_{N_1N_2} \right] dV \quad \text{by (i)}$$

$$= \int \mathrm{tr}\left[\sigma_{R_1B_1'N_1} \otimes \sigma_{R_2B_2'N_2} \; F_{B_1'B_2'} \otimes F_{N_1N_2} \otimes I_{R_1R_2} \right] dV$$

$$= \int \mathrm{tr}\left[V\omega V^\dagger \otimes V\omega V^\dagger \; F \otimes F \otimes I \right] dV$$

form (ii)

$$\vdots$$

$$\leq \frac{1}{d_R} \mathrm{tr}\left[(\omega^2_{BMN}) \right] + \frac{1}{d_N d_{B'}}$$

So
$$\int \| \sigma_{B'N} - \zeta_{B'} \otimes \Phi_N \|_1^2 \, dV$$
$$\leq d_{B'} d_N \cdot \frac{1}{d_R} \mathrm{tr}\left[(\omega^2_{BMN}) \right].$$
$$= \frac{d_{BM} d_N}{d_R^2} \mathrm{tr}\left[(\omega^2_{BMN}) \right].$$

BACK TO OLD BLACK HOLE

DECODE AS SOON AS $r \gtrsim k$.

IF $k = 1$, $r = O(1)$ HAWKING QUANTA.

SUGGESTS $t_{info} \sim r_s$

$$< r_s \log r_s$$

$$\boxed{\text{INCONSISTENT WITH BH COMPLEMENTARITY!}}$$

BUT WAIT: HOW LONG UNTIL RANDOM V BECOMES A GOOD APPROXIMATION?

ie HOW LONG UNTIL MICROSCOPIC HORIZON DEGREES OF FREEDOM THERMALIZED IN STRONG SENSE REQ'D ("SCRAMBLED")?

ESTIMATE RELAXATION TIMES FROM CLASSICAL BH PHYSICS:

- RINGDOWN TIME $\sim r_s$
- CHARGE SPREADING ON STRETCHED HORIZON $\sim r_s \log r_s$

SUGGESTS SCRAMBLING TIME $\gtrsim r_s \log r_s$.

$\boxed{\text{SAFE!}}$ SEE REFS AT END

FIREWALLS

CENSORED!

SCRAMBLING REFS

ORIGINAL "FAST SCRAMBLING" CONJECTURE

SEKINO & SUSSKIND, FAST SCRAMBLERS, 10:065, 2008

PROOF OF ANALOG OF $r_s \log r_s$ LOWER BOUND IN DISCRETE ANALOG

LASHKARI, STANFORD et.al., TOWARDS THE FAST SCRAMBLING CONJECTURE. arXiv: 1111.6580.

ALSO: BLACK HOLES AND THE BUTTERFLY EFFECT. SHENKER & STANFORD arXiv: 1306.0622

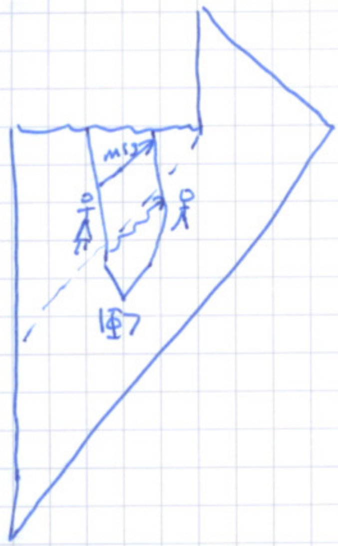
MORE QUANTUM INFORMATION READING

- JOHN PRESKILL'S LECTURE NOTES: BROAD PERSPECTIVE, RELATIVELY PHYSICS-ORIENTED, EXCELLENT ON ERROR CORRECTION, INFO THEORY PART GETTING A BIT OUT OF DATE.

- NIELSEN & CHUANG. QUANTUM COMPUTATION & QUANTUM INFORMATION. GETTING OLD BUT TRULY EXCELLENT.

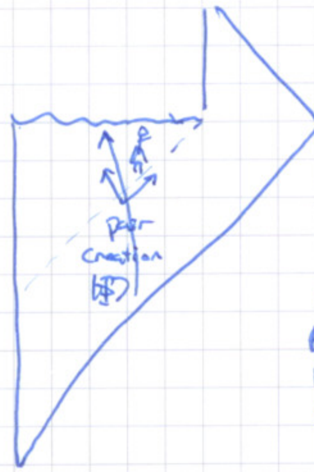
FIREWALL PICTURE STORY

①



OUR EXPERIMENT

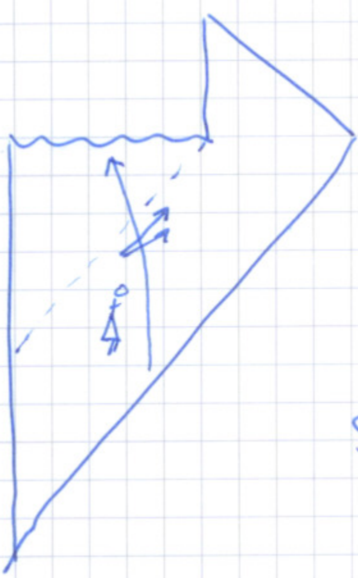
②



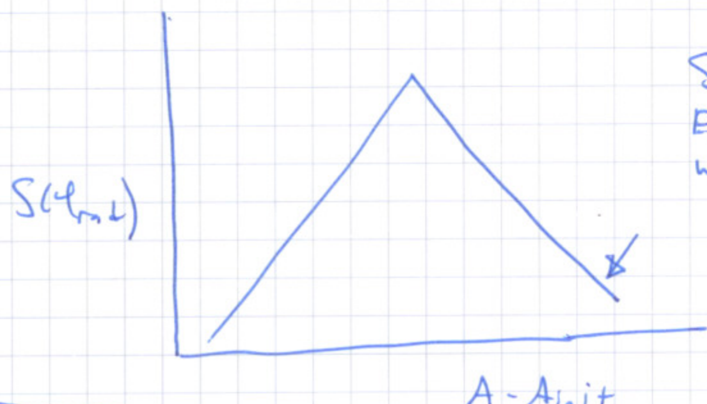
ALICE VERIFIES
ENTANGLEMENT
BY NOT GETTING
COOKED.

AMPS
VARIANT

③



LATE TIME HAWKING
PHOTON:



SHOULD BE
ENTANGLED
WITH EARLY
RADIATION

NOTE:

ALL ACTION
IS REALLY HERE