the gravitational-wave memory

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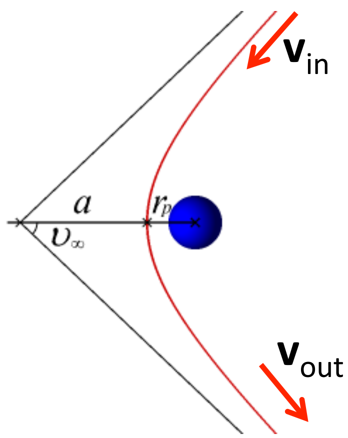
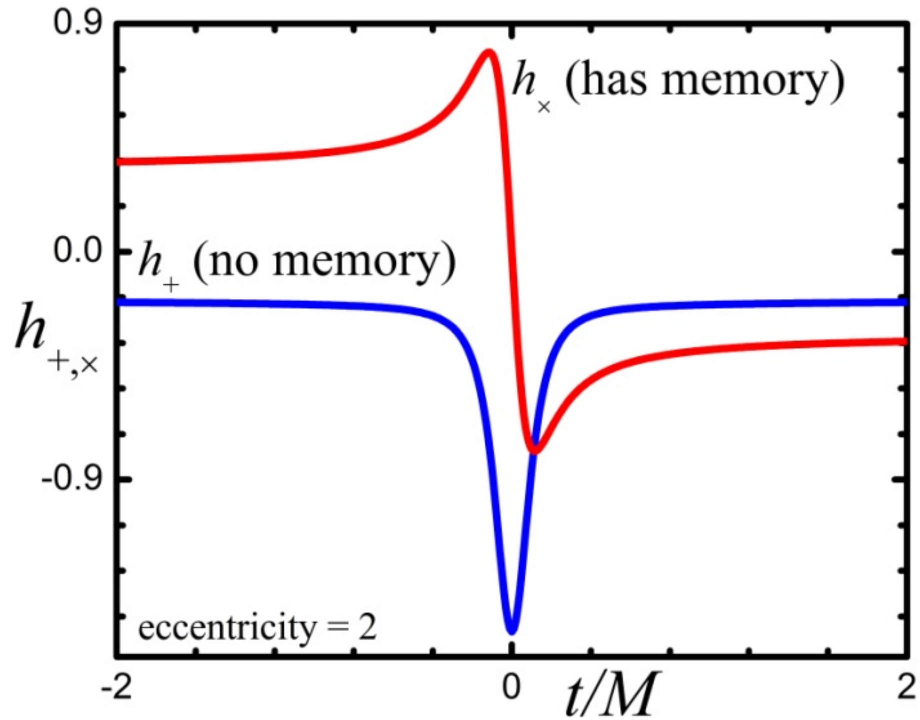
Outline of this talk:

- What is the memory and why is it interesting?
- How do we calculate the memory?
- Is it observable?

Examples of memory:

Two-body scattering/hyperbolic orbits

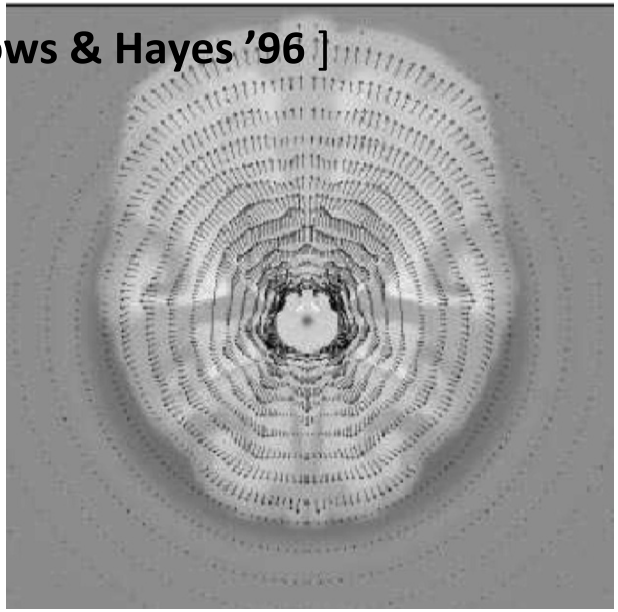


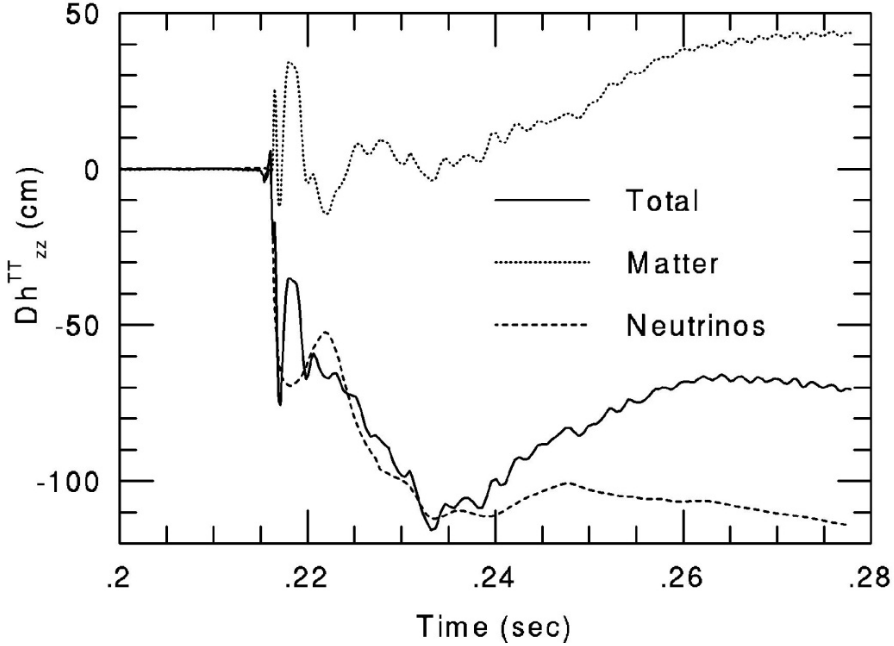


[Turner '77, Turner & Will '78, MF '11]

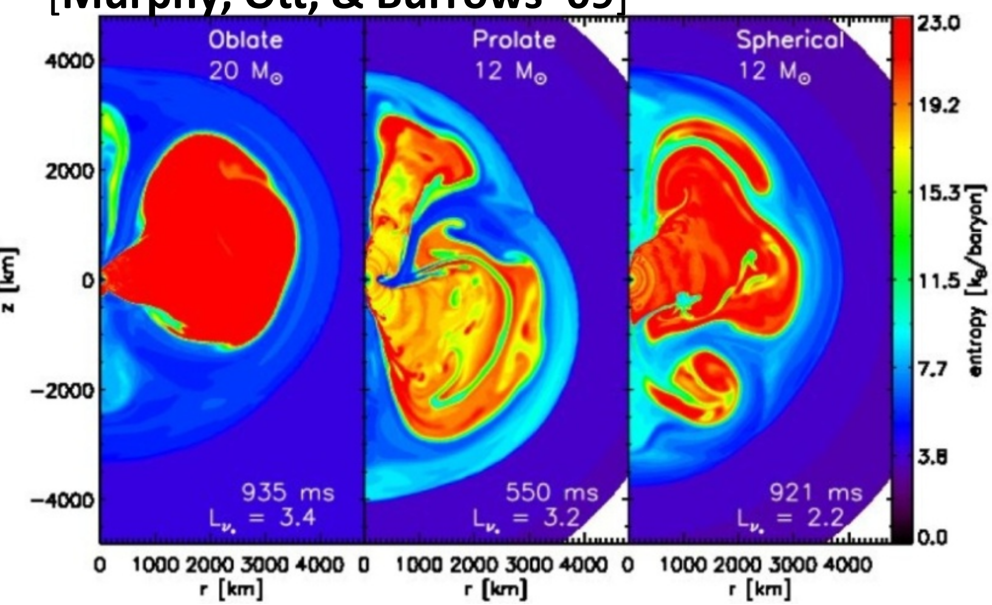
Examples of memory: Core-collapse supernovae

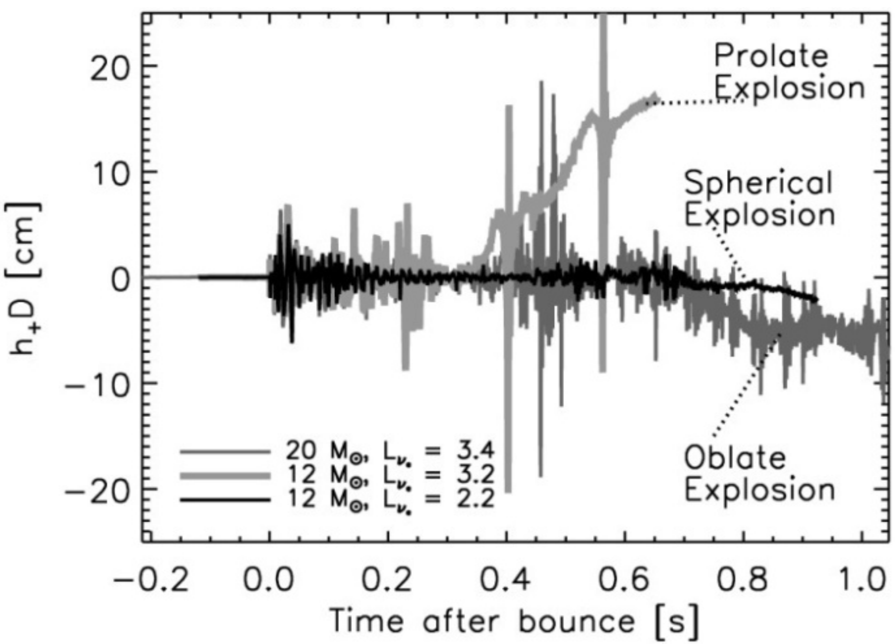
[Burrows & Hayes '96]





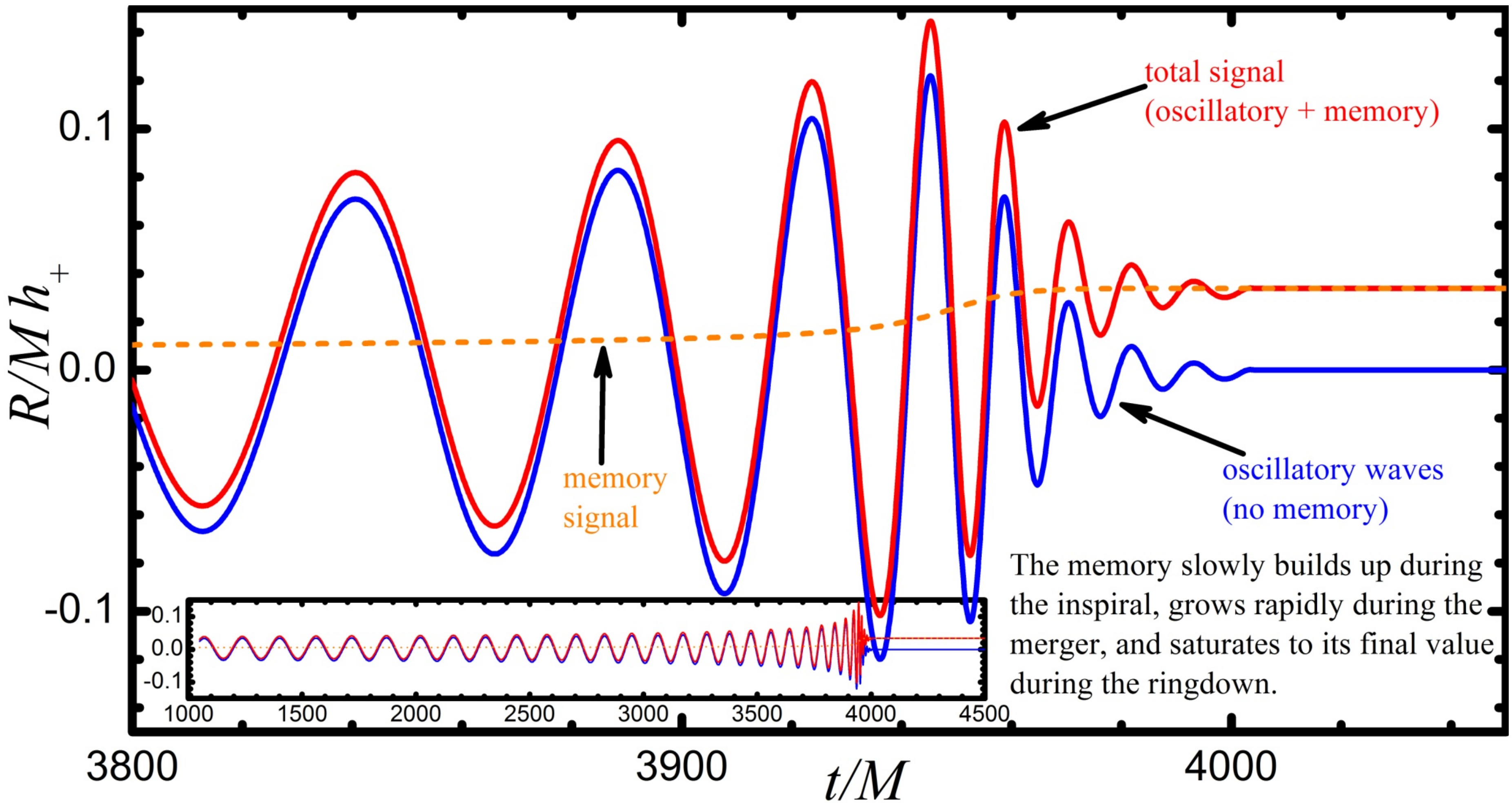
[Murphy, Ott, & Burrows '09]



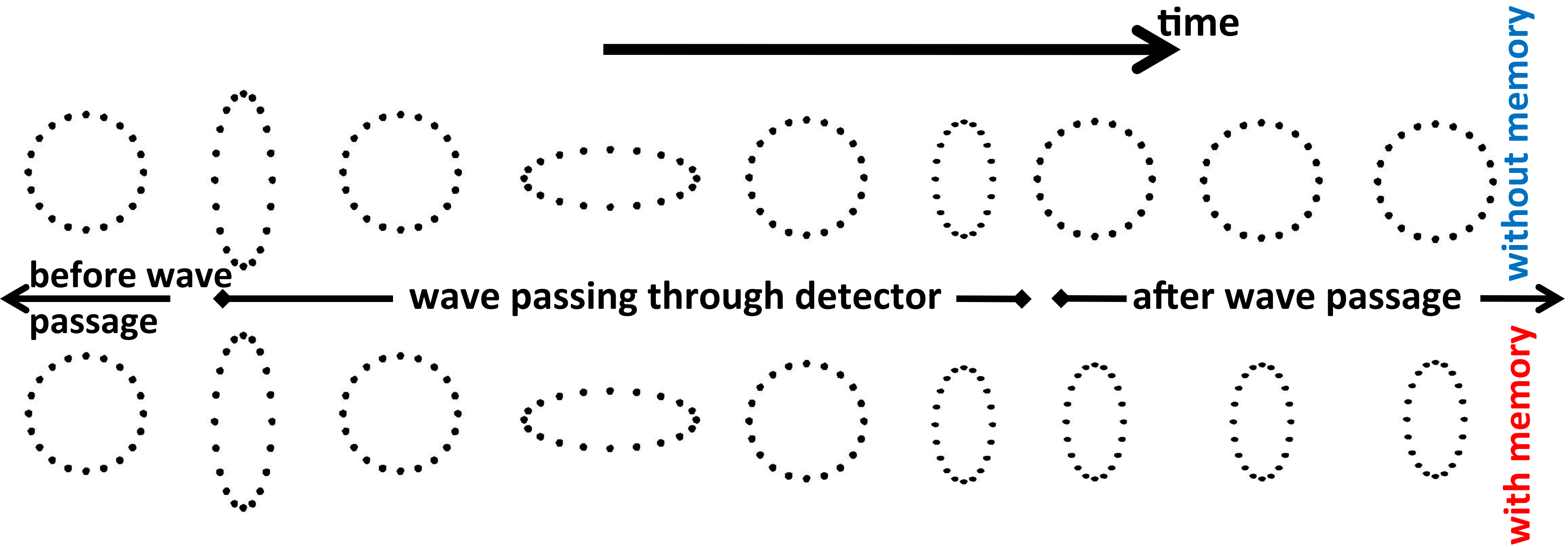


Examples of memory:

Binary black-hole mergers



Why is this called "memory"?



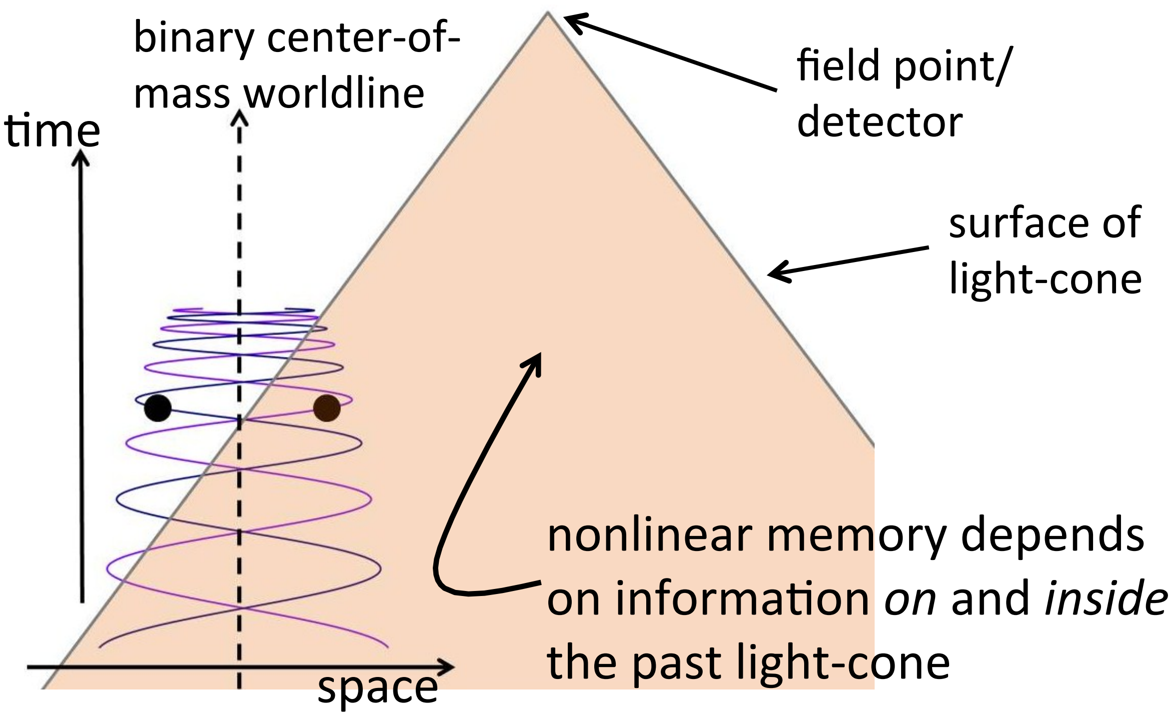
[GW propagating perpendicular to the screen]

Why is this interesting?

The nonlinear memory is unique among the many other nonlinear effects present in the gravitational-wave signal:

- it is non-oscillatory and visually distinctive in the waveform.
- it has a clear interpretation in terms of the GWs produced by GWs (more later).

- like GW “tails”, the nonlinear memory is ***hereditary***:



Why is this interesting?

The nonlinear memory is unique among the many other nonlinear effects present in the gravitational-wave signal:

• Unlike other post-Newtonian corrections, the memory affects the waveform at leading (Newtonian) order. Its "hereditary" nature allows a small effect to build-up to a large value over time.

$$h_+ = -2\frac{\mu}{R} v_{\rm orb}^2(t) \left[(1+\cos^2\iota)\cos[2\varphi(t) - 2\Phi] + \frac{1}{96}\sin^2\iota(17+\cos^2\iota) + O(v_{\rm orb}) \right]$$

[Wiseman & Will '91]

• The nonlinear memory is observable and could serve as a **test of general relativity**.

Understanding the memory:

the linear memory effect [Zel'Dovich & Polnarev '74; Braginsky & Grishchuk '85; Braginsky & Thorne '87]

$$\dot{x}_j(t) \to v^j_\infty$$

$$h^{\rm TT}_{jk} \approx \frac{2}{R} \ddot{\mathcal{I}}^{\rm TT}_{jk} \qquad \mathcal{I}^{\rm TT}_{jk} = \mu [x_j x_k]^{\rm TT}$$

$$\ddot{x}_j = -\frac{M}{r^3} x_j$$

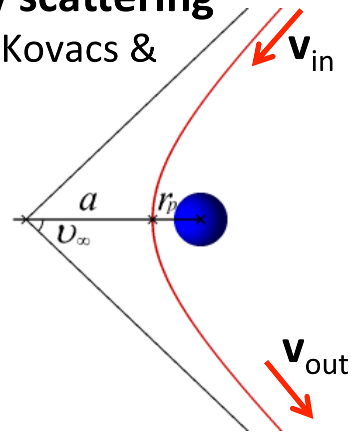
$$\ddot{\mathcal{I}}^{\rm TT}_{jk} = \mu [x_j \ddot{x}_k + \ddot{x}_j x_k + 2 \dot{x}_j \dot{x}_k]^{\rm TT}$$

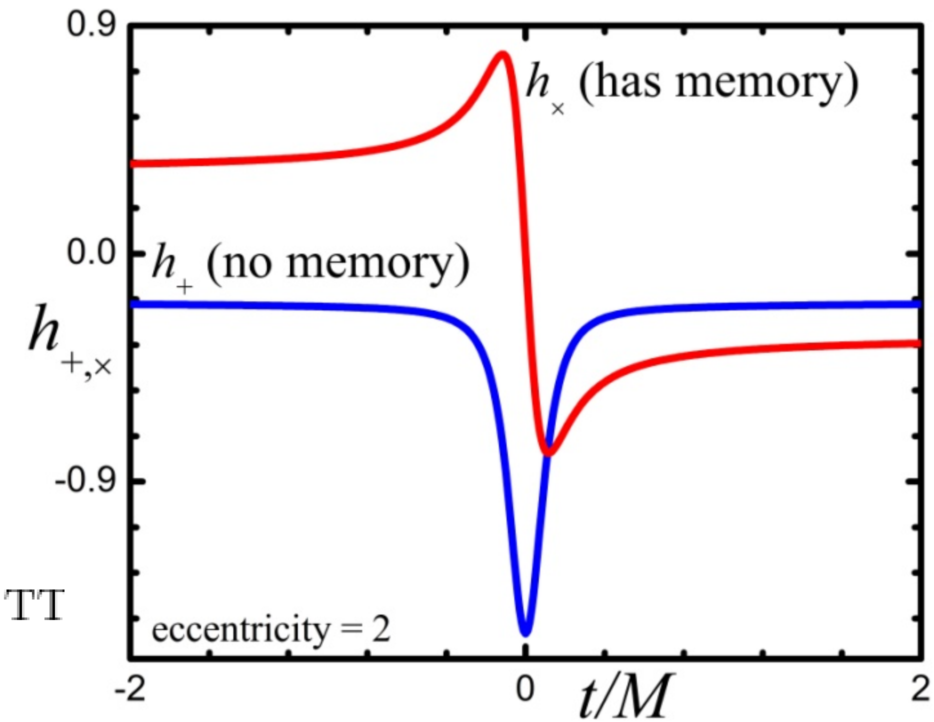
$$= 2\mu \left[\dot{x}_j \dot{x}_k - \frac{M}{r^3} x_j x_k \right]^{\rm TT} \longrightarrow \Delta h^{\rm TT}_{jk} = \frac{4\mu}{R} \Delta [v^j v^k]^{\rm TT}$$

Understanding the memory: the linear memory effect

[Zel'Dovich & Polnarev '74; Braginsky & Grishchuk '85; Braginsky & Thorne '87]

Hyperbolic orbit/two-body scattering
[Turner '77, Turner & Will '78, Kovacs & Thorne '78]





$$h_{jk}^{\rm TT} \approx \frac{2}{R}\ddot{\mathcal{I}}_{jk}^{\rm TT} \qquad \mathcal{I}_{jk}^{\rm TT} = \mu[x_j x_k]^{\rm TT}$$

$$\ddot{x}_j = -\frac{M}{r^3}x_j$$

$$\ddot{\mathcal{I}}_{jk}^{\rm TT} = \mu[x_j\ddot{x}_k + \ddot{x}_j x_k + 2\dot{x}_j\dot{x}_k]^{\rm TT}$$

$$= 2\mu\left[\dot{x}_j\dot{x}_k - \frac{M}{r^3}x_j x_k\right]^{\rm TT} \longrightarrow \Delta h_{jk}^{\rm TT} = \frac{4\mu}{R}\Delta[v^j v^k]^{\rm TT}$$

Understanding the memory:
the linear memory effect

General formula for the memory jump in a system w/ N components [Braginsky & Thorne '87, Thorne '92]

$$\Box \bar{h}_{ij} = -16\pi \sum_{A=1}^{N} T_{ij}^{\mathrm{pp},A}$$

$$\Delta h_{ij} = \lim_{t\to+\infty} h_{ij}(t) - \lim_{t\to-\infty} h_{ij}(t)$$

$$\Delta h_{ij}^{\mathrm{TT}} = \Delta \sum_{A=1}^{N} \frac{4M_A}{R\sqrt{1-v_A^2}} \left[\frac{v_A^j v_A^k}{1 - \boldsymbol{N}\cdot\boldsymbol{v}_A} \right]^{\mathrm{TT}}$$

Understanding the memory: the nonlinear memory

[Christodoulou ‘91; Blanchet & Damour ‘92]

Mathematically, the nonlinear memory arises from the contribution of the **gravitational-wave stress-energy** to Einstein’s equations:

Harmonic gauge EFE…

$$\Box \bar{h}^{\alpha\beta} = -16\pi(-g)(T^{\alpha\beta} + t^{\alpha\beta}_{\rm LL}) - \bar{h}^{\alpha\mu}{}_{,\nu}\bar{h}^{\beta\nu}{}_{,\mu} + \bar{h}^{\mu\nu}\bar{h}^{\alpha\beta}{}_{,\mu\nu}$$

…has a nonlinear source from the GW stress-energy tensor.

$$T^{\rm gw}_{jk} = \frac{1}{R^2}\frac{dE^{\rm gw}}{dt\,d\Omega} n_j n_k$$

Solve EFE:

$$\bar{h}_{jk}(t,\boldsymbol{x}) = 4\int \frac{(-g)[T_{jk}(t',\boldsymbol{x}') + t^{\rm LL}_{jk}(t',\boldsymbol{x}') + \dots]}{|\boldsymbol{x}-\boldsymbol{x}'|}\delta(t'-t-|\boldsymbol{x}-\boldsymbol{x}'|)\, d^4x'$$

[Wiseman & Will ‘91]

$$\delta h^{\rm TT}_{jk} = \frac{4}{R}\int_{-\infty}^{T_R} dt' \left[\int \frac{dE^{\rm gw}}{dt'\,d\Omega'}\frac{n'_j n'_k}{(1-\boldsymbol{n}'\cdot\boldsymbol{N})}d\Omega'\right]^{\rm TT}$$

Understanding the memory: the nonlinear memory

[Christodoulou ‘91; Blanchet & Damour ‘92]

Mathematically, the nonlinear memory arises from the contribution of the **gravitational-wave stress-energy** to Einstein’s equations:

Nonlinear memory can be related to the “linear” memory if we interpret the component masses as the individual radiated gravitons (Thorne’92):

$$\Delta h_{ij}^{\mathrm{TT}} = \Delta \sum_{A=1}^{N} \frac{4M_A}{R\sqrt{1-v_A^2}} \left[\frac{v_A^j v_A^k}{1-\boldsymbol{v}_A\cdot\boldsymbol{N}}\right]^{\mathrm{TT}}$$

$$v_A^j \to c n_A^j$$

$$\frac{M_A c^2}{\sqrt{1-v_A^2}} \to E_A$$

$$\Delta h_{ij}^{\mathrm{TT}} = \Delta \sum_{A=1}^{N} \frac{4E_A}{R} \left[\frac{n_A^j n_A^k}{1-\boldsymbol{n}_A\cdot\boldsymbol{N}}\right]^{\mathrm{TT}}$$

$$\delta h_{jk}^{\mathrm{TT}} = \frac{4}{R}\int_{-\infty}^{T_R} dt' \left[\int \frac{dE^{\mathrm{gw}}}{dt'\,d\Omega'} \frac{n'_j n'_k}{(1-\boldsymbol{n}'\cdot\boldsymbol{N})} d\Omega'\right]^{\mathrm{TT}}$$

Understanding the memory: the nonlinear memory

[Christodoulou '91; Blanchet & Damour '92]

Can also think of it as a nonlinear correction to the multipoles:

$$T^{\rm gw}_{\alpha\beta} \propto \frac{dE^{\rm gw}}{dt d\Omega} \sim O(h^2)$$

$$\ddot{\mathcal{I}}_{jk} \to \ddot{\mathcal{I}}_{jk} + U^{\rm gw}_{jk}$$

$$h^{\rm TT}_{jk} \approx \frac{2}{R} \ddot{\mathcal{I}}^{\rm TT}_{jk}$$

- Memory piece scales like the radiated energy.

$$\Delta h^{(\rm mem)} \sim \frac{\Delta E^{\rm gw}}{R}$$

- So the nonlinear memory is present in *all* GW sources.
- The effect is hereditary (depends on entire past evolution).

Understanding the memory:
the nonlinear memory: inspiralling binaries

Although it arises from a 2.5PN correction to the multipole moments, for inspiralling binaries the nonlinear affects the waveform at ***leading*** (Newtonian) order:

$$h_+ = -2\frac{\mu}{R} v_{\rm orb}^2 \left[(1+\cos^2\Theta)\cos[2\varphi(t) - 2\Phi] + \frac{1}{96}\sin^2\Theta(17+\cos^2\Theta) + O(v_{\rm orb}^{1/2}) \right]$$

[Wiseman & Will '91]

Why?

$$\Delta h^{jk}_{\rm mem} \sim \frac{\Delta E_{\rm GW}}{R} \qquad \Delta E_{\rm GW} \sim \Delta E_{\rm binding} \sim \frac{\mu M}{r} \sim \mu v_{\rm orb}^2$$

$$h^{ij}_{\rm oscil.} \propto \frac{1}{R}\ddot{\mathcal{I}}_{ij} \sim \frac{\mu}{R} v_{\rm orb}^2$$

Computing the nonlinear memory: previous/ongoing work (inspiral only)

✓ Wiseman & Will '91: 0PN memory waveform (circular, nonspinning).

✓ Thorne '92: analogy w/ linear memory; crude detectability estimates.

✓ Kennefick '94: repeats Wiseman-Will; crude detectability estimates.

✓ Wiseman & Will '91: nonlinear memory from high-velocity scattering (e>>1).

✓Arun , Blanchet, Iyer, et al '04, '08: compute 3PN waveform; 0.5PN memory vanishes (circular, nonspinning).

✓ MF '09a: 3PN memory waveform (circular, nonspinning).

✓ MF '11: leading-order nonlinear memory for eccentric binaries (elliptical, hyperbolic, parabolic, radial; nonspinning); crude detectability estimates.

✓ Guo & MF (in prep): 1.5PN memory waveform (spinning binaries).

Computing the nonlinear memory: previous/ongoing work (inspiral only)

Calculation of the inspiral memory is important because it allow us to:

- obtain analytical understanding of how the memory behaves.
- complete our knowledge of PN waveforms consistently to a given order.
- provide accurate initial conditions for the memory in NR calculations.

Computing the nonlinear memory: previous/ongoing work (merging BHs)

For detectability purposes, we need to know the entire build-up and saturation value of the memory (need inspiral + merger/ringdown).

✓MF ‘08, ‘09b: “minimal waveform model” & EOB calc; detectability estimates.

✓ MF (in prep): hybrid NR/PN calculation; improved detectability estimates.

✓ Pollney & Reisswig ’11: extraction from full NR evolutions; aligned spins.

✓Seto ‘09, van Haasteren & Levin ‘10, Pshrikov et al ’10, Cordes & Jenet ‘12: detectability estimates from pulsar timing arrays.

✓ Wang, Hobbs, & Wang ‘13: Parkes PTA search for memory.

✓ Plans for memory search in LIGO.

Computing the nonlinear memory: outline of the calculation

1. Waveform can be expanded in spin-weighted spherical harmonic modes:

$$h_+ - i h_\times = \sum_{l=2}^{\infty} \sum_{m=-l}^{l} h^{lm}(T_R, R)\, {}_{-2}Y^{lm}(\Theta, \Phi)$$

2. The nonlinear memory modes are related to the GW energy flux:

$$h^{(\mathrm{mem})}_{lm} = \frac{16\pi}{R}\sqrt{\frac{(l-2)!}{(l+2)!}} \int_{-\infty}^{T_R} dt \int d\Omega\, \frac{dE_{\mathrm{gw}}}{dt d\Omega}(\Omega) Y^*_{lm}(\Omega)$$

3. The energy flux is related to the oscillating (non-memory) h_{lm} modes:

$$\frac{dE_{\mathrm{gw}}}{dt d\Omega} = \frac{R^2}{16\pi}\langle \dot{h}_+^2 + \dot{h}_\times^2 \rangle = \frac{R^2}{16\pi} \sum_{l',l'',m',m''} \langle \dot{h}_{l'm'} \dot{h}^*_{l''m''} \rangle\, {}_{-2}Y^{l'm'}\, {}_{-2}Y^{l''m''\,*}$$

4. Compute time-derivative of $h_{lm}[v(t), \boldsymbol{L}(t), \boldsymbol{S}_1(t), \boldsymbol{S}_2(t), e(t)]$, substitute equations of motion, plug in and integrate.

Computing the nonlinear memory:
result: 3PN h_{lm} modes and polarization

$$h_{+,\times} = \frac{2\eta M x}{R} H_{+,\times} \;+\; O\left(\frac{1}{R^2}\right), \;\; \text{where} H_{+,\times} = \sum_{n=0}^{\infty} x^{n/2} H^{(n/2)}_{+,\times}.$$

[MF, Phys. Rev. D ‘09]

$$H_+^{(0,\text{mem})} = \alpha \frac{1}{96} s_\Theta^2 (17 + c_\Theta^2),$$

$$H_+^{(0.5,\text{mem})} = 0,$$

$$H_+^{(1,\text{mem})} = \alpha s_\Theta^2 \left[-\frac{354\,241}{2\,064\,384} - \frac{62\,059}{1\,032\,192} c_\Theta^2 - \frac{4195}{688\,128} c_\Theta^4 + \left(\frac{15\,607}{73\,728} + \frac{9373}{36\,864} c_\Theta^2 + \frac{215}{8192} c_\Theta^4 \right) \eta \right],$$

$$H_+^{(1.5,\text{mem})} = 0,$$

$$H_+^{(2,\text{mem})} = \alpha s_\Theta^2 \left[-\frac{3\,968\,456\,539}{9\,364\,045\,824} - \frac{570\,408\,173}{4\,682\,022\,912} c_\Theta^2 + \frac{122\,166\,887}{3\,121\,348\,608} c_\Theta^4 + \frac{75\,601}{15\,925\,248} c_\Theta^6 + \left(-\frac{7\,169\,749}{18\,579\,456} - \frac{13\,220\,477}{18\,579\,456} c_\Theta^2 - \frac{1\,345\,405}{6\,193\,152} c_\Theta^4 - \frac{25\,115}{884\,736} c_\Theta^6 \right) \eta + \left(\frac{10\,097}{147\,456} + \frac{5179}{36\,864} c_\Theta^2 + \frac{44\,765}{147\,456} c_\Theta^4 + \frac{3395}{73\,728} c_\Theta^6 \right) \eta^2 \right],$$

$$H_+^{(2.5,\text{mem})} = -\alpha \frac{5\pi}{21\,504} (1 - 4\eta) s_\Theta^2 \left(509 + 472 c_\Theta^2 + 39 c_\Theta^4 \right),$$

$$H_+^{(3,\text{mem})} = \alpha s_\Theta^2 \left\{ -\frac{69\,549\,016\,726\,181}{46\,146\,017\,820\,672} + \frac{6\,094\,001\,938\,489}{23\,073\,008\,910\,336} c_\Theta^2 - \frac{1\,416\,964\,616\,993}{15\,382\,005\,940\,224} c_\Theta^4 - \frac{2\,455\,732\,667}{78\,479\,622\,144} c_\Theta^6 - \frac{9\,979\,199}{2\,491\,416\,576} c_\Theta^8 + \left[\frac{1\,355\,497\,856\,557}{149\,824\,733\,184} - \frac{3485\pi^2}{9216} + \left(-\frac{3\,769\,402\,979}{4\,682\,022\,912} - \frac{205\pi^2}{9216} \right) c_\Theta^2 + \frac{31\,566\,573\,919}{49\,941\,577\,728} c_\Theta^4 + \frac{788\,261\,497}{3\,567\,255\,552} c_\Theta^6 + \frac{302\,431}{9\,437\,184} c_\Theta^8 \right] \eta + \left(\frac{5\,319\,395}{28\,311\,552} - \frac{24\,019\,355}{99\,090\,432} c_\Theta^2 - \frac{4\,438\,085}{3\,145\,728} c_\Theta^4 - \frac{3\,393\,935}{7\,077\,888} c_\Theta^6 - \frac{7835}{98\,304} c_\Theta^8 \right) \eta^2 + \left(\frac{1\,433\,545}{63\,700\,992} + \frac{752\,315}{15\,925\,248} c_\Theta^2 + \frac{129\,185}{2\,359\,296} c_\Theta^4 + \frac{389\,095}{1\,179\,648} c_\Theta^6 + \frac{9065}{131\,072} c_\Theta^8 \right) \eta^3 \right\},$$

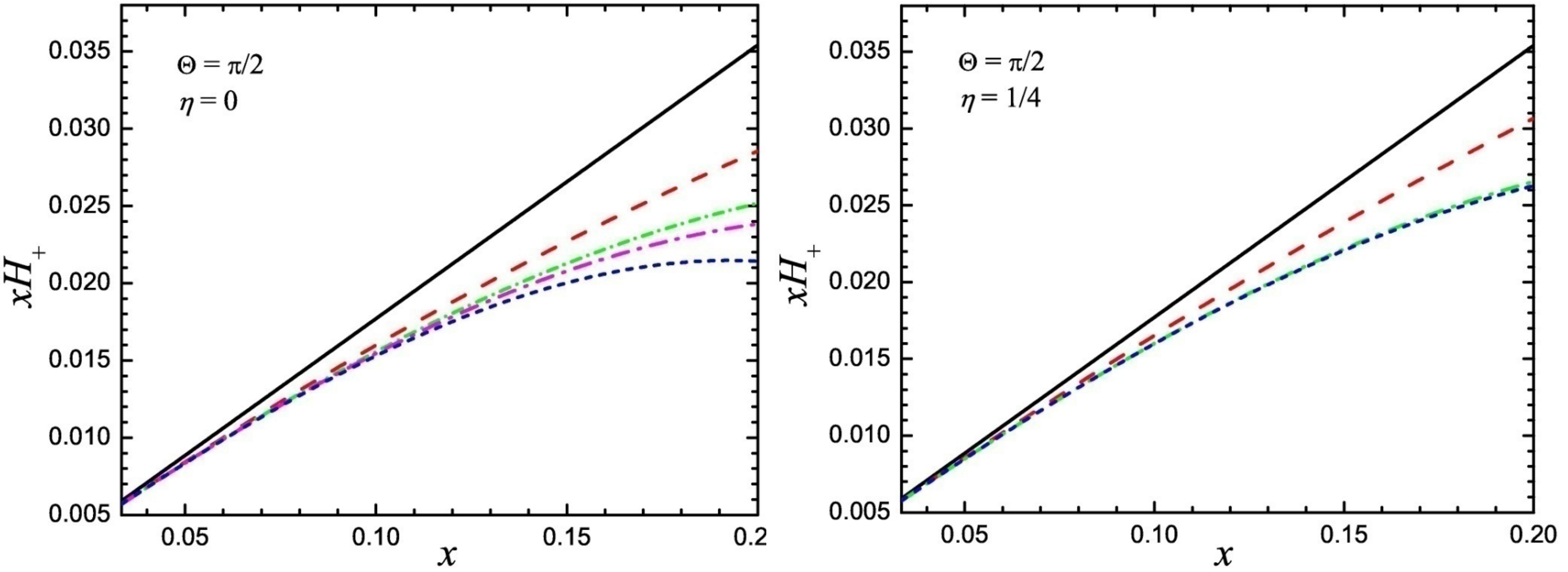
THIS COMPLETES THE WAVEFORM AMPLITUDE TO 3PN ORDER.

Computing the nonlinear memory:
result: 3PN h_{lm} modes and polarization

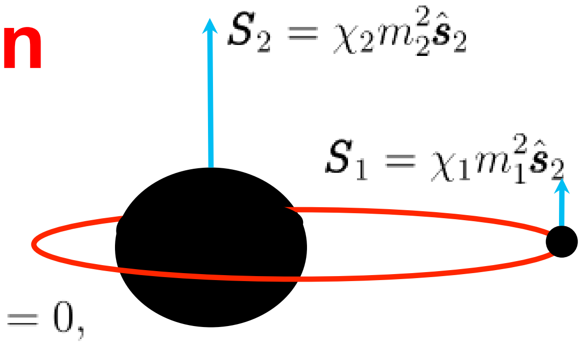
$$h_+^{\text{mem}} = \frac{2\eta M}{R} x H_+$$

$$x = (M\omega)^{2/3} \approx \frac{M}{r}[1 + O(c^{-2})]$$

0PN | 1PN | 2PN | 2.5PN | 3PN



Computing the nonlinear memory:
result: aligned spins, no precession



$$h_+^{(\mathrm{mem})} = \frac{2\eta M v^2}{R} \sum_{n=0}^{\infty} v^n H_+^{(n/2,\mathrm{mem})}.$$

$$H_+^{(0,\mathrm{mem})} = \frac{1}{96} s_\Theta^2 (17 + c_\Theta^2), \qquad H_+^{(0.5,\mathrm{mem})} = 0,$$

$$H_+^{(1,\mathrm{mem,nonspin})} = s_\Theta^2 \left[-\frac{354\,241}{2\,064\,384} - \frac{62\,059}{1\,032\,192} c_\Theta^2 - \frac{4195}{688\,128} c_\Theta^4 + \left(\frac{15\,607}{73\,728} + \frac{9373}{36\,864} c_\Theta^2 + \frac{215}{8192} c_\Theta^4 \right) \eta \right],$$

$$H_+^{(1.5,\mathrm{mem,nonspin})} = 0, \qquad H_+^{(1.5,\mathrm{mem,spin})} = \frac{s_\Theta^2}{768} \sum_{i=1,2} \chi_i \kappa_i \left[369 \frac{m_i^2}{M^2} + 351\eta + c_\Theta^2 \left(23 \frac{m_i^2}{M^2} + 57\eta \right) \right],$$

$$\chi_i = |S_i|/m_i$$

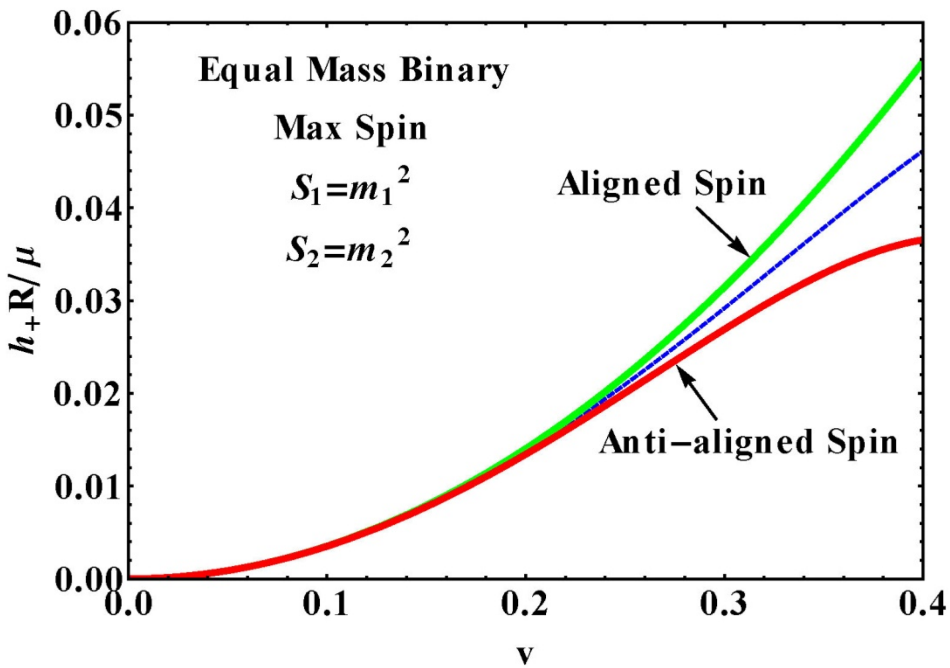
$$\kappa_i = |\hat{s}_i| \cdot \hat{L}$$

$$s_\Theta = \sin\Theta$$

$$c_\Theta = \cos\Theta$$

$$M = m_1 + m_2$$

$$\eta = \frac{m_1 m_2}{M^2}$$



spin contributes a ~20% - 30% correction.

[Guo & MF (in prep)]

Computing the nonlinear memory:
result: precessing spins (small inclination approx.)

$$h_+^{(\mathrm{mem})} = \frac{2\eta M v^2}{R} \sum_{n=0}^{\infty} v^n H_+^{(n/2,\mathrm{mem})}.$$

$$H_+^{(0,\mathrm{mem})} = \frac{1}{96} s_\Theta^2 (17 + c_\Theta^2), \qquad H_+^{(0.5,\mathrm{mem})} = 0,$$

$$H_+^{(1,\mathrm{mem,nonspin})} = s_\Theta^2 \left[-\frac{354\,241}{2\,064\,384} - \frac{62\,059}{1\,032\,192} c_\Theta^2 - \frac{4195}{688\,128} c_\Theta^4 + \left(\frac{15\,607}{73\,728} + \frac{9373}{36\,864} c_\Theta^2 + \frac{215}{8192} c_\Theta^4 \right) \eta \right],$$

$$H_+^{(1,\mathrm{mem,spin})} = -s_\Theta^2 \sum_{i=1,2} \chi_i^2 (1 - \kappa_i^2) \frac{m_i^4}{M^4} \left(\frac{25 + 5c_\theta^2}{192\eta^2} \right)$$

$$H_+^{(1.5,\mathrm{mem,nonspin})} = 0, \qquad H_+^{(1.5,\mathrm{mem,spin})} = \frac{s_\Theta^2}{768} \sum_{i=1,2} \chi_i \kappa_i \left[369 \frac{m_i^2}{M^2} + 351\eta + c_\Theta^2 \left(23 \frac{m_i^2}{M^2} + 57\eta \right) \right],$$

$$\chi_i = |\boldsymbol{S}_i|/m_i$$

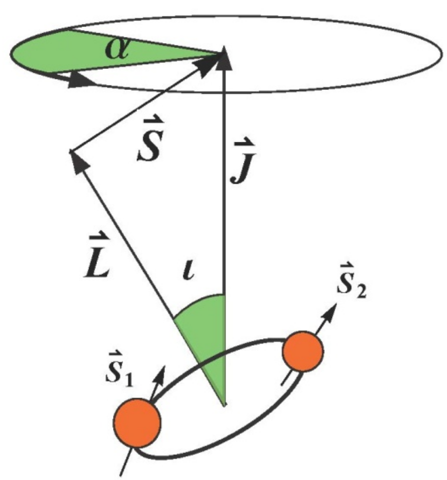
$$\kappa_i = |\hat{\boldsymbol{s}}_i| \cdot \hat{L}$$

$$s_\Theta = \sin\Theta$$

$$c_\Theta = \cos\Theta$$

$$M = m_1 + m_2$$

$$\eta = \frac{m_1 m_2}{M^2}$$

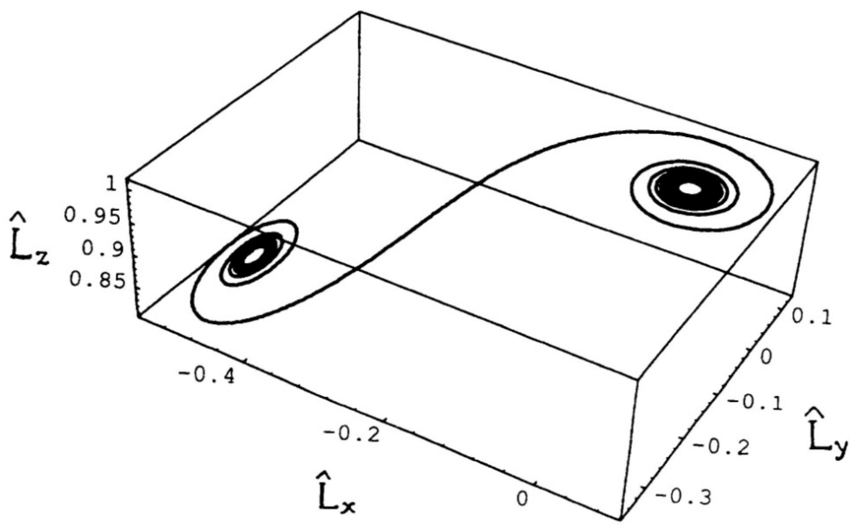


$$\iota \ll 1 \to |\boldsymbol{L}| \gg |\boldsymbol{S}|$$

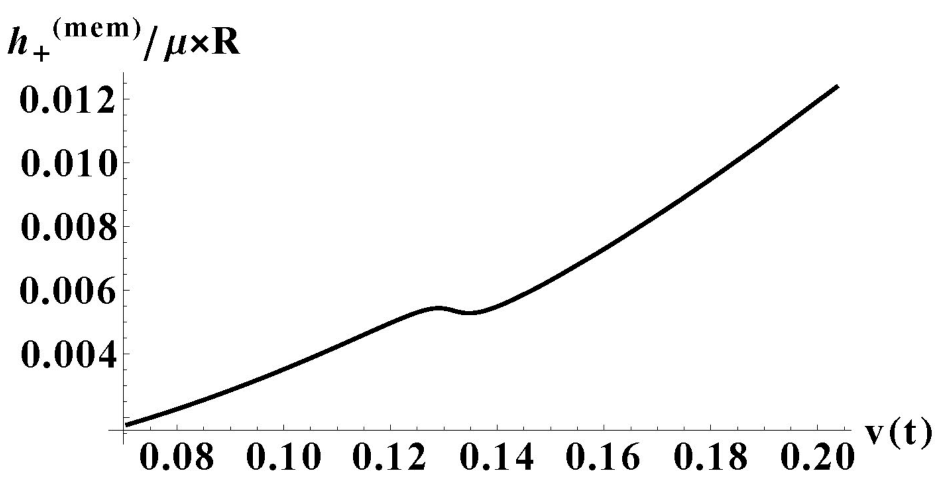
[Guo & MF (in prep)]

Computing the nonlinear memory:
result: arbitrary spins

- Need to solve spin equations numerically.
- Simple precession case: memory monotonically increases as before.
- Transitional precession (preliminary): mostly monotonic increase, except for a single oscillation during the "transition"



[Apostolatos et al'94]



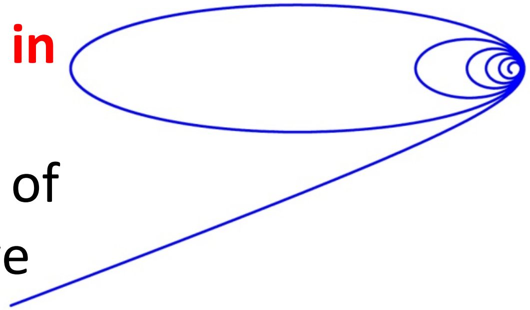
[Guo & MF (in prep)]

Computing the nonlinear memory:
eccentric binaries: motivation

- elliptical orbit (e<1) waveforms:
  - ✓ 0PN (Peters-Mathews '63)
  - ✓ 1PN (Junker-Schafer '91)
  - ✓ 2PN (Gopakumar-Iyer '02)
  - ❑ nonlinear memory correction unknown

- hyperbolic/parabolic orbit (e≥1) waveforms:
  - ✓ 0PN (Mike Turner '77)
  - ✓ 1PN (Junker-Schafer '91)
  - ❑ nonlinear memory only computed for e>>1 case (Wiseman-Will '91)

- **Circularized binaries were eccentric (even hyperbolic) in the past:**
  - ❑ Since the nonlinear memory is hereditary, effects of the (past-growing) eccentricity could potentially have an effect on the value of the (late-time) memory.



Computing the nonlinear memory: eccentric binaries: outline of calculation

1. We want the leading-order nonlinear memory terms: these rely on only the leading-order mass-quadrupole moments for Keplerian orbits:

$$h_{2m}^{\rm N} \approx \frac{\ddot{I}_{2m}}{R\sqrt{2}}$$

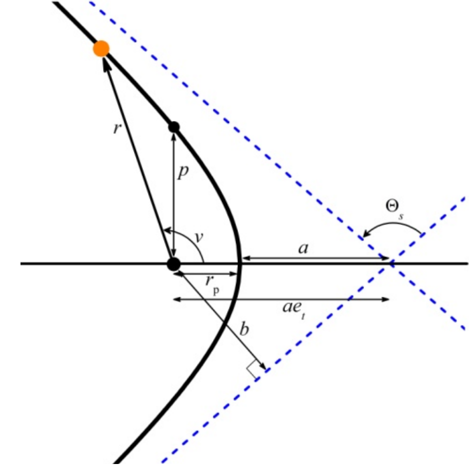
2. These leading-order modes are substituted into an expression for the nonlinear memory modes:

$$h_{lm}^{(\rm mem)} = \frac{16\pi}{R}\sqrt{\frac{(l-2)!}{(l+2)!}}\int_{-\infty}^{T_R} dt \int d\Omega \, \frac{dE_{\rm gw}}{dt d\Omega}(\Omega) Y_{lm}^{*}(\Omega)$$

$$h_{20}^{(\rm mem)} = \int_{-\infty}^{T_R} dt \, \frac{R}{42}\sqrt{\frac{15}{2\pi}} \left\langle 2|\dot{h}_{22}^{\rm N}|^2 - |\dot{h}_{20}^{\rm N}|^2 \right\rangle$$

3. To compute the time integral, use a Keplerian model for the orbit (including radiation-reaction in the elliptical case). Transform time-integral to an integral over the true-anomaly or the eccentricity.

Computing the nonlinear memory: result: eccentric binaries

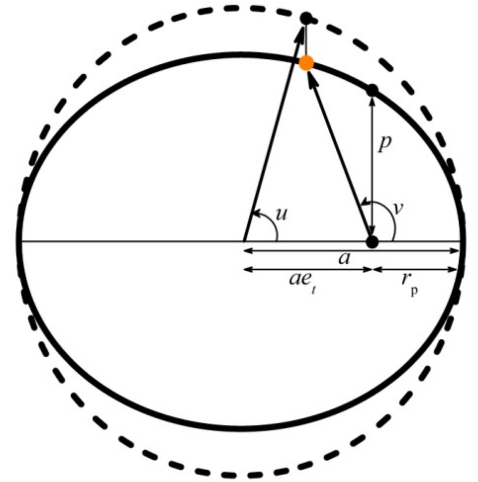


linear memory case:

Hyperbolic orbits: $\Delta h^{(\text{lin. mem})} \propto \eta \left(\frac{M}{R}\right)\left(\frac{M}{p}\right)\frac{(e^2-1)^{3/2}}{e^2}$

Parabolic orbits: no linear memory

Elliptical orbits: no linear memory



Nonlinear memory case:

Hyperbolic/parabolic orbits: $\Delta h^{(\text{mem})} \propto \eta^2 \left(\frac{M}{R}\right)\left(\frac{M}{p}\right)^{7/2} F(e)$

2.5PN

Elliptical orbits:

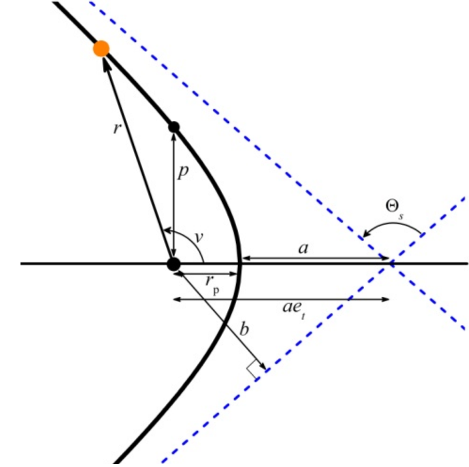
$$h_{20}^{(\text{mem})} = -\frac{2}{7}\sqrt{\frac{10\pi}{3}}\frac{\eta M^2}{R p_0} e_0^{12/19}(304+121e_0^2)^{870/2299}\int_{e_0}^{e(t)} de \frac{1}{e^{31/19}}\frac{(192+580e^2+73e^4)}{(304+121e^2)^{3169/2299}}$$

$$\Delta h^{(\text{mem})} \propto \eta \left(\frac{M}{R}\right)\left(\frac{M}{p_0}\right)\left[1-\left(\frac{e_0}{e(t)}\right)^{12/19}\right]$$

0PN

[MF PRD'11]

Computing the nonlinear memory: result: eccentric binaries



$$\Delta h^{(\mathrm{mem})} = \int_{-\infty}^{T_R} \dot{h}^{(\mathrm{mem})}\, dt$$

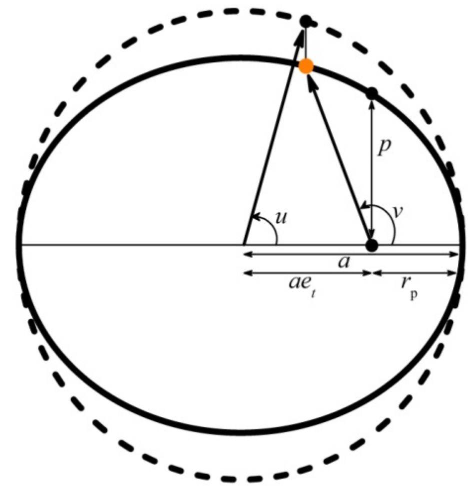
$$\Delta h^{(\mathrm{mem}),\mathrm{hyper.}} \sim \dot{h}^{(\mathrm{mem})} T_{\mathrm{orb}}$$

$$\Delta h^{(\mathrm{mem}),\mathrm{ellip.}} \sim \dot{h}^{(\mathrm{mem})} T_{\mathrm{rr}}$$

$$T_{\mathrm{orb}} \sim M \left(\frac{p}{M}\right)^{3/2}$$

$$T_{\mathrm{rr}} \sim \frac{M}{\eta} \left(\frac{p}{M}\right)^{5}$$

$$\frac{T_{\mathrm{orb}}}{T_{\mathrm{rr}}} \sim \eta \left(\frac{M}{p}\right)^{5/2}$$



Nonlinear memory case:

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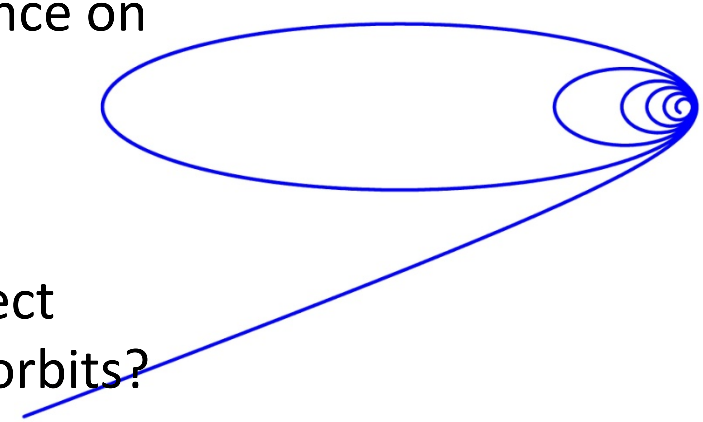
$$\Delta h^{(\mathrm{mem})} \propto \eta \left(\frac{M}{R}\right) \left(\frac{M}{p_0}\right) \left[1 - \left(\frac{e_0}{e(t)}\right)^{12/19}\right]$$

0PN

[MF PRD'11]

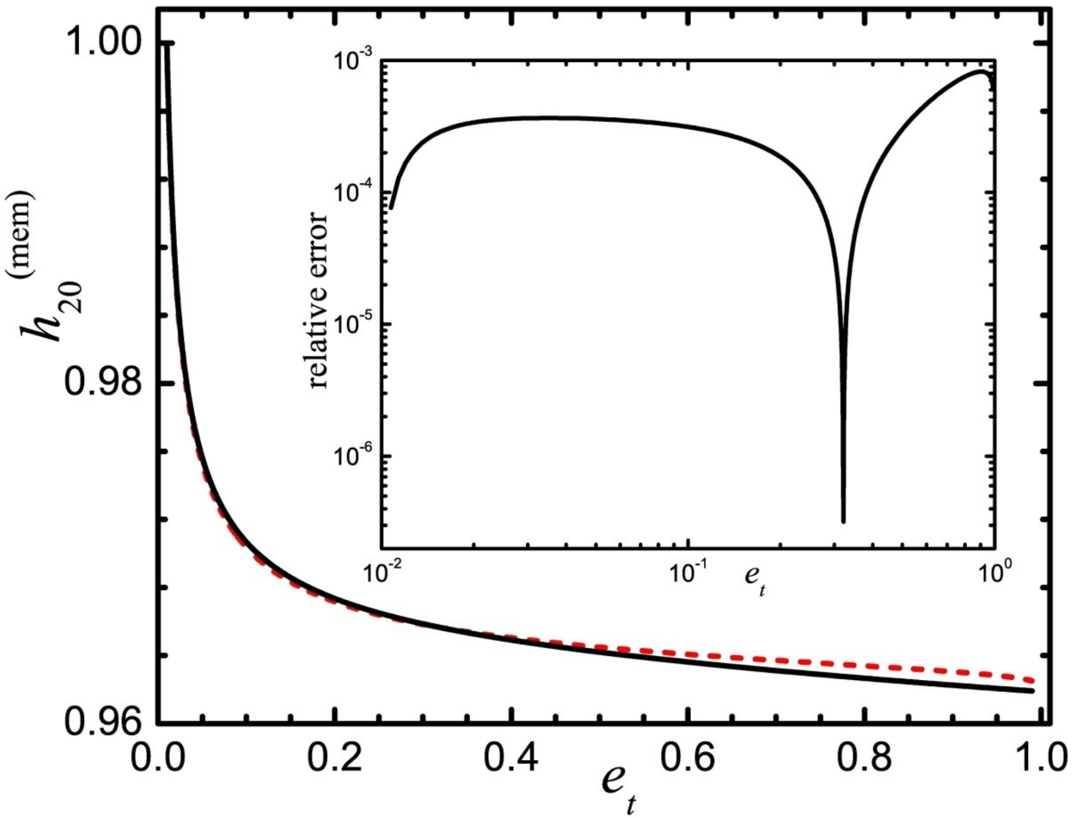
Computing the nonlinear memory: result: sensitivity to early-time eccentricity

- Hereditary nature of memory implies dependence on past orbital evolution.



- BH binaries usually assumed to be circularized.

- Does the binary’s past-growing eccentricity affect memory calculations that assume quasi-circular orbits?



[MF PRD’11]

Summary of results in eccentric case:

• The nonlinear memory is a non-oscillatory component of the gravitational-wave signal; it is due to *gravitational waves produced by gravitational waves*.

• Previous calculations considered only quasi-circular binaries.

• Nonlinear memory waveforms have been computed for binaries with any eccentricity (elliptical, hyperbolic, parabolic, and radial orbits).

• In the hyperbolic/parabolic case, nonlinear memory enters at 2.5PN order.

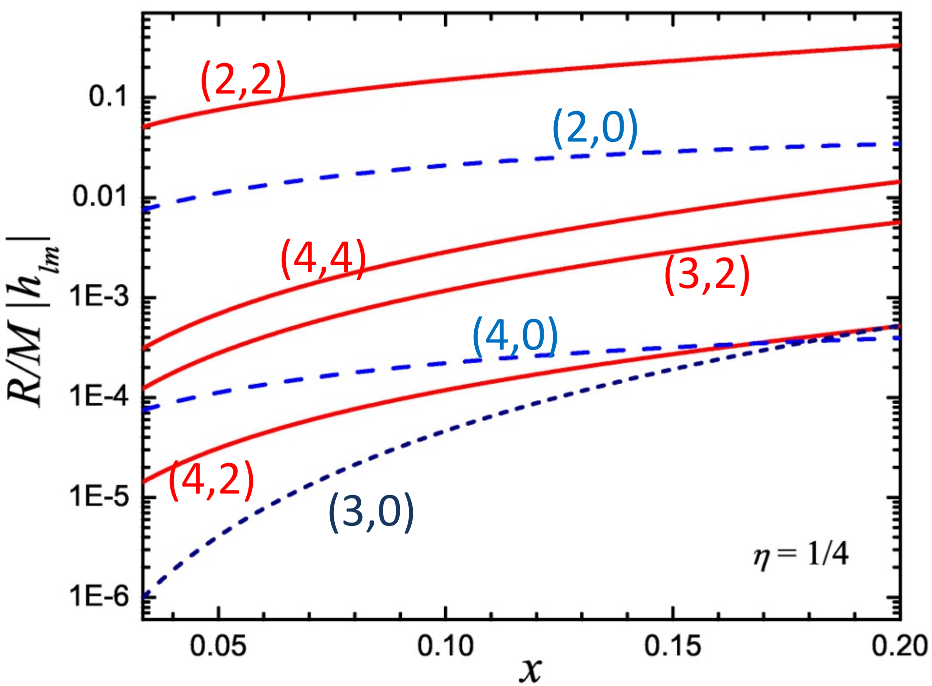
• In the elliptic case, nonlinear memory enters at the same order as the Peters-Mathews waveforms (0PN order, as in the circular case).

• While the nonlinear memory depends on the past-history of the binary, the past-growing eccentricity is only a small correction to the memory for nearly-circularized binaries.

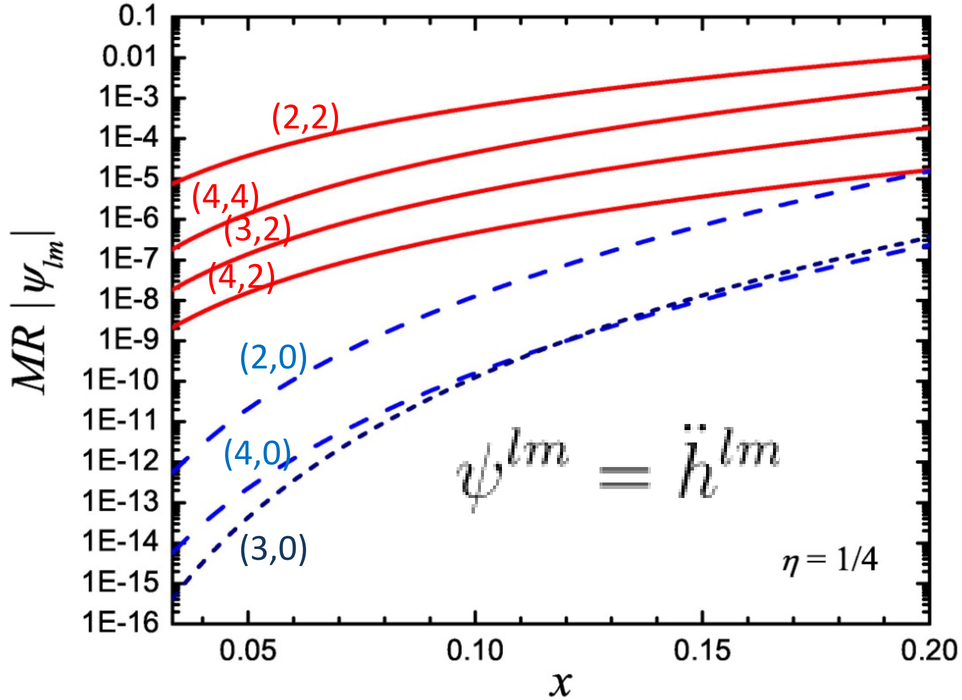
Calculating the nonlinear memory from BH mergers: limitations of numerical relativity

- **No memory in (2,2) mode!** Shows up in m=0 modes in quasi-circular case.
- In Ψ_4 memory is a 5PN order effect; in h_+ it is a 0PN effect.

(2,2), (4,4), (3,2), (4,2) modes are much larger than the memory modes (2,0), (4, 0), etc..



Orbital separation decreasing →



Orbital separation decreasing →

[MF PRD'09]

Calculating the nonlinear memory from BH mergers: limitations of numerical relativity

Other problems with NR computations of the memory:

- Need to choose two integration constants to go from curvature to metric perturbation $\psi_{lm} = \ddot{h}_{lm}$

  Choosing these incorrectly leads to "artificial" memory (Berti et al. '07).
- Memory is sensitive to the past-history of the source; need large initial separation:
  - Consider leading-order (2,0) memory mode, with a finite separation r_0

$$h_{20}^{\rm NR}(T_R) = \frac{4}{7}\sqrt{\frac{5\pi}{6}}\frac{\eta M}{R}\left(\frac{M}{r(t)} - \frac{M}{r_0}\right)$$

$$\frac{|\delta h_{20}^{\rm NR}|}{h_{20}} \approx \frac{r(t)}{r_0}$$

- Errors from gauge effects and finite extraction radius can further contaminate NR waveforms and swamp a small memory signal.

[MF PRD'09]

Calculating the nonlinear memory from BH mergers: an analytic hybrid scheme

Recall that the nonlinear memory contributes a piece to the h_{lm} modes that depends on integrals over the gravitational-wave energy flux…

$$h_{lm}^{(\mathrm{mem})} = \frac{32\pi}{R\sqrt{2}}\sqrt{\frac{(l-2)!}{2(l+2)!}}\int_{-\infty}^{T_R} dt \int d\Omega\, \frac{dE_{\mathrm{gw}}}{dt d\Omega}(\Omega) Y_{lm}^{*}(\Omega)$$

…where this flux is itself a sum over the h^{lm} modes :

$$\frac{dE_{\mathrm{gw}}}{dt d\Omega} = \frac{R^2}{16\pi}\langle \dot{h}_{+}^{2} + \dot{h}_{\times}^{2}\rangle = \frac{R^2}{16\pi}\sum_{l',l'',m',m''}\langle \dot{h}_{l'm'}\dot{h}^{*}_{l''m''}\rangle {}_{-2}Y^{l'm'}\,{}_{-2}Y^{l''m''\,*}$$

The nonlinear memory only appears in the m=0 modes, which can be expanded in terms of the m≠0 modes:

$$h_{20}^{(\mathrm{mem})} = \frac{R}{21}\sqrt{\frac{5}{2\pi}}\int_{-\infty}^{T_R} dt\, \left[|\dot{h}_{22}|^2 + \frac{\sqrt{35}}{4}\left(\dot{h}_{22}\dot{h}^{*}_{32} + \dot{h}^{*}_{22}\dot{h}_{32}\right) + \text{higher order modes} \right]$$

Plug in merger/ringdown description for h_{22} , etc…and then match to 3PN inspiral calculation of h_{20} .

Calculating the nonlinear memory from BH mergers:
an analytic hybrid scheme: minimal waveform model

Only consider leading-order (2,2) contribution to the memory…

$$h_{20}^{(\mathrm{mem})} = \frac{R}{21}\sqrt{\frac{5}{2\pi}}\int_{-\infty}^{T_R} dt\, |\dot{h}_{22}|^2$$

…and use a simple model for the inspiral and ringdown (2,2) modes…

$$h_{22}^{\mathrm{insp}} = -\frac{8}{R}\sqrt{\frac{\pi}{5}}\frac{\eta m^2}{r}e^{-2i\phi(t)} \qquad h_{22}^{\mathrm{ring}} = \frac{1}{R}\sum_{n=0}^{2} A_{22n}e^{-\sigma_{22n}t}$$

$$\sigma_{lmn} = i\omega_{lmn} + \tau_{lmn}^{-1}$$

…matching the multipoles and 2 derivatives at the light-ring (r/m ≈ 3) to get A_{22n} .

$r(t)$ and $\phi(t)$ evolve via the standard leading-order formulas:

$$r(t) = r_m(1 - t/\tau_{rr})^{1/4} \quad \phi(t) = \sqrt{m/r^3}\,t + \phi_0$$

Use NR results to determine the final mass and spin (as a function of η) that enter the quasi-normal mode (QNM) frequencies and damping times.

Get a fully analytic time-domain solution for the full memory:

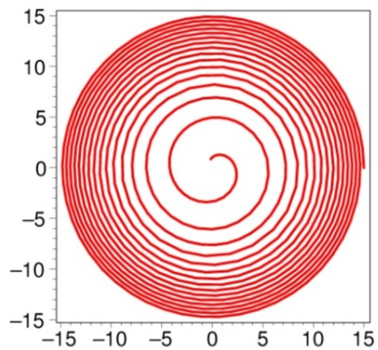
$$\hat{h}_{\mathrm{MWM}}^{\mathrm{mem}} = \frac{8\pi M}{r(T)}H(-T) + H(T)\left\{\frac{8\pi M}{r_m} + \frac{1}{\eta M}\sum_{n,n'=0}^{n_{\max}}\frac{\sigma_{22n}\sigma_{22n'}^{*}A_{22n}A_{22n'}^{*}}{\sigma_{22n}+\sigma_{22n'}^{*}}\left[1 - e^{-(\sigma_{22n}+\sigma_{22n'}^{*})T}\right]\right\}$$

[MF ApJL '09]

Calculating the nonlinear memory from BH mergers: an analytic hybrid scheme: EOB model

Only consider leading-order (2,2) contribution to the memory…

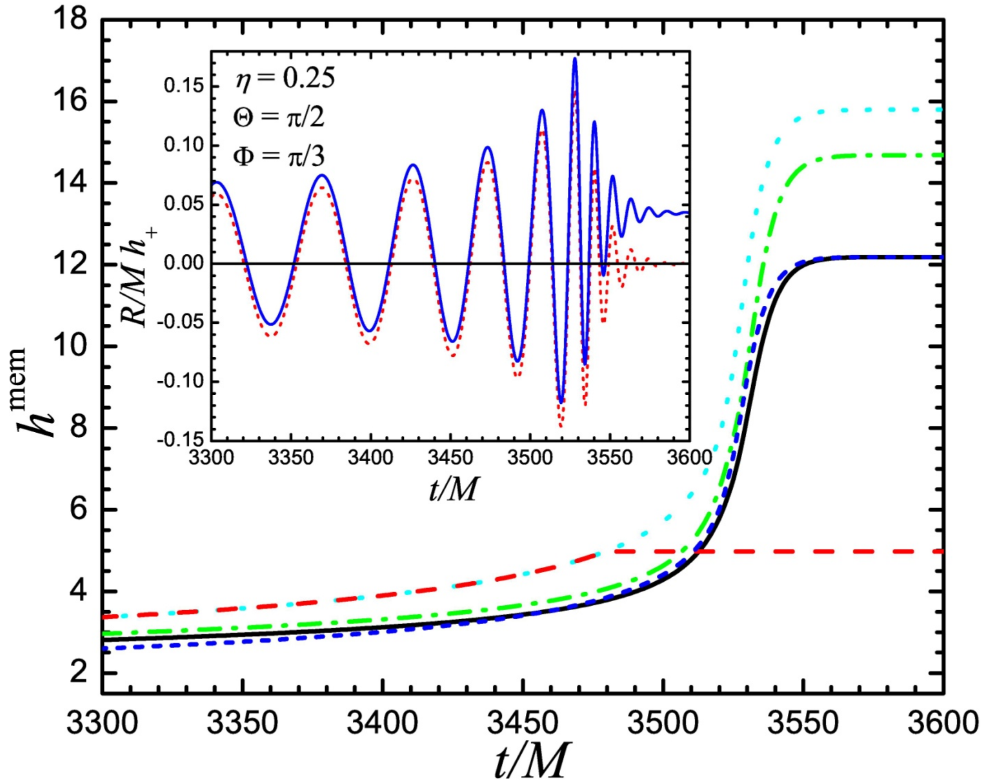
$$h_{20}^{(\mathrm{mem})} = \frac{R}{21}\sqrt{\frac{5}{2\pi}}\int_{-\infty}^{T_R} dt\,|\dot{h}_{22}|^2$$



Or use an EOB model calibrated to NR simulations:

$$h_{22}^{\mathrm{EOB,insp}} = -\frac{8M}{R}\sqrt{\frac{\pi}{5}}(r_\omega \Omega)^2 e^{\mp 2i\varphi} F_{22} f_{22}^{\mathrm{NQC}}$$

Solve EOB Hamilton's equation for particle motion, and match inspiral modes to 5 QNMs.



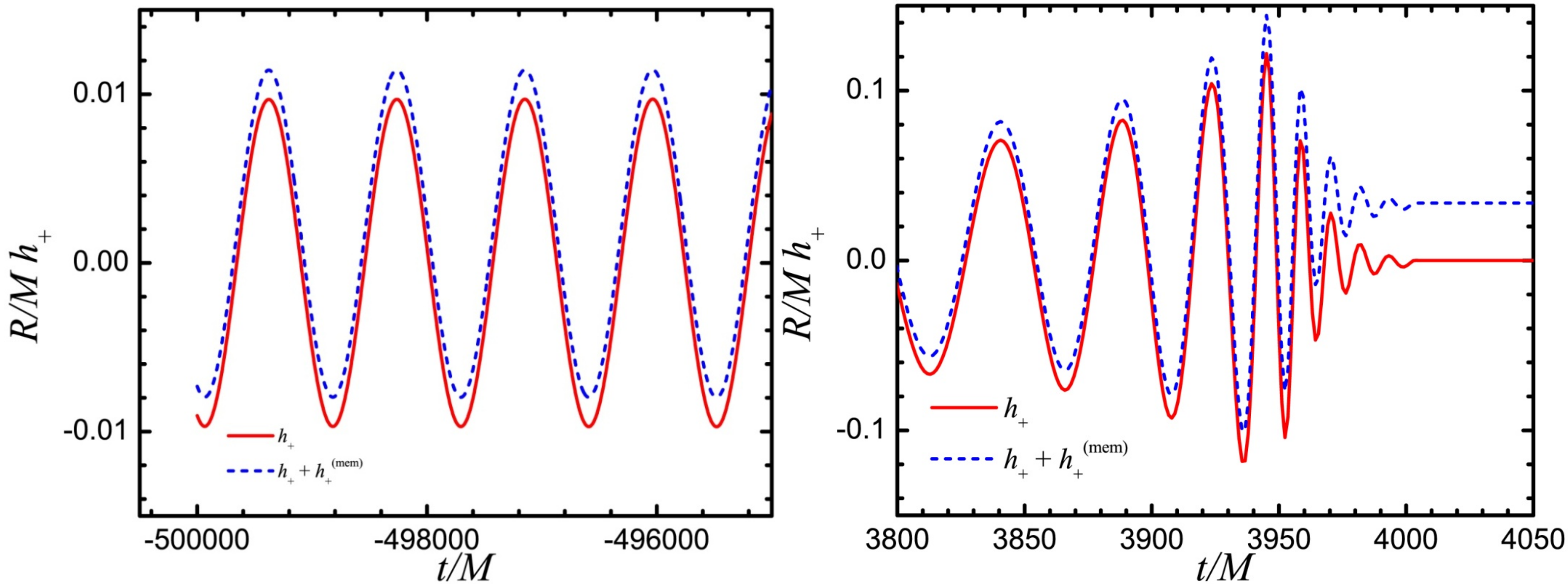
[MF, ApJL, '09]

Calculating the nonlinear memory from BH mergers: an analytic hybrid scheme: direct NR-hybrid

Only consider leading-order (2,2) contribution to the memory…

$$h_{20}^{(\mathrm{mem})} = \frac{R}{21}\sqrt{\frac{5}{2\pi}}\int_{-\infty}^{T_R} dt\, |\dot{h}_{22}|^2$$

Or directly use the numerical NR (2,2) mode [Caltech/Cornell/CITA]:

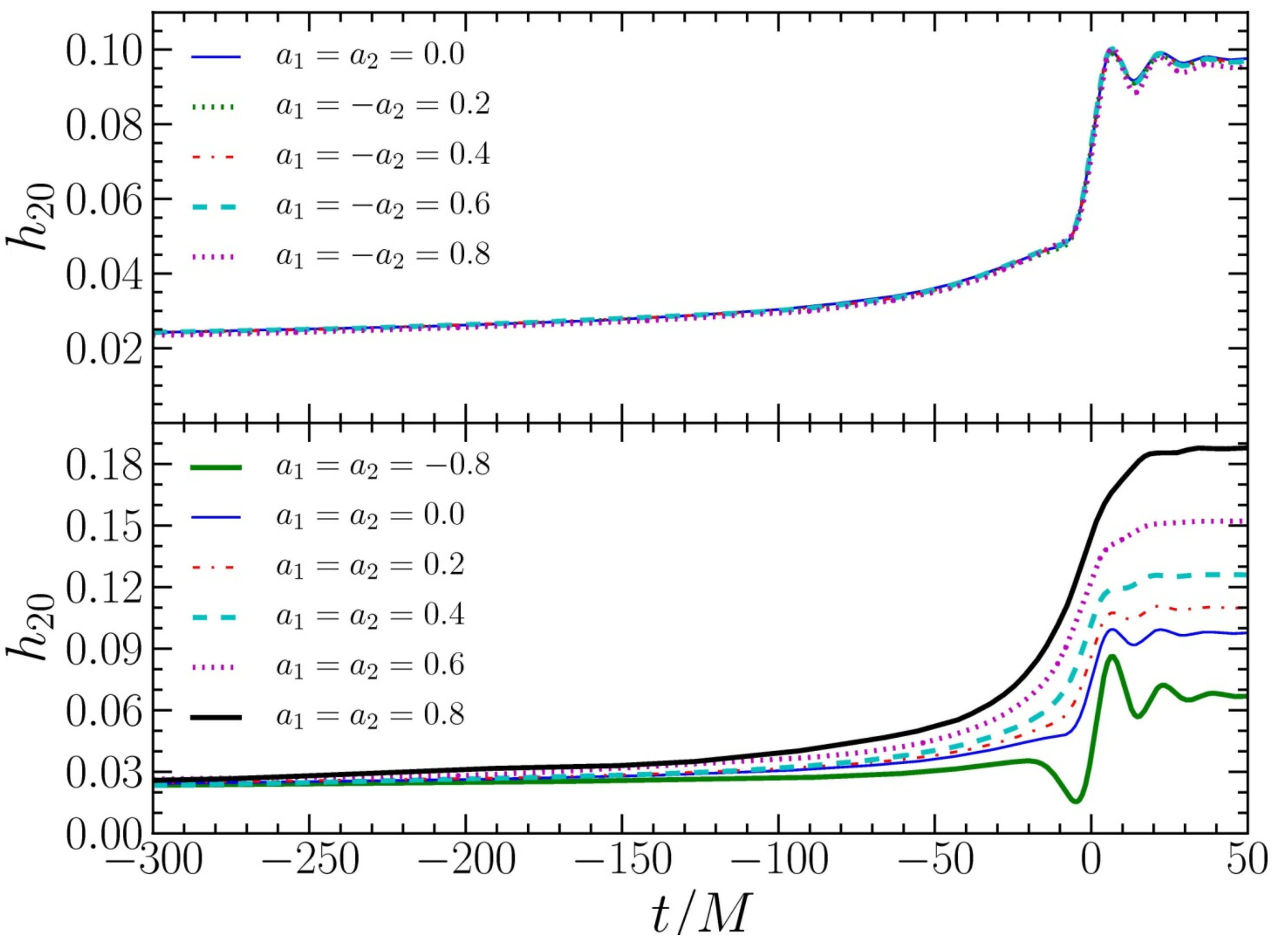


- ~27% smaller than previous EOB calculation.
- smooth matching to 3PN order inspiral allows evaluation to arbitrarily early times.

Memory during the merger/ringdown: NR results

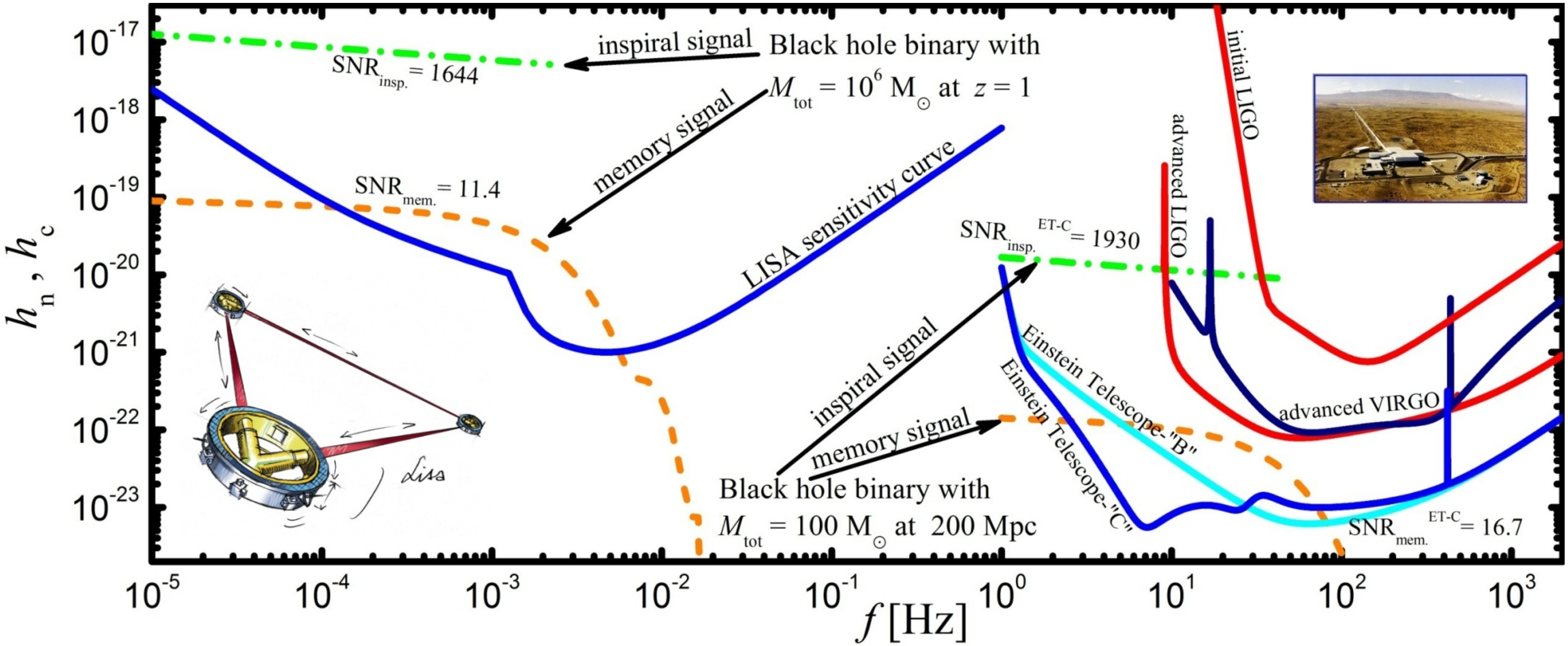
Pollney & Reissweg [arXiv:1004.4209] extract memory directly from NR simulation:

- causally-disconnected outer boundary
- Cauchy characteristic extraction to compute GWs at Scri+

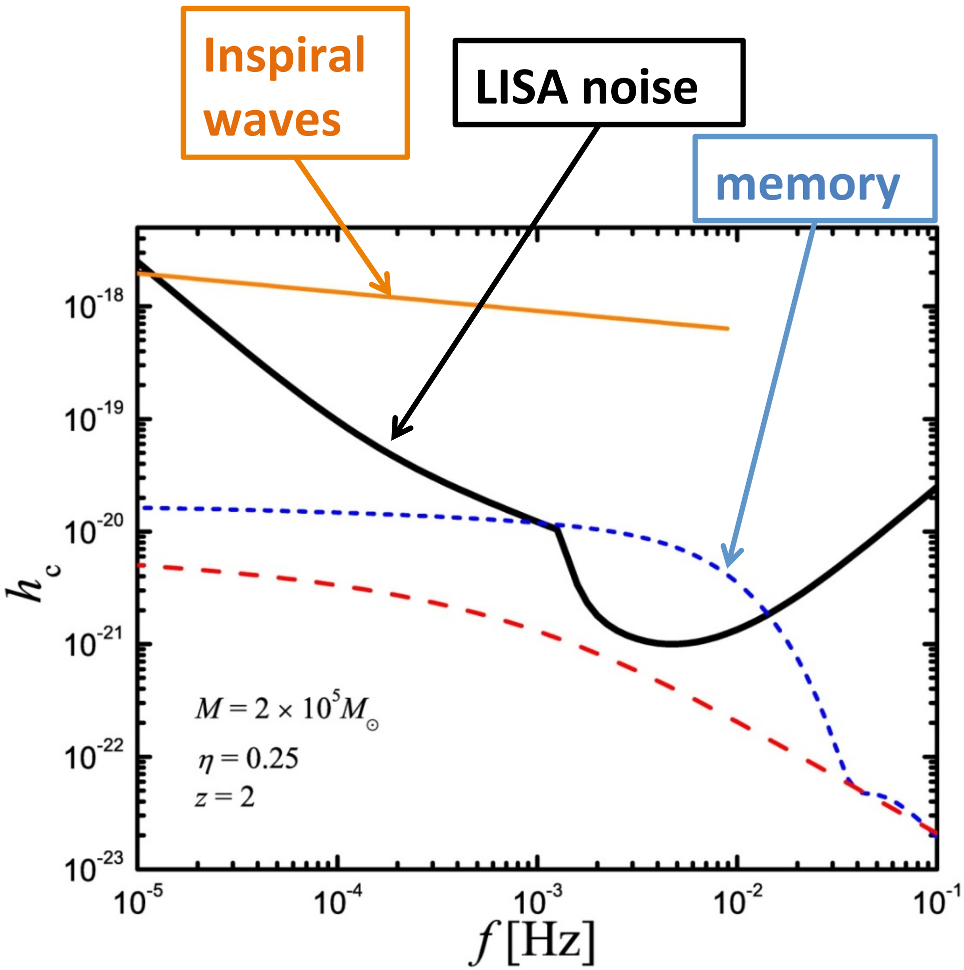


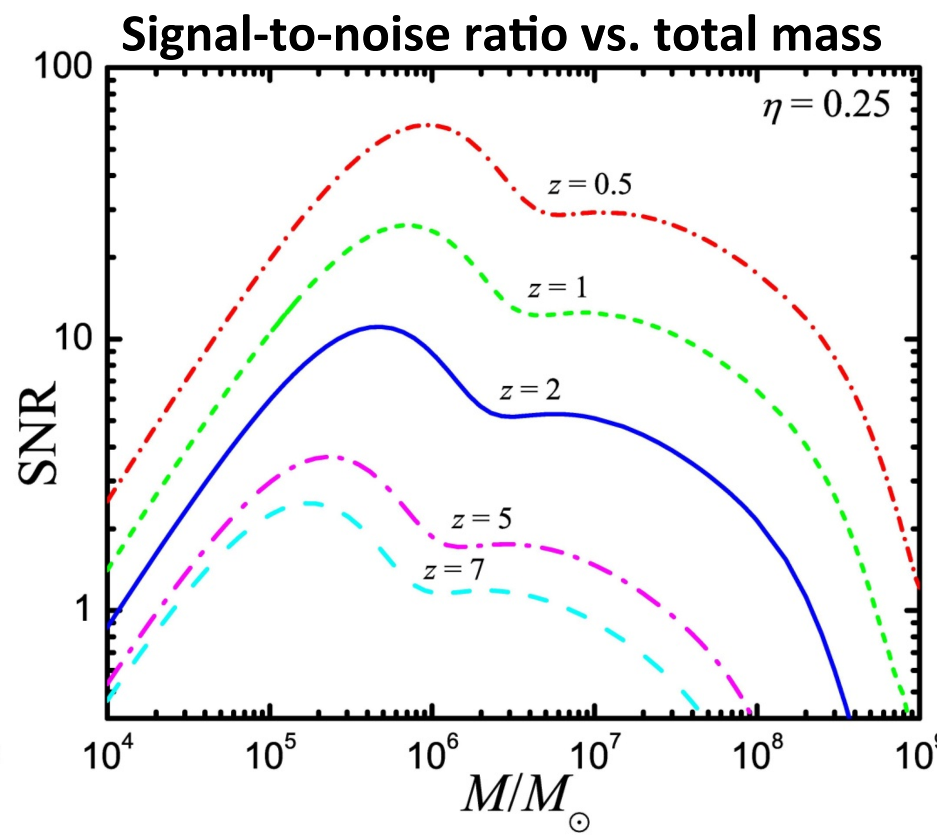
- equal-mass, equal spins aligned or anti-aligned.
- excitation of (2,0) QNM.
- accurate 3PN inspiral memory crucial to getting correct magnitude.

Detectability of the nonlinear memory: interferometers



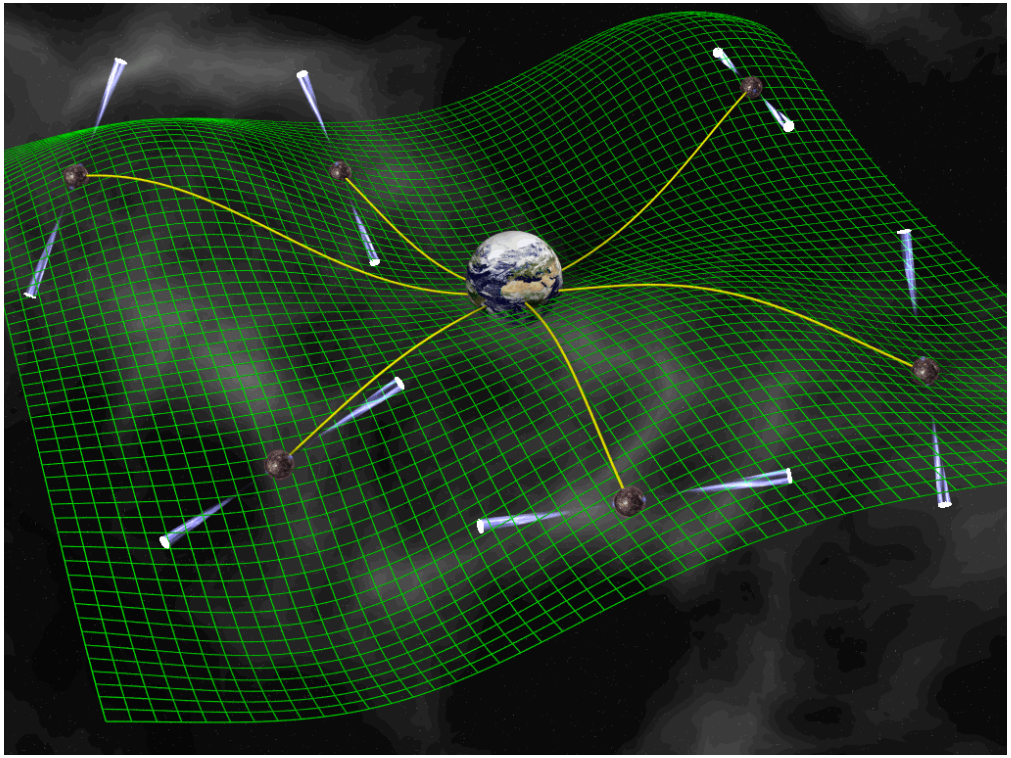
Detectability of the nonlinear memory: LISA





[MF, 0902.3660, ApJL, 696, 159]

Detectability of the nonlinear memory: pulsar timing



$$\Delta T_{\rm resid.} = \int_0^{T_{\rm obs}} dt' \, \frac{\Delta\nu(t')}{\nu_0}$$

$$\frac{\Delta\nu(t)}{\nu_0} \propto h(t)$$

- **Memory burst detectable out to z ~ 0.1 for M=10^8 $M_\odot$ (observable universe for M=10^{10} $M_\odot$).**

- **But rates are low: 0.1 – 0.01 detections in 10 yrs w/ near-term Pulsar-Timing-Arrays.**

Seto, MNRAS '09
Pshirkov, Baskaran, Postnov, MNRAS, '10
van Haasteren & Levin, MNRAS '10

Additional DC contributions to the waveform:

- Arun et al ’04 found a **nonlinear, non-hereditary** DC contribution to the “×” polarization at 2.5PN order; it originates from nonlinear corrections to the radiative current octupole moment V^{3m}:

$$H_{\times}^{(2.5,\mathrm{mem})} = -\alpha \frac{6}{5} s_{\Theta}^{2} c_{\Theta} \eta$$

- In addition, there are **linear, zero-frequency** terms that arise from the m=0 pieces of the source mass and current moments. For example:

$$I_{lm} \propto \eta M r^{l}(t) e^{-im\varphi(t)}[1+O(2)]$$

$$I_{2\pm2}^{(2)} \propto \eta M x e^{\mp 2i\varphi(t)}[1+O(2)]$$

$$I_{20}^{(2)} \propto \eta^{3} M x^{6}[1+O(2)]$$

…which leads to a 5PN, non-oscillatory correction to the waveform.

- Not clear if these effects produce a memory that continues to build up during the merger/ringdown.

[MF PRD’09]

Summary:

1. The linear memory arises from systems w/ unbound masses (hyperbolic orbits, explosions).

2. The nonlinear memory arises from the GWs produced by GWs, and is a generic feature of ***all* GW sources**.

3. For quasi-circular and elliptical inspiraling binaries, the nonlinear memory causes a slowly-growing, non-oscillatory amplitude correction to the waveform at ***leading-(Newtonian)-order***.

4. Modeling the memory in BH mergers is difficult for most NR codes, but can be accomplished using quasi-analytic techniques.

5. The memory is detectable by LISA for large SNR mergers. Detection prospects are poor for the upcoming generation of ground-based detectors, but not substantially worse than other classes of sources that we routinely try to detect.

6. Observation of the nonlinear memory provides a means to confirm an interesting strong-field prediction of GR.