

Bangalore Statistical Physics Summer School  
Lectures:  
**Nonequilibrium Stochastic Dynamics of  
Particle Systems**

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These lectures will give a general introduction to the stochastic dynamics of nonequilibrium and interacting particle systems and an overview of some models of general interest in a variety of contexts.

General references the subject are:

Stochastic Processes in Physics and Chemistry (North-Holland Personal Library, 3rd Edition 2007) N. G. Van Kampen

A Kinetic View of Statistical Physics Pavel L. Krapivsky, Sidney Redner, Eli Ben-Naim (Cambridge University Press, 2010)

## **1 Introduction to Stochastic Dynamics**

**A** Recap of thermal equilibrium; Examples of nonequilibrium systems.

**B** Transition rates and Master equation (forward); Detailed Balance and Kolmogorov Criterion; Nonequilibrium Stationary States (NESS)

**C** Example of random walk

**D** General spectral theory

**E** Diffusive processes: forward and backward Fokker Planck (diffusion) equations

References for this lecture:

Stochastic Processes in Physics and Chemistry (North-Holland Personal Library, 3rd Edition 2007) N. G. Van Kampen

Nonequilibrium dynamics in low-dimensional systems, M.R. Evans and R.A. Blythe, *Physica A: Statistical Mechanics and its Applications* Volume 313, Issues 1–2, 1 October 2002, Pages 110–152

## **2 Diffusive Processes and Mean First Passage times**

**A** Diffusive Processes; mean first passage times

**B** Survival probability of diffusive particle with trap; asymptotics from Laplace Transform; mean time to absorption

**C** Diffusion in higher dimensions and return probabilities

A Guide to First-Passage Processes (Cambridge University Press, 2001) Sidney Redner

## **3 Diffusive with Resetting**

**A** Search problems; intermittent search strategies

**B** Diffusion with resetting; forward and backward equilibrium stationary state

**D** Survival probability of resetting diffusive particle with trap; mean time to absorption; asymptotics of survival probability

**C** Connection with extreme values statistics

References for this lecture:

*Journal of Physics A: Mathematical and Theoretical* Volume 42 Number 43, 30 October 2009 Special Issue: The Random Search Problem: Trends and Perspectives

M. R. Evans and S. N. Majumdar, Diffusion with Stochastic Resetting *Phys. Rev. Lett.* 106, 160601 (2011)

## **4 Asymmetric Exclusion Process**

**A** Definition, motivations, boundary conditions

**B** Dynamical exponent and scaling

**C** Correlation functions

**D** Mean field theory; Phase diagram for open boundary conditions

**E** Matrix product exact stationary state

**F** Domain wall theory

References for this lecture:

D. Mukamel 2000 Phase transitions in nonequilibrium systems, Chapter in Soft and Fragile Matter: Nonequilibrium Dynamics, Metastability and Flow ed ME Cates and MR Evans (Institute of Physics Publishing 2000), <http://arxiv.org/pdf/cond-mat/0003424.pdf>

R A Blythe and M R Evans 2007 Nonequilibrium steady states of matrix-product form: a solver's guide J. Phys. A: Math. Theor. **40** R333 doi:10.1088/1751-8113/40/46/R01 (First two sections give overview) <http://arxiv.org/pdf/0706.1678.pdf>

T Chou, K Mallick and R K P Zia 2011 Non-equilibrium statistical mechanics: from a paradigmatic model to biological transport Rep. Prog. Phys. **74** 116601 doi:10.1088/0034-4885/74/11/116601 (Excellent review of biological applications) <http://arxiv.org/pdf/1110.1783.pdf>

## 5 Zero-range Process

**A** Definition, motivations, connection with ASEP

**B** Factorised Stationary States (FSS)

**C** Real space condensation

**D** Large deviations of sums of random variables

**E** Dynamics: ZRP -like and Explosive

References for this lecture:

M R Evans and T Hanney 2005, Nonequilibrium statistical mechanics of the zero-range process and related models J. Phys. A: Math. Gen. **38** R195 doi:10.1088/0305-4470/38/19/R01 <http://arxiv.org/pdf/cond-mat/0501338.pdf>

S N Majumdar 2010 Real-space Condensation in Stochastic Mass Transport Models", Les Houches lecture notes for the summer school on "Exact Methods in Low-dimensional Statistical Physics and Quantum Computing" (Les

Houches, July 2008), ed. by J. Jacobsen, S. Ouvry, V. Pasquier, D. Serban and L.F. Cugliandolo and published by the Oxford University Press (2010).  
<http://arxiv.org/abs/0904.4097>

C. Godrèche 2007, From Urn Models to Zero-Range Processes: Statics and Dynamics in Ageing and the Glass Transition, Lecture Notes in Physics Volume 716, 2007, pp 261-294  
<http://arxiv.org/pdf/cond-mat/0604276.pdf>