QUANTUM QUENCH & HOLOGRAPHY

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LARGE N FIELD THEORIES → GAUGE GRAVITY DUALITY

Prehistory of Gauge-Gravity Duality
- Large N limit → strings
- → gravity
- Black Holes & Holography

't Hoofts Large N limit

$$S = \int d^dx \left[\mathrm{Tr} \tfrac{1}{2}(\partial M)^2 + V(M) \right]$$

$$M \to M_{ij}(x^\mu) \qquad i,j = 1 \dots N$$

Two related features
- $1/N$ appears as a coupling constant of the effective theory of singlets
- Feynman diagrams with the same power of N have same topology — they are discretizations of 2d surfaces. The 2d surface can be considered as worldsheet of strings

$$V(M) = \frac{g}{\sqrt{N}} \mathrm{Tr}\, M^3 + \frac{\lambda}{N} \mathrm{Tr}\, M^4$$

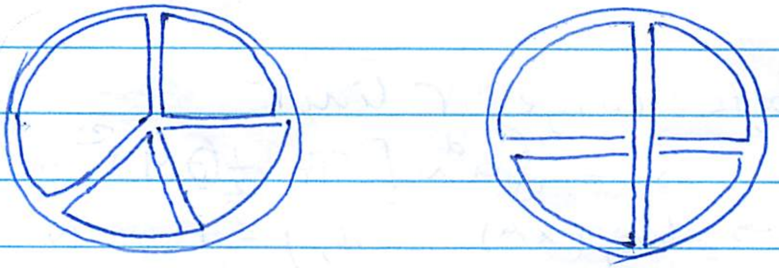
Rescale $\quad M \to \sqrt{N}\, M$

$$S = N \int d^dx \left[\mathrm{Tr} \left[\tfrac{1}{2}(\partial M)^2 + g M^2 + \lambda M^4 \right] \right]$$

• Feynman diagrams

 propagator P

 Vertex V



PLANAR $\chi = 2$ NON-PLANAR $\chi = 0$

For a given diagram

$$N^{V+E-P} = N^{\chi}$$

$$\chi = 2 - 2g \quad \text{Euler characteristic}$$

$$g = \#\text{ of handles}$$

• The same counting holds for an action

$$S = S_0 + N\mathcal{O}J$$

$\mathcal{O} \to$ Some operator which is invariant under $U(N)$

This is generating functional

$$e^{-W[J]} = \int \mathcal{D}M\, e^{-S}$$

and to leading order

$$W[J] \sim N^2$$

Thus

$$\langle \underbrace{\mathcal{O}\mathcal{O}\mathcal{O}\mathcal{O}\ldots}_{n} \rangle_c = N^{2+n}$$

$$\langle \mathcal{O}\mathcal{O} \rangle_c = N^4 \qquad \langle \mathcal{O} \rangle = N^3$$

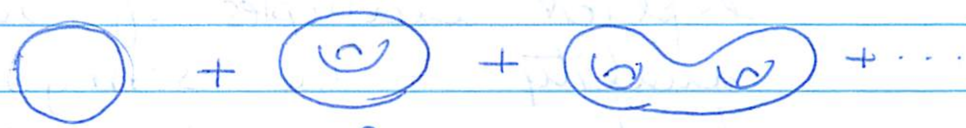
Thus

$$\frac{\langle \mathcal{O}\mathcal{O} \rangle_c}{\langle \mathcal{O} \rangle^2} = \frac{N^4}{N^6} = O\left(\frac{1}{N^2}\right)$$

FACTORIZATION

- This means that the large-N limit is some kind of classical limit $\hbar \to 1/N$

- The second feature — the $1/N$ expansion is a topological expansion. Exactly like perturbation expansion of a theory of strings

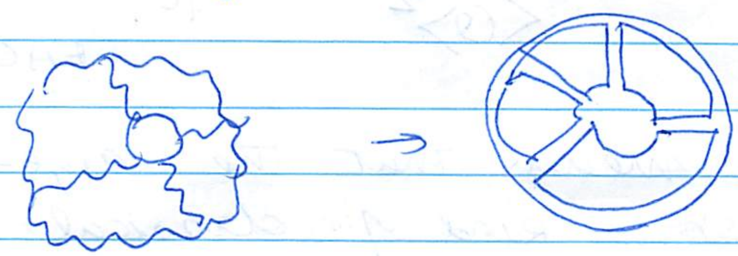


$$g_s^2 \qquad g_s^4 \qquad \cdots$$

$$g_s = 1/N$$

→ A theory of closed strings

- By the same token if we had fields in FUNDAMENTAL representation ψ_i $(i = 1 \ldots N)$
  - diagrams would have BOUNDARIES
  - Theory of closed and open strings



- Closed string theories ~~[illegible]~~ contain gravity — hint of a gauge-gravity duality.

- Surprisingly when people found explicit examples of such duality — the string theory lived in ONE MORE DIMENSION

Consider eg. 0+1 dimensional field theory of a single matrix $M_{ij}(t)$

States which are SINGLETS →

$\psi[\lambda_i, t]$ $\quad \lambda_i$: Eigenvalues.

Introduce a density of eigenvalues

$$\rho(x) \equiv \frac{1}{N} \sum_i \delta(x-\lambda_i)$$

Then

$$\Psi[\lambda_i, t] \rightarrow \Psi[\rho(x), t]$$

- It turns out that because of the jacobian of transformation

$$\prod_{i,j} dM_{ij} \rightarrow \prod_i d\lambda_i \; U^\dagger dU \; \prod_{i<j} (\lambda_i - \lambda_j)^2$$

λ_i : thought of as position coordinate of fermion in one dimension

N fermions

$\rho(x)$: Bosonized density of fermions

- As a field theory this is a 1+1 theory of a single bosonic scalar field

In a suitable (double scaling) limit the theory has 1+1 dim Lorentz invariance

Turns out - EXACTLY 1+1 dimensional bosonic string theory.

(in 1+1 dim the only dynamical mode of a string is the center of mass)

- This is an explicit and earliest example of HOLOGRAPHY

  1+1 dim string theory does not, of course, have a graviton - but it has "coulombic" gravity, through massive

  The "collective field theory" of $\rho(x)$ contains such gravitational interactions in a subtle way.

  — THIS THEORY IS CLASSICAL IN $N = \infty$ LIMIT.

- More interesting instances of holography came from thinking about BLACK HOLES in string theory.

- The features of large-N limit which will be important for us are

~~(a)~~ $\boxed{\text{Factorization} - N=\infty \text{ limit is a classical limit}}$

- Is it possible that a factorization property holds in a field theory for some other reason?
  - If so it should be possible to ~~pose~~ rephrase non-perturbative questions in this theory as classical questions in some other theory.

Black Holes ~~in String Theory~~ & Holography.

~~D-branes & non-abelian gauge~~ Theory
~~→ Some collection become black holes~~

~~(8)~~ 't Hooft + Susskind: Bekenstein bound strongly suggests

Theory of gravity in d+1 dimensions
≡ Theory without gravity in d dimensions

SYMMETRIES

$$ds^2 = \frac{1}{z^2}\left[-dt^2 + dz^2 + d\vec{x}^2\right]$$

Isometries $SO(d,2)$

$$D = x^\mu \frac{\partial}{\partial x^\mu} + z\frac{\partial}{\partial z}$$

UV-IR

$$T^\mu = \frac{\partial}{\partial x^\mu}$$

$$M^{\mu\nu} = x^\mu \partial_\nu - x^\nu \partial_\mu$$

$$S^\mu = 2x^\mu x^\nu \partial_\nu - x^2 \partial_\mu - z^2\partial_\mu + 2x^\mu z \partial_z$$

$z=0$ restriction $\rightarrow$ these become generators of CONFORMAL SYMMETRY on the plane

For some field with some spin the action of these transformations suitably derived.

For scalars, e.g.

$$D\phi(x,z) = \left(x^\mu\partial_\mu + z\partial_z\right)\phi(x,z)$$

Suppose $\phi(x,z) \sim z^{d-\Delta} f(x)$ as $z \to 0$

$$D\phi(x,z) = z^{d-\Delta}\left[x^\mu\partial_\mu + (d-\Delta)\right]f(x)$$

Thus $f(x)$ has dimensions $(d-\Delta)$

GKPW tells us

$f(x) \rightarrow$ Coupling corresponding to operator $\mathcal{O}(x)$

$\Rightarrow$ Dimension of operator of $\mathcal{O}(x)$ is Δ

This is the reason why

$$e^{-S}\Big|_{\phi_0} = \left\langle \exp \int d^d x\, \epsilon^{-\Delta} \bar{\phi}_0(x)\, \mathcal{O}(x) \right\rangle$$

EXPLICIT AdS/CFT CALCULATION

Scalar field in bulk with mass m

$$\nabla^2\phi - m^2\phi = 0$$

$$ds^2 = \frac{1}{z^2}\left[-dt^2 + dz^2 + d\vec{x}^2\right] \quad (R_{AdS} = 1)$$

Pure AdS calculation

Solution of the wave eqn

$$\phi(z,\vec{x},t) = z^{d/2}\left[A\, H_\nu^{(1)}(\alpha z) + B\, H_\nu^{(2)}(\alpha z)\right] e^{-i(\omega t - \vec{k}\vec{x})}$$

where $\nu = \sqrt{\left(\frac{d}{2}\right)^2 + m^2}$ $\quad \alpha^2 = \omega^2 - \vec{k}^2$

Need two pieces of data to get the solution

(1) Specify value of field on the cutoff boundary at $z = \epsilon$

$$\phi(\epsilon, k) = \phi_0(k) \qquad k \equiv (\vec{k}, \omega)$$

(2) Impose some kind of regularity conditions in the interior — at the Poincare horizon $z = \infty$

· In Euclidean signature,

$$H_\nu^{(1)}, H_\nu^{(2)} \rightarrow K_\nu\left(\sqrt{\omega_E^2 + k^2}\, z\right), I_\nu$$

As $\sqrt{\ldots}\, z \rightarrow \infty$

$$K_\nu \sim e^{-kz} \qquad I_\nu \sim e^{kz}$$

$\Rightarrow$ Choose K_ν

- For $\omega^2 < k^2$ same holds in the Lorentzian signature

- For $\omega^2 > k^2$ the condition which describes a ground state at early times $\longrightarrow$ Purely ingoing wave at horizon.

$$e^{-i\omega t} H_\nu^{(1)}(\alpha z) \longrightarrow \frac{1}{\sqrt{z}} e^{-i\omega(t - \frac{\alpha}{\omega} z)}$$

— this is ingoing

TURNS OUT TO BE ANALYTIC CON. OF K_ν

A purely ingoing wave is consistent with the nature of a horizon.

- Given these two conditions the solution is now fixed. $\rightarrow$ we need to calculate the on-shell action.

- $$S = \frac{1}{2}\int dz\, d\vec{x}\, dt \sqrt{g} \left[-g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - m^2\phi^2\right]$$

$$= -\frac{1}{2}\int_{\partial M} dn^\mu \sqrt{g}\, g^{\mu\nu} \phi\, \partial_\nu \phi$$

$$+ \frac{1}{2}\int dz\, d\vec{x}\, dt \sqrt{g}\, (\phi)(\nabla^2\phi - m^2\phi)$$

$$= -\frac{1}{2}\int_{\partial M} dn^\mu \sqrt{g}\, g^{\mu\nu} \phi\, \partial_\nu \phi$$

- For us there are 2 boundaries
  $z = \infty$ and $z = 0$
  At $z = \infty$ the solution dies off

  $\rightarrow$ Only contribution is from the $z=0$ boundary – the boundary of AdS space

- Computation in Euclidean signature:

$$S = \int d\vec{k}\, d\omega_E \left[\sqrt{g}\, g^{zz} \phi(k,z)\, \partial_z \phi(k,z) \right]_{z=\epsilon}$$

$$\left.\begin{aligned} \sqrt{g} &\rightarrow \frac{1}{z^{d+1}} \rightarrow \frac{1}{\epsilon^{d+1}} \\ g^{zz} &\rightarrow z^2 \end{aligned}\right] \sqrt{g}\, g^{zz} = \epsilon^{1-d}.$$

$$\phi(k,\epsilon) = \phi_0(k)$$

The solution with this boundary condition is

$$\phi(k,z) = \left(\frac{kz}{k\epsilon}\right)^{d/2} \frac{K_\nu(kz)}{K_\nu(k\epsilon)}\, \phi_0(k).$$

The series expansion of $K_\nu(kz)$ is

$$K_\nu(kz) = \frac{1}{2}\Gamma(\nu)\left(\frac{kz}{2}\right)^{\nu}\left(1 + O(z^2k^2)\right) + \frac{1}{2}\Gamma(-\nu)\left(\frac{kz}{2}\right)^{-\nu}\left(1 + O(z^2k^2)\right)$$

Substitute and calculate.

Asymptotically AdS

The nature of the expansion of solution near the boundary is independent of the details in the large z region
– so valid for the soln in any asymptotically AdS space
Generally for $z \to 0$, $\Delta_\pm = \frac{d}{2} \pm \nu$

$$\phi(k,z) \to z^{\Delta_-}(\alpha(k) + O(z^2)) + z^{\Delta_+}(\beta(k) + O(z^2))$$

For pure AdS

$$\alpha(k) = \frac{1}{\epsilon^{d/2-\nu}} \phi_0(k) \qquad (\epsilon \to 0)$$

$$\beta(k) = \epsilon^{-\Delta_-} \frac{\Gamma(\nu)}{\Gamma(-\nu)} \left(\frac{k}{2}\right)^{2\nu} \phi_0(k)$$

However if the interior is not AdS, one would still have the same form, but $\beta(k)$ will be different.

- With this asymptotic form

$$\partial_z \phi \big|_\epsilon = \frac{\Delta_- \alpha\, \epsilon^{\Delta_- -1}(1+O(\epsilon^2)) + \Delta_+ \beta\, \epsilon^{\Delta_+ -1}(1+O(\epsilon^2))}{\alpha\, \epsilon^{\Delta_-}(1+O(\epsilon^2)) + \beta\, \epsilon^{\Delta_+}(1+O(\epsilon^2))}$$

Assume first $m^2 > 0$ Then

$$\Delta_+ = \Delta_- + 2\nu$$

Require $\Delta_- + 2 < \Delta_+$

$$\Rightarrow \quad \nu < 1$$

This means $\frac{d^2}{4} + m^2 < 1$

$$m^2 < 1 - \frac{d^2}{4}$$

Then $\Delta_{\phi -} = \frac{d}{2} - \nu > \frac{d}{2} - 1$

which is the unitarity bound

Then

$$\partial_z \phi \sim \frac{1}{\epsilon} \left\{ \Delta_- + 2\nu \frac{\beta(k)}{\alpha(k)} \epsilon^{2\nu} \right\} \phi_0(k)$$

and the action is

$$S = \frac{1}{2} \int dk \, \frac{1}{\epsilon^d} \phi_0(k) \phi_0(-k) \left[\Delta_- + 2\nu \frac{\beta(k)}{\alpha(k)} \epsilon^{2\nu} \right]$$

Reexpress in terms of

$$\chi(k) = \epsilon^{-\Delta_-} \phi_0(k)$$

$$S = \frac{1}{2} \int dk \, \epsilon^{-2\nu} \Delta_- \chi^2 + \frac{1}{2} \int dk \, G(k) \chi(k) \chi(-k)$$

Renormalization:

If we now compute

$$\frac{\delta}{\delta\chi(k)}\frac{\delta}{\delta\chi(-k)}e^{-S_{cl}}$$

we get an infinite answer in the $\epsilon \to 0$ limit

This should be so. We are calculating the correlator in a strongly coupled theory — That should also be infinite due to UV divergences.

$\Rightarrow$ Identify ϵ with UV cutoff of the theory

To obtain the correct answer we need to introduce counterterms in the field theory action

$\Rightarrow$ In the holographic context this means we need to introduce boundary terms in the action — which respect boundary diffeo.

- Clearly these terms can be only quadratic in the field. They can be:

$$\int d^dx \sqrt{\gamma}\left[\gamma^{ij}\partial_i\phi\,\partial_j\phi + r_0\phi^2\right]$$

where

$\gamma_{ij} \rightarrow$ induced metric

$$\gamma_{ij} = \frac{1}{\epsilon^2}\delta_{ij}$$

$$\sqrt{\gamma} = \frac{1}{\epsilon^d} \qquad \sqrt{\gamma}\,\gamma^{ij} = \frac{1}{\epsilon^{d-2}}$$

The term $\sqrt{\gamma}\,\phi^2$ is exactly $\frac{1}{\epsilon^d}\phi_0^2$

$\rightarrow$ Hence may add a counterterm to cancel this

The term $\sqrt{\gamma}\,\gamma(\partial\phi)^2$, written in term of χ is

$$\epsilon^{2(1-\nu)}k^2\chi^2$$

For $\nu<1$ this vanishes in the $\epsilon\to 0$ limit — which is why we did not explicitly write this.

- Renormalized on-shell action

$$S_{ren} = \frac{1}{2}\int dk\; G(k)\,\chi(k)\,\chi(-k)$$

$$G(k) = \tau 2\nu\,\frac{\beta(k)}{\alpha(k)}$$

- Similarly the 1 point fn. is

$$\langle \mathcal{O} \rangle = \cancel{2}\nu \frac{\beta(k)}{\alpha(k)} \chi(k)$$

But $\quad \chi(k) = \epsilon^{-\Delta_-} \phi_0(k)$

$$= \alpha(k)$$

$$\boxed{\langle \mathcal{O} \rangle = \nu \beta(k)}$$

For the same reason the action may be written as

$$\boxed{S = \frac{1}{2} \int dk \, (2\nu) \, \beta(k) \alpha(k)}$$

- We have done calculations ~~for~~ assuming that signature is Euclidean. The calculation in Lorentzian is very similar

For $\omega \neq 0$ Regularity $\rightarrow$ Ingoing conditions

$G(k)$ is RETARDED CORRELATOR

$$\langle \mathcal{O} \rangle = \nu \beta(k) = G(k) \alpha(k)$$ LINEAR RESPONSE

SOURCES & STATES

- If initial state is a vacuum – such is AdS – causality demands that $\langle \mathcal{O} \rangle \neq 0$ only if a source is turned on. Thus in this case, we have response
$$\beta(k) = G_R(k)\,\alpha(k).$$

- However, if the state is excited we can have a nonzero $\langle \mathcal{O} \rangle$ in the absence of a source.
$$\beta(k) \neq 0 \quad \text{even if} \quad \alpha(k) = 0$$

Simplest example: a static solution AdS Black brane

AdS_{d+1}
$$ds^2 = -\left(1 - \frac{r_0^{\,d}}{r^d}\right) r^2 dt^2 + \frac{dr^2}{r^2\left(1 - \frac{r_0^{\,d}}{r^d}\right)} + r^2 d\Omega^2$$

For large $r = 1/z$ expand for $d=4$
$$g_{\mu\nu} = g_{\mu\nu}^{AdS} + h_{\mu\nu}$$

~~$h_{\mu\nu}dx^\mu dx^\nu$~~ It turns out one can always choose coordinates near the boundary such that
$$ds^2 = \frac{dz^2}{z^2} + \frac{1}{z^2}\, g_{ij}(x,z)\, dx^i dx^j$$

· Pretty much like the scalar field there is an expansion

$$g_{ij}(x,z) = g_{ij}^{(0)}(x,\cancel{z}) + z^2 g_{ij}^{(2)}(x) + z^4 g_{ij}^{(4)}(x) + \cdots$$

~~In our meth~~ Also like the scalar field

$g_{ij}^{(2)}$ determined by $g_{ij}^{(0)}$

But $g_{ij}^{(4)}$ has a part which is not.

$$T_{ij} \sim g_{ij}^{(4)}$$

→ For the black brane one finds

$g_{ij}^{(0)} = \eta_{ij}$ No source

$g_{ij}^{(4)} \neq 0$ $\langle T_{ij} \rangle \neq 0$

What kind of state? – thermal state with a temperature

$$T = \frac{r_0}{4\pi} d.$$

- Charged black brane

$$f(r) = 1 - \left[1 + \frac{d-2}{2(d-1)}\left(\frac{\mu}{r_+}\right)^2\right]\left(\frac{r_+}{r}\right)^{d} + \frac{d-2}{2(d-1)}\left(\frac{\mu}{r_+}\right)^2\left(\frac{r_+}{r}\right)^{2(d-1)}$$

$$A_t = \mu\left[1 - \left(\frac{r_+}{r}\right)^{d-2}\right]$$

This is a solution of Einstein-Maxwell

$$S = \int d^{d+1}x \sqrt{g}\left[\frac{1}{2k^2}\left(R + \frac{d(d-1)}{L^2}\right) - \frac{1}{4g^2}F_{\mu\nu}F^{\mu\nu}\right]$$

- Gauge symmetry in the bulk becomes a global symmetry on boundary

Boundary Coupling should be of the form

$$\int d^dx \sqrt{\gamma}\, A_i J_i$$

Gauge invariance $A_i \to A_i + \partial_i \alpha$

$$\Rightarrow \quad \partial_i J_i = 0$$

- μ therefore represents the CHEMICAL POTENTIAL of the boundary theory

- The term

$$\mu (r_+)^{d-2}$$

is the subleading term in the expansion of A_t

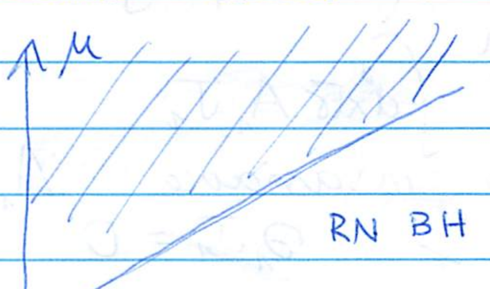
This should be therefore $\langle Q \rangle$

$$\rho = \mu (r_+)^{d-2}$$

- The temperature is now modified

$$T = \frac{r_+}{4\pi}\left[d - \frac{(d-2)^2}{2(d-1)} \left(\frac{\mu}{r_+}\right)^2 \right]$$

Thus there is a line in the μ-r_+ plane where $T=0$



· AdS soliton:

If one of the spatial directions θ is made compact with some radius R — the bulk solution has to be different

From field theory point of view — theory, states with momentum in the compact direction gapped

From the bulk point of view the proper radius of θ is
$(r\,R)$
This vanishes as $r \to 0$ developing a singularity

Rather the geometry caps off.

$$ds^2 = \frac{dr^2}{r^2 f_s(r)} + r^2\left(-dt^2 + d\vec{x}^2\right) + r^2 f_s(r)\, d\theta^2$$

$$f_s(r) = 1 - \left(\frac{r_0}{r}\right)^d$$

and

$$r_0 = \frac{4\pi}{d\,R}$$

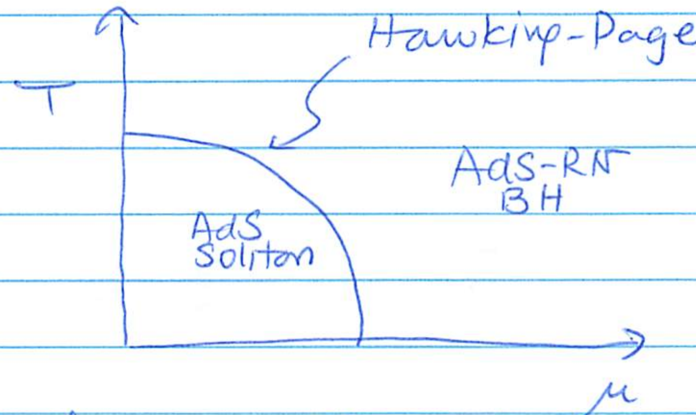
— AdS SOLITON.

- Like the black hole this is a solution which has nonzero energy — but this is the lowest energy state

- One can introduce a chemical potential as well
  The modification is trivial
  $$A_t = \mu$$
  No $F_{\mu\nu}$, no charge density

- In addition one can introduce a temperature — Euclideanize and compactify the euclidean time.
  — Temperature can be arbitrary.

- These are not both stable



$$r_0^{\,d} = r_+^d \left[1 + \frac{d}{2(d-1)} \left(\frac{\mu}{r_+} \right)^2 \right]$$

SIMPLE AdS/CFT EXAMPLES
— PROBES

N_c D3 branes produces a metric

$$ds^2 = + H^{-1/2}(r)\left[-dt^2 + d\vec{x}^2\right] + H^{1/2}(r)\left[dr^2 + r^2 d\Omega_5^2\right]$$

$$H(r) = 1 + \left(\frac{R}{r}\right)^4 \qquad R^4 = 4\pi g_s N l_s^4$$

Then for $r \ll R$

$$ds^2 = \left(\frac{r}{R}\right)^2\left[-dt^2 + d\vec{x}^2\right] + \left(\frac{R}{r}\right)^2 dr^2 + R^2 d\Omega_5^2$$

R is the AdS_5 radius.

In the 10 dim asymptotically Minkowskian space
D3 along $x_1\ x_2\ x_3$
r, Ω_5 $x_4, x_5, x_6, x_7, x_8, x_9$

- Consider absorption
  — From gravity point of view
  Conversion of asymptotic region modes $\longrightarrow$ Near horizon modes

From brane point of view
Conversion

closed strings far away $\rightarrow$ open strings in brane
$\downarrow$
gluons

Thus

Open strings joining branes $\rightleftharpoons$ Closed strings in $AdS \times S$

$\downarrow$ Low energy

Gauge fields on branes A^{μ}_{ij} $\rightleftharpoons$ closed strings in $AdS \times S$

$$g_{YM}^2 = g_s$$

$$\left(\frac{R}{\ell_s}\right)^4 = 4\pi g_s N$$

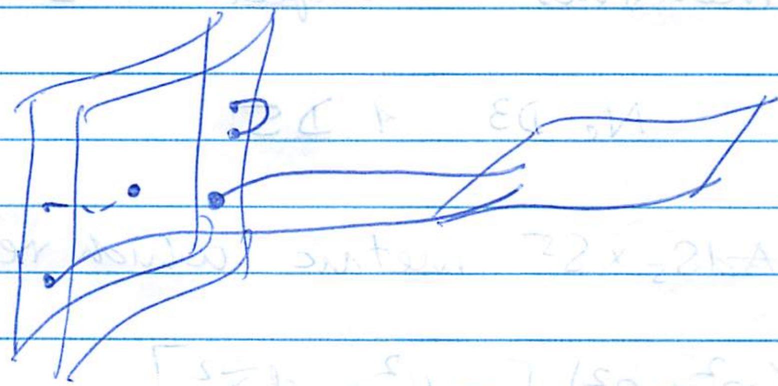
$\Rightarrow$ $N \to \infty$ $g_{YM} \to 0$

$g_{YM}^2 N = \text{fixed}$

Classical String Theory in AdS

$g_{YM}^2 N \gg 1 \rightarrow$ Sugra.

- How do we introduce fields which carry fundamental rep. ?



Put a set of other branes in the background — N_f in number

Open strings carry single index

- Argument similar to Maldacena
  Open – Open duality
  Theory of quarks $\longrightarrow$ Brane action.

- Low energy Brane action is the DBI action.

- This way one can obtain sectors of field theory which live in lower dimensions — defect CFTS

Example N_c D3 1 D5

Write $AdS_5 \times S^5$ metric which replaces the D3

$$ds^2 = (r^2+\rho^2)\left[-dt^2 + d\vec{x}^2\right] + \frac{1}{r^2+\rho^2}\left[d\rho^2 + \rho^2 d\Omega_2^2 + dr^2 + r^2 d\tilde{\Omega}_2^2\right]$$

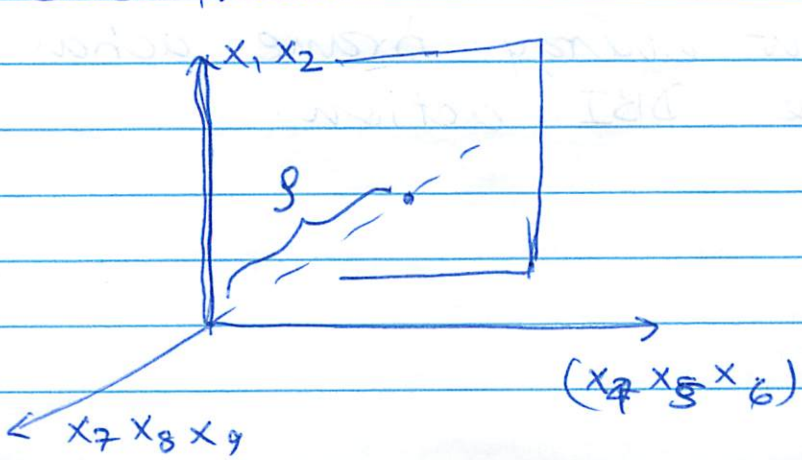
(ρ, Ω_2) : $x_7\ x_8\ x_9$

$(r, \tilde{\Omega}_2)$: $x_4\ x_5\ x_6$

Wrap D5 on

$x_1, x_2 ; x_4, x_5, x_6$

Then DBI fields



In this configuration

ρ : mass of the quarks

However since the open strings which come from D5 and end on D3 can only move along (x_1, x_2)

$\longrightarrow$ 2+1 dim theory of quarks

- When $N_f \ll N_c$ - the branes may be considered separately with back-reaction suppressed
  - — PROBE APPROXIMATION
  - — Usual 't Hooft limit

- $N_f \sim N_c \rightarrow$ Veneziano limit

(5)

BLACK HOLE FORMATION.

Chesler & Yaffe:

The Poincare patch boundary metric is flat, but may be written as

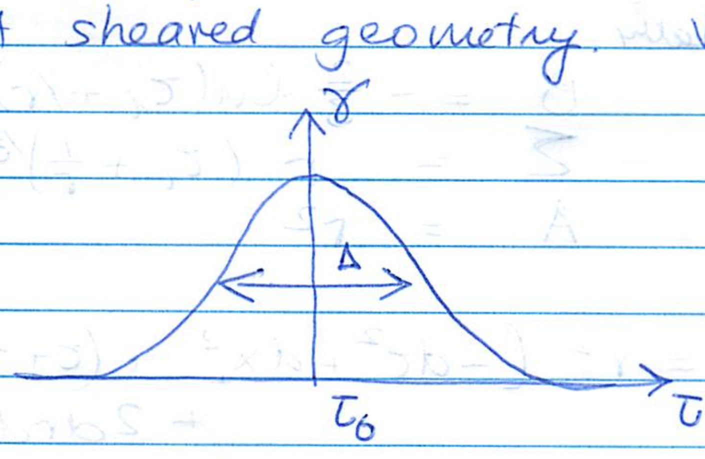
$$ds^2 = -d\tau^2 + dx_\perp^2 + \tau^2 dy^2$$

where $\quad dx_\perp^2 = dx_1^2 + dx_2^2 \qquad (AdS_5)$

Consider a deformed boundary metric

$$ds^2 = -d\tau^2 + dx_\perp^2 e^{\gamma(\tau)} + \tau^2 e^{-2\gamma(\tau)} dy^2$$

An-isotropic, but boost-invariant
— A sheared geometry. $\sqrt{h}$ same



To compute the bulk metric
— τ becomes a EF time coordinate
ansatz

$$ds^2 = -A(r,\tau)d\tau^2 + \Sigma^2(r,\tau)\left[e^{B}dx_\perp^2 + e^{-2B}dy^2\right] + 2\,dr\,d\tau$$

Boundary conditions

$$\lim_{r\to\infty} \frac{\Sigma}{r} = \tau^{1/3}$$

$$\lim_{r\to\infty} e^{B} = \frac{e^{\gamma(\tau)}}{\tau^{2/3}}$$

$$\Sigma^2 e^{B} \to r^2 e^{\gamma}$$

$$\Sigma^2 e^{-2B} \to r^2 \tau^2 e^{-2\gamma}$$

At an initial time τ_i → choose geometry to be AdS
Specifically

$$B = -\frac{2}{3}\ln(\tau_i + 1/r)$$

$$\Sigma = r\,(\tau_i + \tfrac{1}{r})^{1/3}$$

$$A = r^2$$

$$ds^2 = r^2\left[-d\tau^2 + dx_\perp^2 + \left(\tau + \tfrac{1}{r}\right)^2 dy^2\right] + 2\,dr\,d\tau$$

. Solve the equations of motion numerically ~~for~~ with these boundary and initial conditions.

- Look for formation of APPARENT HORIZON

Apparent Horizon : Boundary of a region of trapped surfaces

Consider null geodesics purely radial – they cross constant r surfaces

In flat space

"Outgoing" $\rightarrow$ r increases

"Incoming" $\rightarrow$ r decreases

For outgoing area of r=const surface increases/decreases.

Apparent horizon : $dA_r = 0$ along a null geodesic

Here

$$A_r = \Sigma^3$$

$$dA_r = 3\Sigma^2\left(\frac{\partial \Sigma}{\partial r}dr + \frac{\partial \Sigma}{\partial \tau}d\tau\right)$$

Null lines (1) $d\tau = 0$

(2) $A\,d\tau = 2dr$

Thus

$$dA_r = 3\Sigma^2\left(\frac{\partial \Sigma}{\partial \tau} + \frac{1}{2}A\frac{\partial \Sigma}{\partial r}\right)d\tau$$

Apparent horizon

$$\partial_\tau \Sigma + \frac{1}{2}A\,\partial_r \Sigma = 0$$

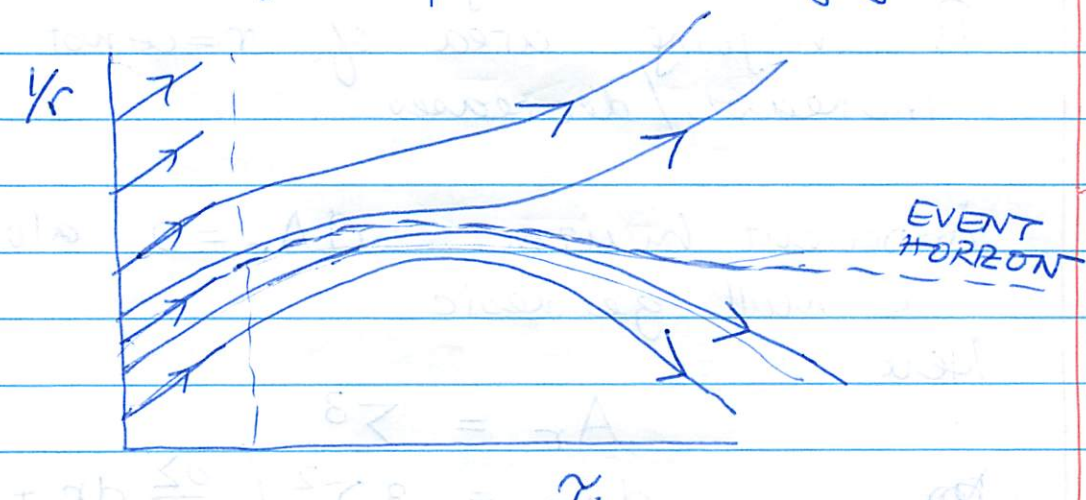
To look for EVENT HORIZON map our the "outgoing" null geodesics

$$\frac{dr}{d\tau} = \frac{1}{2}A$$

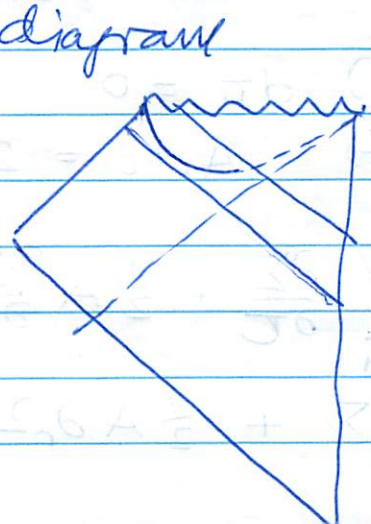
For $\tau < \tau_i$ $A = r^2$

$$\tau + \frac{2}{r} = \text{const.}$$

For $\tau > \tau_i$ they start changing

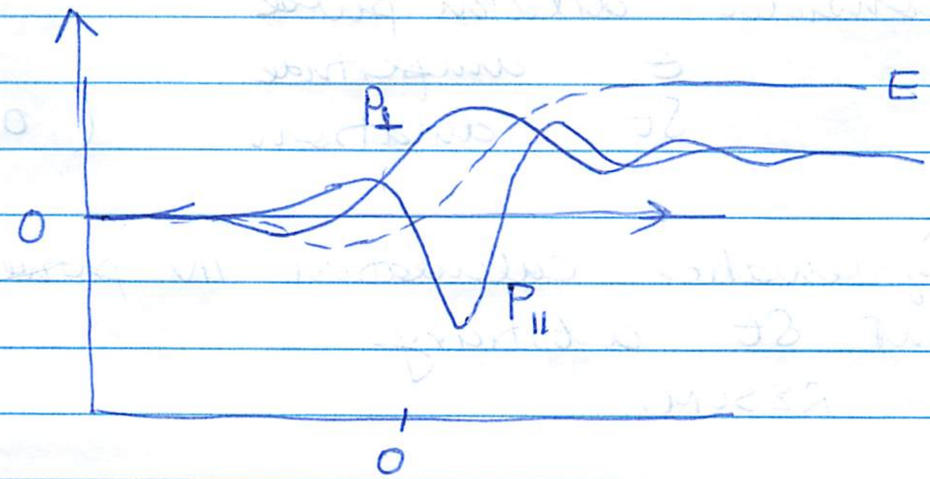


Penrose diagram



- Calculate $\langle T_{\mu\nu} \rangle$ by looking at the subleading term in the metric

$$T^{\mu}_{\ \nu} = \frac{N_c^2}{2\pi^2} \, \mathrm{diag}\left(-\mathcal{E}, P_\perp, P_\perp, P_\parallel\right)$$



The fact that the system becomes ISOTROPIC — signature of passage to hydrodynamic limit.

Bhattacharya & Minwalla

Consider dilaton pulse

ϵ : amplitude

δt : duration $\quad (0 < \tau < \delta t)$

Systematic calculation in powers of ϵ, but δt arbitrary

— RESUM.

Initial condition $\tau < 0$ AdS.

For $\tau > 0$

and $\quad r \gg \frac{1}{\delta t} \epsilon^{\frac{2}{d-1}}$

$$ds^2 = 2 dr d\tau - \left(r^2 - \frac{M(v)}{r^{d-2}} \right) d\tau^2 + r^2 d\vec{x}^2$$

$$M(v) \sim \frac{\epsilon^2}{(\delta t)^d}$$

and $\quad M(v) \to M \quad$ as $\quad \tau > \delta t$

$$M \sim \epsilon^2 / (\delta t)^d$$

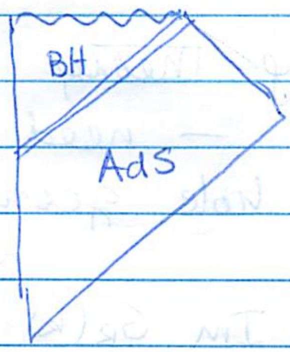
Temperature $\quad T_H = \frac{\epsilon^{\frac{2}{d}}}{(\delta t)}$

6

VAIDYA & THERMALIZATION

$$ds^2 = \frac{1}{z^2}\left[-(1-m(v)z^d)\,dv^2 - 2\,dz\,dv + d\vec{x}^2\right]$$

$$m(v) = \tfrac{1}{2}M\left(1+\tanh\frac{v}{v_0}\right)$$



In $v_0 \to 0$ limit the geometry for $v>0$ is Black brane

$$T = \frac{d}{4\pi} M^{1/d}$$

For $v<0 \to$ AdS.

Compute

- Correlation functions of local operators
- Wilson loops
- Entanglement entropy

Need to specify meaning of approach to equilibrium.

Consider the Wightman function

$$G^{>}_{\mathcal{O}}(t,\vec{x};t'\vec{x}') = \langle \mathcal{O}(t,\vec{x})\,\mathcal{O}(t',\vec{x}')\rangle$$

Example: For 2d free Theory

$$\mathcal{O} = \partial_{\pm}\phi$$

Then

$$G \sim \frac{1}{\beta^2 \sinh^2\left(\frac{\pi X_+}{\beta}\right)}$$

Result expected from conformal symmetry

Map Complex plane $\Rightarrow$ Cylinder

' For a strongly interacting theory which has a AdS_3 dual — need to compute in BTZ black hole geometry.

$$G^{>}(k) = -\frac{2i}{e^{\beta\omega}-1} \, \mathrm{Im}\, G_R(k)$$

For $\Delta=1$ operator

$$-2\,\mathrm{Im}\, G_R(\omega,k) = \frac{\sinh\frac{\beta\omega}{2}}{\cosh\frac{\beta\omega}{2}+\cosh\frac{\beta k}{2}}$$

Consider vacuum subtracted Wightman function in equilibrium

$$G^{>}_{sub}(\Delta t,k,\beta) = \int\frac{d\omega}{2\pi}e^{-i\omega\Delta t}\left[G^{>}_{\omega k}(\beta)-G^{>}(\infty)\right]$$

Equal time subtracted Wightman fn

$$G^{>}_{sub}(k,T) = -\frac{i}{2\pi}\frac{k}{e^{\beta\omega}-1}$$

Idea: Use similar definition in a time dep situation

Now $G^{>}_{sub}$ dependence on t

$G^{>}_{sub}(t,k)$

One measure of thermalization

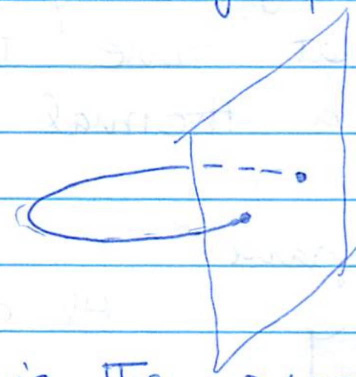
$$G^{>}_{sub}(t,k) \longrightarrow G^{>}_{sub}(k,T)$$

Geodesic Computation of Correlators.

$$\langle \mathcal{O}(t,\vec{x})\, \mathcal{O}(t,\vec{x}')\rangle = \int \mathcal{D}P\, e^{i\Delta L(P)}$$

Δ = dimension of operator.

Saddle point
— geodesic



where $L(P)$ is the proper length
For space-like geodesics, $L(P)$ is purely imaginary

Along some path $X^{\mu}(\lambda)$

$$L = \int d\lambda \sqrt{-G_{\mu\nu}\dot{X}^{\mu}(\lambda)\dot{X}^{\nu}(\lambda)}$$

(In flat space static gauge $1-\dot{x}^2$.
For spacelike $\dot{x}^2 > 1$).

Thus

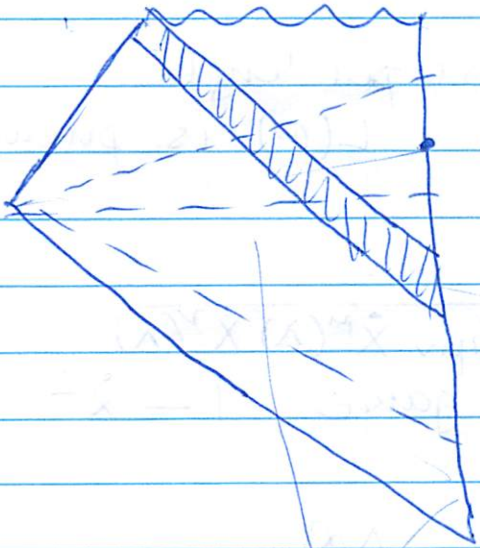
$$\langle \mathcal{O}\mathcal{O}\rangle = \sum_{\text{geodesics}} e^{-\Delta \mathcal{L}} \qquad L = i\mathcal{L}.$$

These geodesic lengths diverge due to the boundary

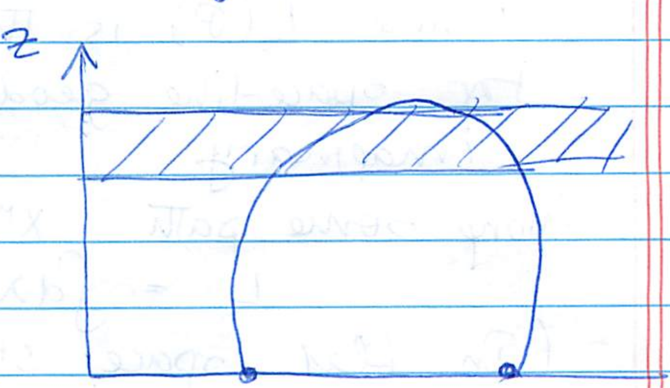
- subtract the answer for pure AdS
- Call this $\delta\mathcal{L}_{\text{thermal}}$

Compute same object in the time dependent geometry and find out around what time the answer approaches a thermal answer.

General diagram



At a given time slice



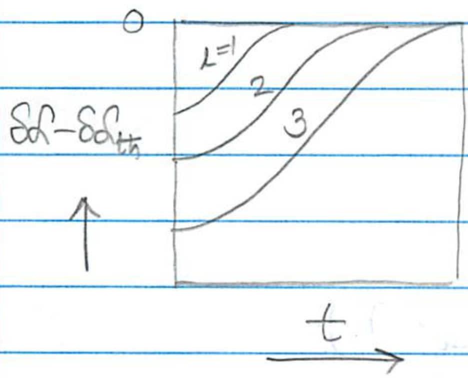
At early time $\delta\mathcal{L} < \delta\mathcal{L}_{\text{thermal}}$

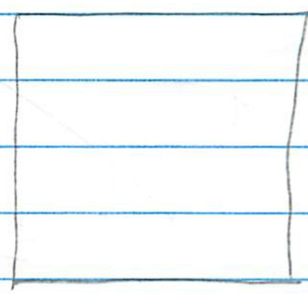
For a given separation of 2 point fn., define a τ_{crit}

τ_{crit} = Time at which the geodesic grazes the center of the shell at $v=0$

Results:

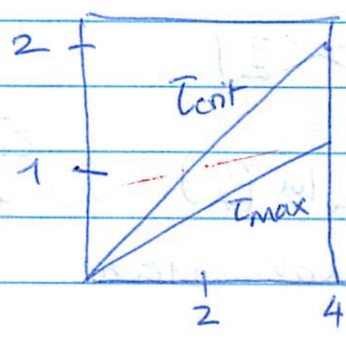
(1) $\delta\mathcal{L} - \delta\mathcal{L}_{thermal}$





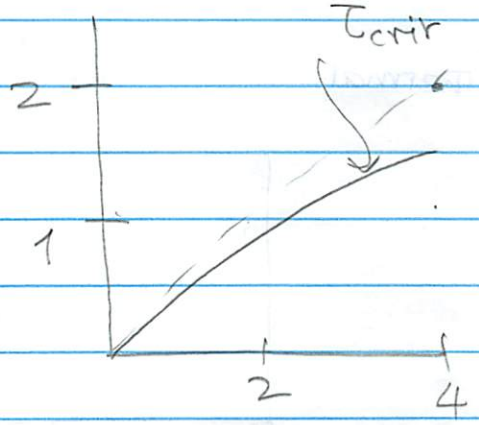
τ_{max} : Time at which thermalization proceeds most rapidly
→ time at which the curves are steepest

d = 2 result for time scales



Consistent with general arguments of Calabrese & Cardy

- However in $d = 3, 4$ they deviate from linearity



(Comment on superluminal?)).

Entanglement Entropy

Consider a quantum system in some state $|\Phi\rangle$ — Divide into two disjoint parts A and B

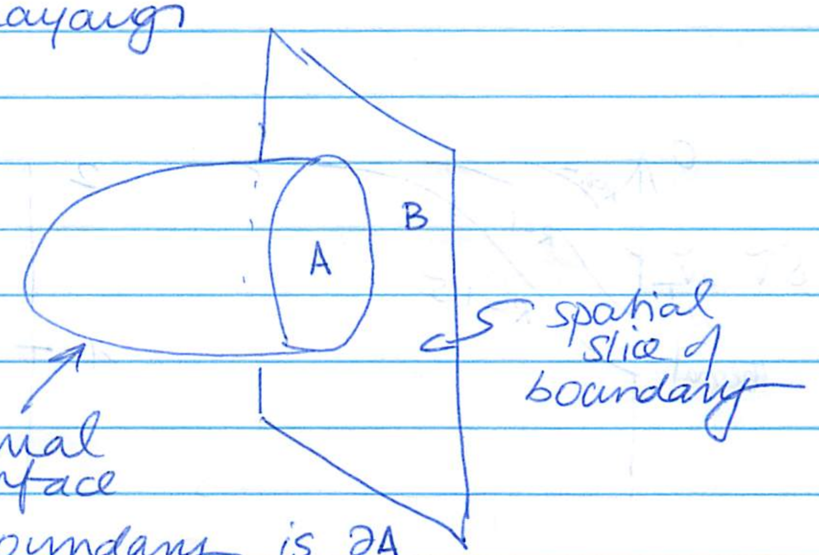
$$\rho_A \equiv \mathrm{tr}_B |\Phi\rangle\langle\Phi|$$

Then

$$S(A) = -\,\mathrm{tr}_A(\rho_A \ln \rho_A) \qquad \text{Entanglement entropy}$$

Similar for thermal state

- Ryu & Takayangi



whose boundary is ∂A

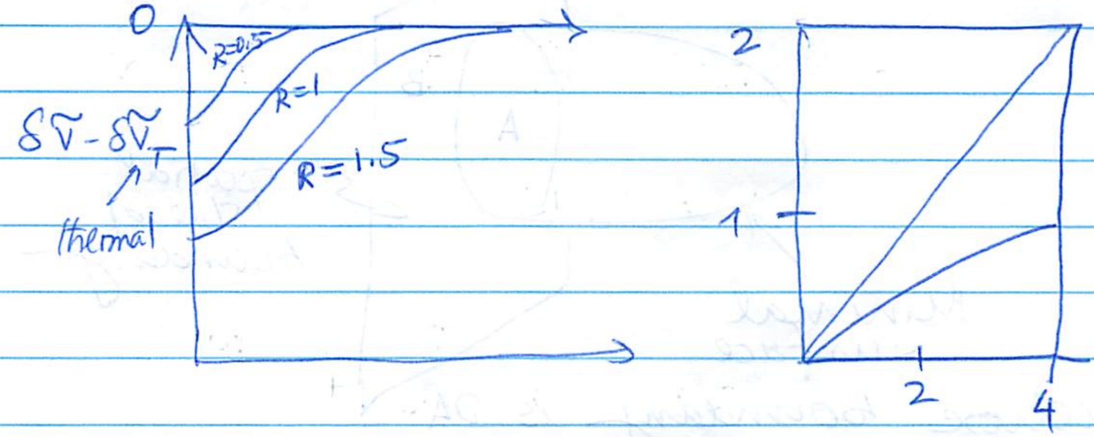
$$S = \frac{\text{Area}(\partial A)}{4G_N}$$

Note: In AdS_{d+1}
Space $\to$ $(d-1)$ dimensional
"Surface" $\Rightarrow$ $(d-1)$ dimensional surface

- For time dependent backgrounds
minimal $\Rightarrow$ extremal

- For AdS_5 the surface is 3 dim
— volume
δV = Subtracted by the divergent part of vol. in pure AdS.

$$\delta \tilde{V} = \frac{\delta V}{\text{volume of region on boundary}} = \frac{\delta V}{\frac{4}{3}\pi R^3} \quad \text{for spherical}$$



Inclusion of Chemical potential
(Caceres & Kundu
Galante & Schvellinger)

l: Typical length scale
(Corr. length, radius of W loop etc).

$Tl \lll 1$ Thermalization time
nonmonotonic as a fn. of μ/T

μ/T small – τ_{crit} decreases as we increase μ/T
(faster thermalization)

μ/T large – τ_{crit} ~~decreases~~ increases as fn of μ/T

- $d\tau_{crit}/d\mu > 0$ for large μ/τ interpreted as
  - increasing μ, encourages BE condensation — preventing excitation
  - increasing μ for fermions increases number of available states.

HOLOGRAPHIC QUANTUM QUENCH

AdS/CFT relates CFT's or CFT's + deformations to a bulk theory

$$S = S_{CFT} + \int d^d x \, \epsilon^{d-\Delta} \phi_0(x) \mathcal{O}(x)$$

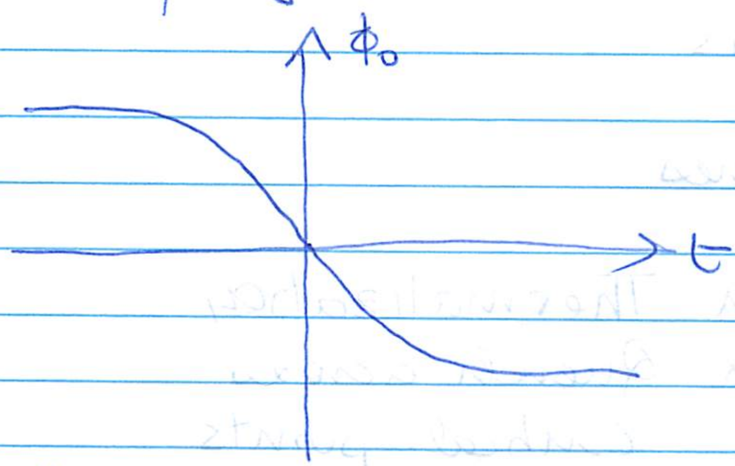
includes time

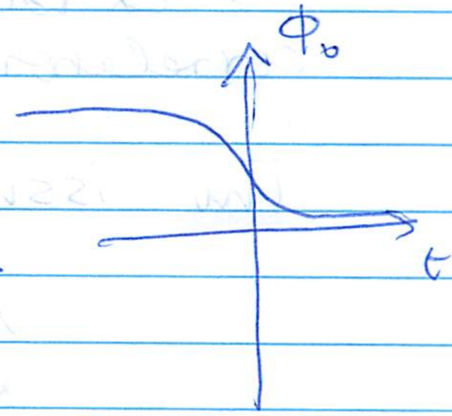
Then

$$\int \mathcal{D}M \, e^{-S} = \left\langle \exp \int d^d x \, \epsilon^{d-\Delta} \phi_0(x) \mathcal{O}(x) \right\rangle$$

$$= \exp\left[-S_{bulk}\right]\Big|_{\phi(x,z) = \phi_0(x)}$$

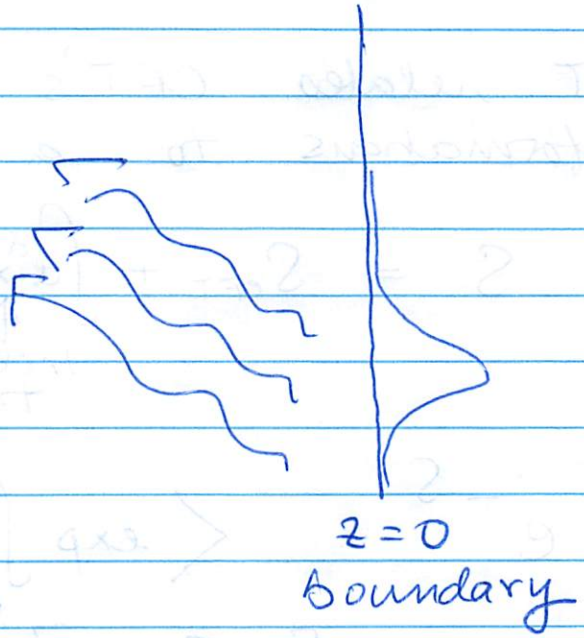
Suppose we ~~start with~~ have a coupling





And we start with ground state (or equilibrium state)
— what happens to the system

In the bulk



Find the bulk fields as a function of time

→ Extract from it various correlators.

Two issues

x Thermalization
x Quench across critical points

(k)

BOTTOM-UP MODELS & PHASE TRANSITIONS

- Much of the properties of AdS/CFT stem from the fact that

$$\text{Isometries of Bulk} \iff \text{Symmetries on boundary}$$

- Thus may eschew String Theory and consider interesting models in the bulk and ASSUME they have a dual description as a local field theory on the boundary which is strongly coupled
- These modes have been very useful to explore ISOLATED CRITICAL POINTS

- No hope of using these to get details of this assumed field theory – but may be able to understand UNIVERSAL behavior

- So useful to explore universal behavior in QUANTUM QUENCH

$d+2 \to D+1$
$d \to D-1$

- Simplest example

$$S = \int d^{d+1}x \sqrt{g} \Big[\frac{1}{2\kappa^2}\Big(R + \frac{(d-1)d}{L^2}\Big)$$
$$- \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$
$$+ \frac{1}{2}\Big(-(D\phi)^2 - m^2\phi^2 - \frac{\lambda}{2}\phi^4\Big)\Big]$$

Gravity + Maxwell + Charged Scalar

- $q \neq 0$, $\lambda = 0$ → Conventional description of holographic SUPERCONDUCTORS (superfluids)

- $q = 0$, $\lambda \neq 0$ → Antiferromagnets?

- First consider ~~neutral~~ charged scalar, $\lambda = 0$

$$h_{\mu\nu} \Leftrightarrow T_{\mu\nu}$$
$$A_\mu \Leftrightarrow J_\mu$$

~~Coupling is J_μ~~

- As will be seen soon.

$$A_{\mu}(\vec{x}, r=\infty) = J_{\mu} i$$

A current — global

$$\partial_i J_i = 0$$

- If $q \neq 0$ may use q to rescale
  $\phi \to \phi/q \qquad A_\mu \to A_\mu/q.$

  $\frac{1}{q^2}$ comes out of Maxwell + scalar action

- Suppose $q^2 \gg k^2$. Then the Maxwell + scalar system may be considered in isolation — backreaction can be ignored
  PROBE APPROXIMATION

- Look for static solutions
  $\phi = 0$ always a solution.

- Gubser: When the background is a charged black hole — particularly close to extremality the trivial solution is UNSTABLE

- In that case we need to look for a stable solution.

- Hartnol, Herzog & Horowitz — indeed there is.
  If we take this nontrivial solution

$$\phi = z^{\Delta_-}\alpha + z^{\Delta_+}\beta$$

There is a solution
- regular in interior
- $\alpha = 0$

$\Rightarrow$ $\langle \mathcal{O} \rangle \neq 0$ without a source
But $\langle \mathcal{O} \rangle$ is a charged operator — SSB of $U(1)$.

- This solution can be found numerically.

- A second example concerns

$$q = 0 \qquad \lambda \neq 0.$$

But $\lambda \gg k^2$

Once again the scalar field may be treated as a probe

Background AdS-RN black hole

Here the instability of the trivial soln can be understood rather easily.

- Consider scalar field in AdS_4-RN black hole

Dual Theory has some (T, μ)

Eqn. of motion

$$\Box\phi - m^2\phi - \phi^3 = 0$$

By changing coordinates and fields

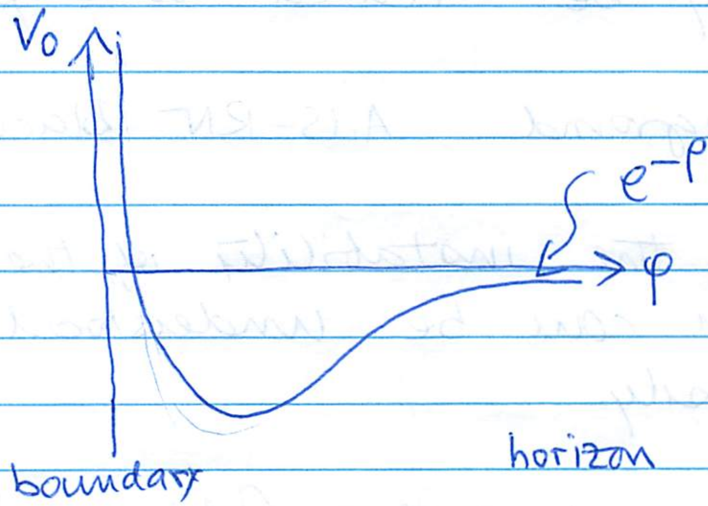
$$d\rho = \frac{dr}{r^2 f(r)} \qquad \phi = \frac{\chi}{r}$$

$$\boxed{-\partial_t^2\chi = -\partial_\rho^2\chi + V_0(\rho)\chi - r^2 f(r)\chi^3}$$

$$V_0(\rho) = r^2 f(r)\left[(m^2+2) - \frac{6\eta}{r^4} + \frac{1+3\eta}{r^3}\right]$$

Look at linearized perturbations
$\chi \sim e^{-i\omega t}$

$$\omega^2 \chi = -\partial_\rho^2 \chi + V_0(\rho)\chi$$



$$-\frac{9}{4} < m^2 < -\frac{3}{2}$$

As m^2 becomes more negative
the well becomes deeper
At some $m^2 = m_c^2$ $\rightarrow$ a zero
energy bound state forms.

For $m^2 < m_c^2$ – a true bound state
$\omega^2 < 0$ $\Rightarrow$ instability

- The boundary condition at the boundary has $\alpha = 0$
  $\Rightarrow$ ZERO MODE at $m^2 = m_c^2$.

- For $m^2 < m_c^2$ the full nonlinear eqn has a smooth solution

$$\langle \mathcal{O} \rangle_{\alpha=0} \sim (m_c^2 - m^2)^{1/2}$$

$$\left.\frac{d\langle \mathcal{O} \rangle}{d\alpha}\right|_{\alpha=0} \sim (m_c^2 - m^2)^{-1}$$

while

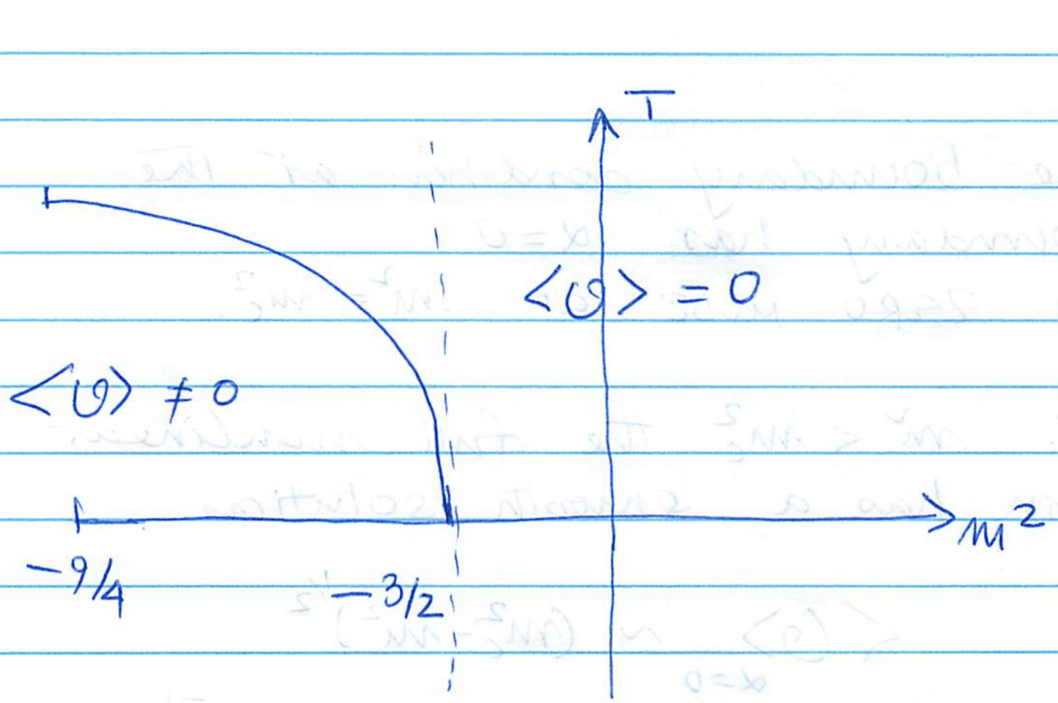
$$\langle \mathcal{O} \rangle_{m^2 = m_c^2} \sim \alpha^{1/3}$$

- However at $T = 0$

$$\langle \mathcal{O} \rangle_{\alpha=0} \sim \exp\left[-\frac{\sqrt{6}\,\pi}{2\sqrt{m_c^2 - m^2}}\right]$$

(Iqbal, Liu, Mezei, Si)

- The critical value of m^2 depends on (T, μ)



T
⟨O⟩ = 0
⟨O⟩ ≠ 0
m²
−9/4
−3/2

HOLOGRAPHIC SUPERCONDUCTOR

$$S = \int d^d x \sqrt{g}\, \frac{1}{2\kappa^2}\left[R + \frac{d(d-1)}{L_{AdS}^2}\right]$$
$$+ \int d^4 x \sqrt{g}\, \frac{1}{4q^2} F_{\mu\nu} F^{\mu\nu}$$
$$+ \int d^4 x \sqrt{g}\, \left[|D\Phi|^2 + m^2 \Phi^2 + \lambda \Phi^4\right]$$

- First study $\lambda = 0$
  Background is AdS/RN Black hole
  Q, M, T_H
  Field theory has (μ, T_H)

- Now consider effect of Φ.
  Have to solve coupled eqns

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 2\kappa^2 \left(\frac{1}{q^2} T_{F\,\mu\nu} + T_{\Phi\,\mu\nu}\right)$$
$$\nabla_\mu F^{\mu\nu} = J^\nu$$
$$\nabla^2 \Phi - m^2 \Phi = 0$$

- Clearly $\Phi = 0$
  and $g_{\mu\nu}, F_{\mu\nu}$ as in AdS/RN
  is a solution

- However it turns out that for given $m^2 < 0$
  μ = fixed
  For some $T < T_c$ this solution is both dynamically and thermodynamically unstable

- Rather there is a new soln. which has
  - no normalizable mode at ∞
  - regular at horizon

$\Rightarrow \quad \langle \mathcal{O} \rangle \neq 0$

U(1) broken.

7

Bhaseen, Gauntlett, Simons, Sonner, Wiseman

$\boxed{\text{THERMAL QUENCH : HOLOGRAPHIC SUPERCONDUCTOR}}$

$$S = \frac{1}{2k^2}\int d^4x \sqrt{g}\left[R + \frac{6}{\ell^2} - \frac{1}{4}F^2 - |D\psi|^2 - m^2|\psi|^2\right]$$

where
$$D\psi \equiv \partial\psi - iqA\psi$$

$$\boxed{k^2 = \frac{\ell^2}{N^{3/2}}}$$

This fractional dependence follows from a microscopic realisation in terms of a theory of M2 branes (ABJM theory)

Metric for N M2 branes in 11 dim

$$ds^2 = f^{-2/3}\left[-dt^2 + dx_1^2 + dx_2^2\right] + f^{1/3}\left[dr^2 + r^2 d\Omega_7^2\right]$$

where
$$f(r) = 1 + \frac{32\pi^2 N \ell_p^6}{r^6}$$

(ℓ_p : 11d Planck length).

for $r \ll N^{1/6}\ell_p$

$$ds^2 = \left(\frac{r}{\ell_p}\right)^4 \frac{1}{N^{2/3}}\left[-dt^2 + dx_1^2 + dx_2^2\right] + \left(\frac{\ell_p}{r}\right)^2 N^{1/3}\left[dr^2 + r^2 d\Omega_7^2\right]$$

Radius of S^7 : $R_7 = N^{1/6}\ell_p$

To get to standard AdS define

$$r^2 = \frac{R_7^2}{R_{AdS}}\rho = R_7\rho$$

$$\frac{r^4}{\ell_p^4}\frac{1}{N^{2/3}} = \frac{R_7^2\rho^2}{\ell_p^4 N^{2/3}} = \frac{N^{1/3}\rho^2\ell_p^2}{\ell_p^4 N^{2/3}} = \frac{\rho^2}{\ell_p^2 N^{1/3}}$$

Thus $\quad R_{AdS} \sim N^{1/6} l_p$

The 4d Gravitational coupling is

$$\frac{1}{\kappa^2} = \frac{R_7^7}{l_p^9}$$

Thus

$$\frac{\kappa^2}{l^2} = \frac{\kappa^2}{R^2_{AdS}} = \frac{l_p^9}{R_7^7} \frac{1}{N^{1/3} l_p^2} = \frac{1}{N^{1/3}} \left(\frac{l_p}{R_7}\right)^7$$

$$= \frac{1}{N^{1/3}} \frac{1}{N^{7/6}} = \frac{1}{N^{9/6}} = \frac{1}{N^{3/2}}$$

- In equilibrium the solutions – for a fixed m^2 and μ

$T > T_c \qquad$ AdS/RN BH $\qquad \psi = 0$

$T < T_c \qquad$ AdS/RN + Hair $\qquad \psi \neq 0$

- From the field theory viewpoint the quench is as follows
  - Start in condensed phase at some $T < T_c$
    – with some equilibrium soln
  - Turn on a a source for the order parameter $J(t)$

$$J(t) = \bar{\delta} \exp\left[-t^2/\bar{\tau}^2\right]$$

- Study time evolution of the system by calculating $\langle \mathcal{O}(t) \rangle$

<u>Results:</u> For $m^2 = -2$,

$$ds^2 = \frac{1}{z^2}\left[-F(t,z)\,dt^2 - 2\,dt\,dz + S^2(t,z)\,(dx_1^2 + dx_2^2)\right]$$

$$A_t = \mu(t) - z\,\rho(t)$$

$$\psi = z\,\psi_1(t) + z^2\,\psi_2(t) + \cdots\cdots \quad \text{(asympt. expansion)}$$

$$\psi_1(t) = \bar{\delta} \exp\left[-t^2/\bar{\tau}^2\right]$$

Calculate $\langle \mathcal{O}(t) \rangle = \frac{1}{2k^2}\left[2\psi_2(t) + 2i\mu(t)\psi_1(t)\right]$

Initial $\mu = \mu_i$ sets a scale

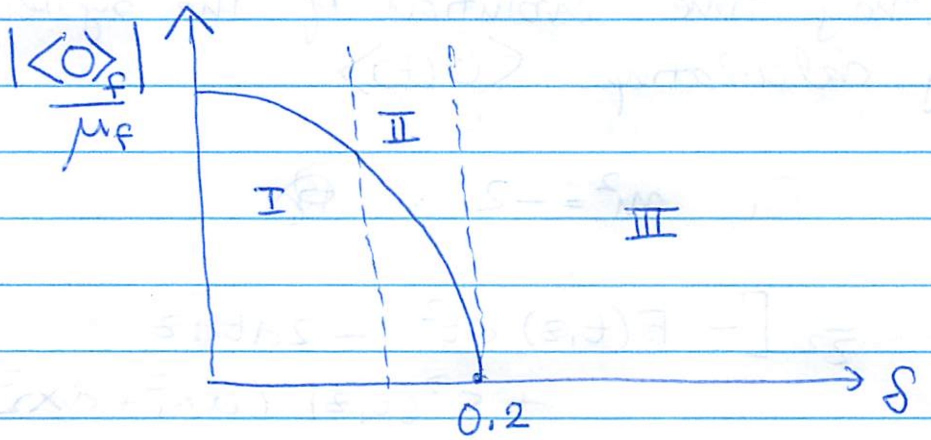
Define

$$\delta = \bar{\delta}/\mu_i \qquad \tau = \mu_i \bar{\tau}$$

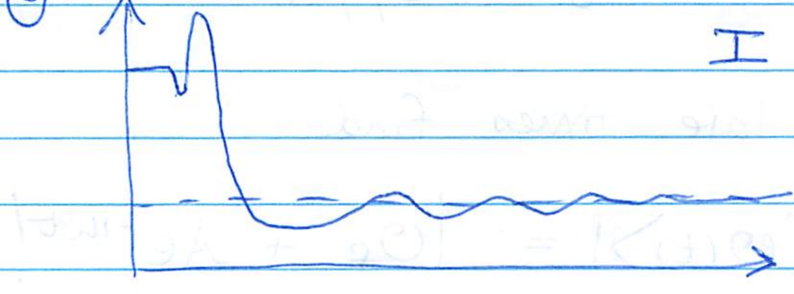
- At late times find

$$\left|\langle \mathcal{O}(t) \rangle\right| = \left|O_f + A e^{-i\omega t}\right|$$

- For a fixed τ, study dependence on δ.



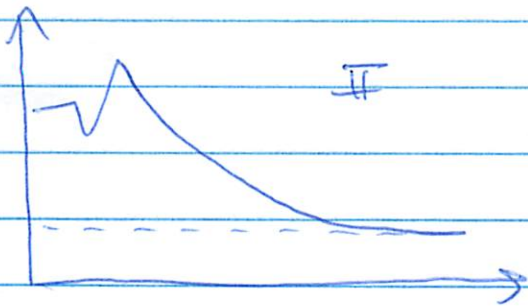
- For small δ system stays in the condensed phase with a final temperature (obtained from the event horizon)
  - $\langle O(t)\rangle$ shows oscillation and decay to a nonzero $\langle O\rangle_f$
  Decay is exponential

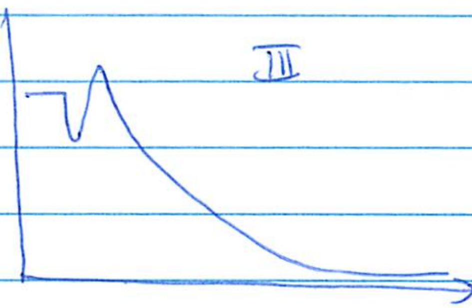


→ damped oscillation to final value.

- For intermediate δ – region II – the approach to a nonzero $\langle O_f \rangle$ is OVERDAMPED

- For large enough δ, order is eventually destroyed





- Transition between I and II $\rightarrow$ A NEW DYNAMICAL TRANSITION

QUENCH ACROSS A HOLOGRAPHIC CRITICAL POINT

$$S = \int d^{d+1}x \sqrt{g} \Big[\frac{1}{2\kappa^2} \left(R + \frac{d(d-1)}{L^2} \right) - \frac{1}{4g^2} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2\lambda} \left[|D_\mu \Phi|^2 - m^2 |\Phi|^2 - \frac{1}{2} |\Phi|^4 \right] \Big]$$

$\lambda \gg g^2 \qquad \lambda \gg \kappa^2$

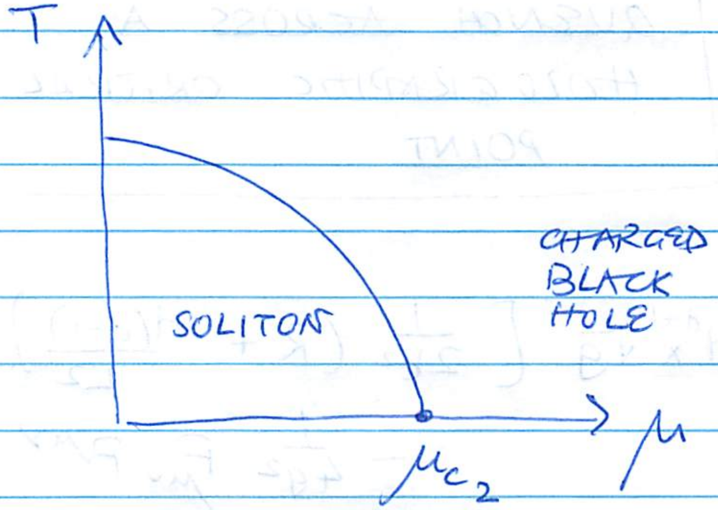
One of the spatial directions is a compact circle of radius R at infinite — call this θ

$$A_t \xrightarrow[r \to \infty]{} \mu$$

$T \neq 0$.

- The dual theory has $T \neq 0$ and $\mu \neq 0$

First consider just gravity + Maxwell



In Soliton phase

- Now consider the scalar with

$$-4 \leq m^2 \leq -3$$

As usual ~~...~~ at ∞ ~~in either~~

$$\Phi \sim r^{-\Delta_-} J(t) \left[1 + O(1/r^2)\right] + r^{-\Delta_+} A(t) \left[1 + O(1/r^2)\right]$$

$J(t)$: Source

$A(t) = \langle \mathcal{O} \rangle$

- To find a solution which has

$J = 0$

Regular at tip.

Eqn to be solved

$$-\frac{1}{r^2}(\partial_t - iq\mu)^2\Phi + \frac{1}{r^3}\partial_r(r^5 f_s(r)\partial_r)\Phi$$
$$- m^2\Phi - |\Phi|^2\Phi = 0$$

- Once again $\Phi = 0$ is a soln ~~in~~ — this is just the AdS soliton with $A_t = \mu$

- With the transformation

$$\rho(r) = \int_r^\infty \frac{dx}{x^2 f_s^{1/2}(x)} \rightarrow \frac{1}{r} \quad r\to\infty$$
$$\rightarrow \rho_* + \sqrt{r-1} \quad \text{as } r\to 1$$

$$\rho_* = 1.311$$

$$\Phi(r) = \frac{1}{[r(\rho)]^{1/2}}\left(\frac{d\rho}{dr}\right)\Psi(\rho)$$

$$\boxed{(-\partial_t^2 + 2i\mu\partial_t)\Psi = (\mathcal{P} - \mu^2)\Psi + \frac{1}{r^2 f_s^{1/2}(r)}|\Psi|^2\Psi}$$

$$\mathcal{P} = -\partial_\rho^2 + V_0(\rho)$$

$$V_0(\rho) = m^2 r^2 + \frac{15r^8 - 18r^4 - 1}{4r^2(r^4-1)}$$

- For relevant values of m^2, $\mathcal{P}$ has a positive spectrum

- This means that
$$\mathcal{D} = \mathcal{P} - \mu^2$$
At some value $\mu^2 = \bar{\mu}^2$.

- In fact for $\mu^2 > \bar{\mu}^2$ there is a nontrivial solution of the NONLINEAR equation which is
  - x regular at tip
  - x vanishes with $J=0$ at the boundary.

- For this solution $\langle O \rangle = A \neq 0$
  $\rightarrow U(1)$ symmetry broken.

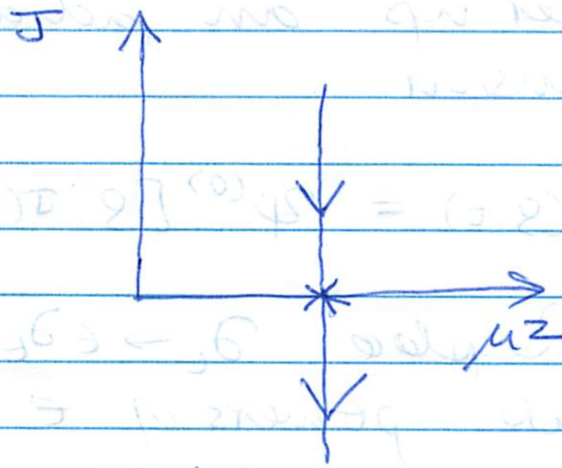
$$\langle O \rangle \sim (\mu - \bar{\mu})^{1/2}$$

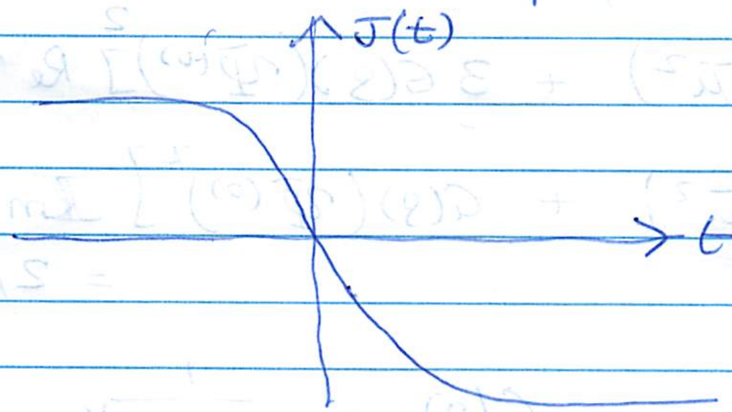
$$\langle O \rangle \Big|_{\mu = \bar{\mu}} \sim J^{1/3}$$

Adiabaticity

Want to study the system with

$$J(t) = -J_0 \tanh(vt)$$





At $t=0$ J crosses an equilibrium critical point

- For any $J \neq 0$ there is a gap
  In our units choose

$$v \ll 1$$

- The lowest order adiabatic solution is given by

$$\psi^{(0)} = \psi(r, J(t))$$

To set up an adiabatic expansion

$$\psi(\rho, t) = \psi^{(0)}[\rho, J(t)] + \epsilon \Psi^{(1)}(\rho, t) + \cdots$$

Replace $\partial_t \to \epsilon \partial_t$ ~~set $\epsilon = 1$~~

equate powers of ϵ

$$[(P - \bar{\mu}^2) + 3G(\rho)(\Psi^{(0)})^2] \, \mathrm{Re}\, \Psi^{(1)} = 0$$

$$[(P - \bar{\mu}^2) + G(\rho)(\Psi^{(0)})^2] \, \mathrm{Im}\, \psi^{(1)} = 2\mu \partial_t \psi^{(0)}$$

$$G(\rho) = \frac{1}{r f_s(r)^{1/2}}$$

Write $\quad \partial_t \psi^{(0)} = \dot{J} \dfrac{\partial \psi^{(0)}}{\partial J}$

- If $\mathcal{G}(\rho,\rho')$ is the Green's fn

$$\left[(\mathcal{P}-\bar{\mu}^2) + G(\rho)(\psi^{(0)})^2\right]\mathcal{G}(\rho,\rho') = \delta(\rho-\rho')$$

The solution is

$$\operatorname{Im}\Psi^{(1)} = 2\mu\,\mathring{J}(t)\int_0^{\rho_*} d\rho'\,\mathcal{G}(\rho,\rho')\,\frac{\partial\Psi^{(0)}}{\partial J}$$

- We already argued that
$\mathcal{D} \equiv \mathcal{P}-\bar{\mu}^2$
has a zero mode.

Since $\psi^{(0)} \sim J^{1/3}$
This means that at $J \to 0$
ie near $t \sim 0$ the Green's function diverges and thus $\Psi^{(1)}$ becomes large

- For small J may use perturbation theory

$\Rightarrow$

$$\operatorname{Im}\Psi^{(1)} \sim \frac{1}{J^{4/3}}\,\mathring{J}(t)$$

Thus adiabaticity breaks when

$$\cancel{\text{or}} \quad \dot{J}(t) \sim J^{5/3}$$

$$\left(\mathrm{Im}\, \Psi^{(1)} \sim \Psi^{(0)} \right)$$

Since near $t \sim 0$ $\quad J \sim vt$

$$\boxed{t \sim v^{-2/5}}$$

and here $\boxed{\Psi^{(0)} \sim (vt)^{1/3} \sim v^{1/5}}$

- After adiabaticity breaks there is no ~~expa~~ Taylor series expansion in v any more

  However we now show that there is an expansion in $v^{2/5}$ IN THE CRITICAL REGION

- In critical region

$$J \sim J_0(vt)$$

- Separate out the source term

$$\psi(\rho,t) = \rho^{\alpha} J(t) + \psi_s(\rho,t).$$

we know

$$\psi_s \sim \rho^{1-\alpha} \qquad \alpha = \Delta_- - d/2$$

- Now ~~write~~ rescale

$$t = v^{-2/5}\eta \qquad \psi_s = v^{1/5}\chi$$

- Eqn becomes $\quad \mathcal{D} = P - \bar{\mu}^2$

$$\mathcal{D}\chi = v^{2/5}\left[2i\mu\,\partial_\eta\chi - G(\rho)|\chi|^2\chi - \eta(\mathcal{D}\rho^{\alpha})\right] + O(v^{4/5})$$

- Now decompose the field in eigenfunctions of $\mathcal{D}$

$$\mathcal{D}\varphi_n = \lambda_n\varphi_n(\rho)$$

We know that there is a soln with $\lambda_n = 0$

$$\chi(\rho,\eta) = \sum_n \chi_n(\eta)\varphi_n(\rho)$$

- The equations now become

$$\lambda_n x_n = v^{2/5}\Big[2i\mu(\partial_\eta x_n) - \sum_{n_1 n_2 n_3} C^{n}_{n_1 n_2 n_3} x^*_{n_1} x_{n_2} x_{n_3} + \mathcal{J}_n \eta\Big] + O(v^{4/5})$$

$$C^{n}_{n_1 n_2 n_3} = \int d\rho\, \varphi_n^*(\rho)\, \varphi_{n_1}\varphi_{n_2}\varphi_{n_3}\, G(\rho)$$

$$\mathcal{I}_n = J_0 \int d\rho\, \varphi_n^*(\rho)\, (\alpha\theta\rho^{\alpha}) -$$

- Clearly the zero mode

$$x_0(\eta) = \xi_0(\eta) + O(v^{2/5})$$

where

$$\boxed{2i\mu \frac{\partial \xi_0}{\partial \eta} = C^{0}_{000}\, |\xi_0|^2 \xi_0 + \mathcal{J}_0 \eta}$$

while

$$n \neq 0 \qquad x_n(\eta) = v^{2/5}\, \xi_n(\eta) + O(v^{4/5})$$

- Thus in the critical region the dynamics is dominated by the zero mode

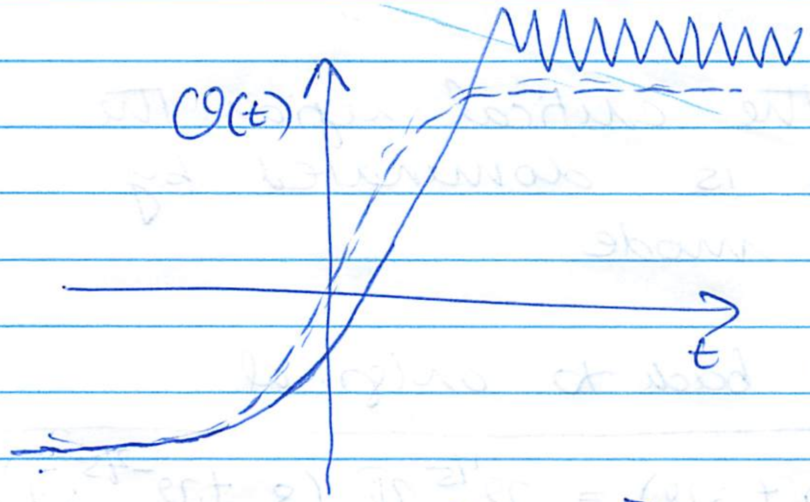
- Reverting back to original

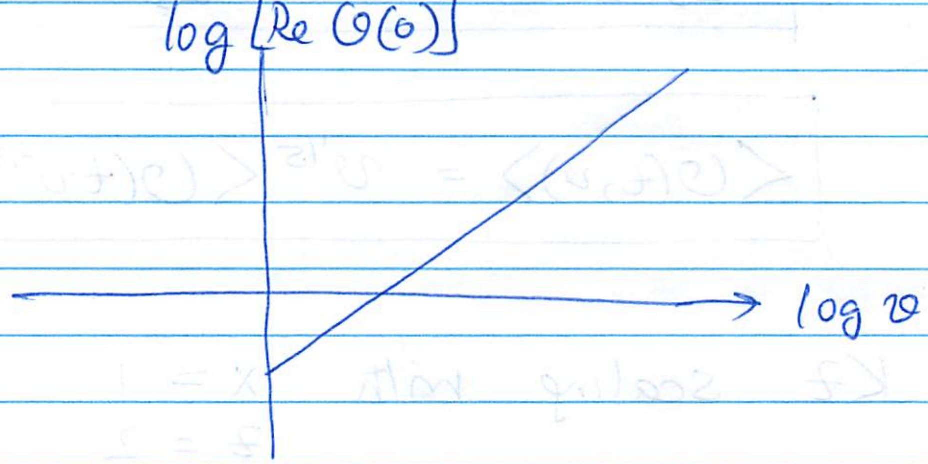
$$\boxed{\Psi_s(s,t;v) = v^{1/5}\,\Psi_s(s, t v^{-2/5}; 1)}$$

$$\boxed{\langle \mathcal{O}(t,v)\rangle = v^{1/5}\langle \mathcal{O}(t v^{2/5}, 1)\rangle}$$

KZ scaling with
$x = 1$
$z = 2$
$\nu = 1/3$

- These results have been verified with numerical calculations.





$$\log[\mathrm{Re}\,\Theta(0)] = -0.79 + 0.206 \log v$$

$$\Theta(0) \sim v^{1/5}$$

KZ argument

$$\mathcal{O} \sim J^{1/3} \qquad \nu = 1/3, \quad x = 1$$

gap $\Delta \sim J^{2/3}$ $\qquad \nu z = \frac{2}{3}$

Thus $\mathcal{O} \sim \Delta^{1/2}$ $\qquad x = 1$ since $z = 2$

$$\frac{x\nu}{\nu z + 1} = \frac{1/3}{\frac{2}{3}+1} = \frac{1}{5} \qquad t \sim l^2$$

$$\left(\Delta \sim \xi^{-\nu z} \right) \qquad \varepsilon^{1/2} \sim t^{-1/2} \sim l^{-1}$$