Implications of Gravitational Radiation from Rapidly Spinning Neutron Stars

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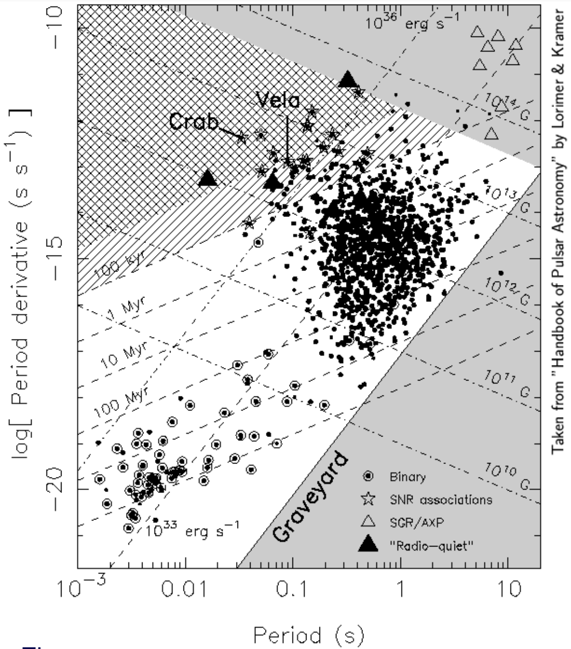

Figure : The period and its derivative for various kind of neutron stars.

Spin frequency distribution of AMXPs

sharp cutoff near 730 Hz

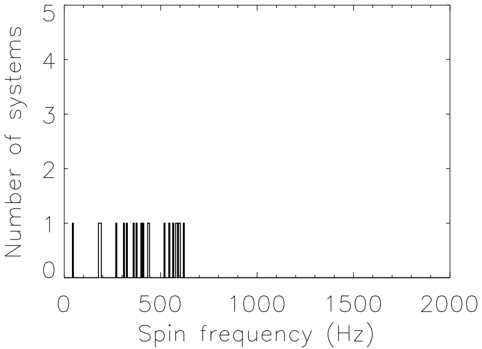

Figure : The spin frequency distribution of accreting millisecond X-ray pulsars. **There is a sharp cutoff in the population for spins above 730 Hz.** RXTE has no significant selection biases against detecting oscillations as fast as 2 kHz, making the absence of fast rotators extremely statistically significant (Ref: D. Chakrabarty astro-ph:0809.4031v1).

Neutron star break-up and equation of state (EoS)

Constraining the M-R space for several EoSs

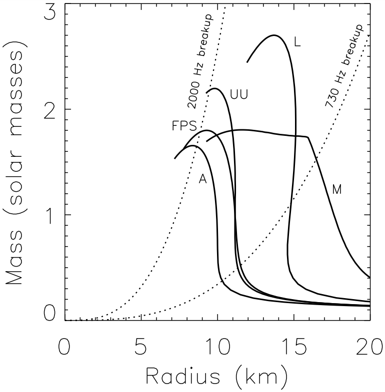

Figure : The solid curves are theoretical mass-radius relations for a variety of models for the equation of state for ultradense matter from. The dashed curves show the limits arising from breakup spin rates of 730 Hz and 2 kHz; the allowed phase space is to the right of the appropriate dashed curve. **While a 2 kHz breakup rate is consistent with most equations of state, a 730 Hz breakup rate is inconsistent with most models and excludes the 8-12 km radius range usually inferred for a 1.4 $M_\odot$ neutron star** (Ref: D. Chakrabarty astro-ph:0809.4031v1).

Gravitational radiation as a natural reason

Spin frequency distribution of AMXPs

To describe the sharp cutoff in frequency distribution of AMXPs, Gravitational radiation turns out to be a natural reason. There are atleast three well known phenomena which can potentially be source of GW from an rotating (*isolated*) neutron star :

- r-mode oscillations
- magnetically confined mountain
- bulk and/or crustal deformation (non axis-symmetric)

Here, we will consider the effect of bulk deformation on GW radiation and its implication in LMXBs.

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GW from an isolated neutron star
Rapidly spinning neutron star

The gravitational wave strain :

$$h_0 = \frac{4\pi^2 G}{c^4} \frac{I_3 {f_{gw}}^2}{r} \varepsilon \qquad (1)$$

with $f_{gw} = 2\pi\omega_{rot}$ and ellipticity $\varepsilon = \frac{I_1 - I_2}{I_3}$

$$h_0 = 1.06 \times 10^{-25} \left(\frac{\varepsilon}{10^{-6}}\right)\left(\frac{I_3}{10^{38} kgm^2}\right)\left(\frac{10kpc}{r}\right)\left(\frac{f_{gw}}{1kHz}\right)^2 \qquad (2)$$

NS typically have $M \simeq 1.4 M_\odot$, $R \simeq 10$ km and $I_3 \sim 10^{38}$ kg-m^2, $r = 10$ kpc.

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GW from an isolated neutron star

Dependence on EoSs?

Energy loss of a rotating NS due to GW wave emission :

$$\frac{dE_{rot}}{dt} = \frac{-32G}{5c^5}\varepsilon^2 I_3^2 \omega_{rot}^6 \tag{3}$$

In the limit of small asymmetry for homogeneous ellipsoid, i.e., $R_1 \simeq R_2 \Rightarrow \varepsilon = \frac{R_1 - R_2}{R_3} + O(\varepsilon^2)$

Each of R_1, R_2 and R_3 depends on the specific EoS, which can potentially be used to find any implication of the cutoff spin frequency for neutron star.

GW from an isolated neutron star

Dependence on several parameters (EoSs?)

Spin down rate for a rotating NS due to GW wave emission and corresponding torque is :

$$\dot{\omega}_{rot} = \frac{-32G}{5c^5}\varepsilon^2 I_3 \omega_{rot}^5 \tag{4}$$

$$\tau_{GW} = I_3\dot{\omega}_{rot} = \frac{-32G}{5c^5}\varepsilon^2 I_3^2 \omega_{rot}^5 \tag{5}$$

notice that : $\tau_{GW} \propto I_3^2\varepsilon^2\omega_{rot}^5$.

Therefore, I have tried to estimate the effect of different EoSs on I_3.

Using the “Rapidly Rotating Neutron Star”(an open source code), I have computed the stellar structure to find out the I_3 along the spin-axis for a stellar spin-frequency $\simeq 700$ Hz.

Equation of state (EoS)	I_3 (in 10^{38} kg m^2)
eosA	0.49
eosB	0.25
eosC	1.28
eosFPS	0.92

eosA -> PANDHARIPANDE NEUTRON: A&B EOS A
eosB -> PANDHARIPANDE HYPERON: A&B EOS B
eosC -> BETHE-JOHNSON MODEL 1: A&B EOS C
eosFPS -> Lorenz, Ravenhall and Pethick, 1993, PRL 70,379

In this results, I DID NOT consider any magnetic field.

Using the “Lorene”(another open source code), I have computed the stellar structure to find out the R_1 (along) and R_2 (perpendicular to) the magnetic dipole-axis for a non-rotating neutron star.

Central B-field	Magnetic/fluid pressure	$(R_1 - R_2)/R_3$
2.15×10^{17} (G)	1.0×10^{-02}	1.378×10^{-02}
2.18×10^{16} (G)	1.0×10^{-04}	1.378×10^{-04}
2.18×10^{14} (G)	1.0×10^{-08}	1.378×10^{-08}
2.18×10^{12} (G)	1.0×10^{-12}	numerical error

In this results, I DID NOT consider any spin of neutron star.

Spin down rate for a rotating NS due to GW wave emission and corresponding torque is :

$$\dot{\omega}_{rot} = \frac{-2M_B^2\omega_{rot}^3}{3c^3 I_3} sin^2\theta \tag{6}$$

$$\tau_{EM} = I_3\dot{\omega}_{rot} = \frac{-2M_B^2\omega_{rot}^3}{3c^3} sin^2\theta \tag{7}$$

notice that : $\tau_{EM} \propto M_B^2\omega_{rot}^3$.

Therefore, smaller the magnetic-field smaller the τ_{EM}.

Magnetars Vs LMXBs

Comparison between τ_{EM} and τ_{GW}

- Note that, the M_B of a magnetar is generally $\gtrsim 10^5$ times larger than that of a NS in LMXBs.
- But, the spin frequency of magnetar is generally $\lesssim 10^{-3}$ times smaller than that of a NS in LMXBs.
- τ_{EM} (magnetar)/τ_{EM} (LMXB) $\sim 10^{10} \times 10^{-9} \simeq 10$
- τ_{GW} (magnetar)/τ_{GW} (LMXB) $\sim 10^{10} \times 10^{-15} \simeq 10^{-5}$

The result is quite conservative; since inside the bulk of a NS in LMXB, magnetic field can be still higher although on surface it is quite small due to field burial.

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Summary

GW may be a dominant mechanism

- As opposed to magnetars, GW may be a dominant mechanism to limit the large spin of the NS in a LMXB.
- In such case, EoSs play a dominant role in determining the cutoff frequency for a distribution of rapidly spinning NSs, iff other effects of GW is negligible.
- If the discussed scenario holds good, we can expect to potentially rule out some of the EoSs by matching the predicted cutoff frequency for each EoS with the observed value.

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THANK YOU for your kind attention.