Abstracts

Mini Courses

p-adic families of modular forms – R. Sujatha. The first example of a p-adic family of modular forms was the Einsenstein family studied by Serre as part of his larger study on congruences. Hida's work carried this much further, integrating it with the theory of Galois representations and Hecke algebras. We shall provide a survey of these topics in the course.

References

[1] M. Emerton: *p-adic families of modular forms [after Hida, Coleman, and Mazur]*. Seminaire Bourbaki, 2009/2010, expose 1013, Asterisque 339 (2011), 31-61.

[2] H. Hida: *Iwasawa modules attached to congruences of cusp forms.* Ann. Scient. Ec. Norm. Sup. 4th series 19 (1986), 231-273.

[3] H. Hida: *Elementary Theory of L-functions and Eisenstein series.* Cambridge University Press, (1993), Book.

[4] D. Banerjee, E. Ghate, V.G.N Kumar: *Λ-adic forms and the Iwasawa main conjecture.* `http://www.math.tifr.res.in/~eghate/lectures.pdf`

Families of Siegel modular forms and Galois representations – Jacques Tilouine. We first recall Hida's geometric construction of p-adic Siegel modular forms and the existence of families of ordinary Siegel modular forms (following Hida and Pilloni), together with the Galois representations associated to these families. Then, we will discuss the finite slope case, and the existence of families of overconvergent Siegel modular forms, based on the theory of the canonical subgroup (by Andreatta-Iovita-Pilloni), resp. on the overconvergence of the monodromy (by Brinon-Mokrane-T.) together with the Galois representations associated to these families.

Ordinary families for definite unitary groups – Baskar Balasubramanyam. In these lectures, we study families of autormorphic forms on definite Unitary groups. The material will divided as follows:

(1) In the first lecture, we will define automorphic forms on definite unitary groups.
(2) We study the Hecke algebras associated to these forms.
(3) We construct Hida families via a control theorem for the ordinary Hecke algebras.
(4) We associate Galois representations to these automorphic forms.

The main reference for these lectures is [1, §2].

References

[1] D. Geraghty. *Modularity lifting theorems for ordinary Galois representations*, Preprint available at `https://www2.bc.edu/david-geraghty/`

Modularity lifting theorems for ordinary Galois representations – David Geraghty. This is a series of four lectures on modularity lifting, with emphasis on the ordinary case. Baskar Balasubramanyam will give four lectures covering the necessary background on Hida theory. In this course, our lectures will be organised as follows:

(1) Introduction to modularity lifting theorems. We will explain the general ideas behind the Taylor-Wiles-Kisin method for proving modularity of Galois representations.
(2) Galois deformation rings. In the first lecture, we will see that the applicability of modularity lifting theorems is dependent on the structure of local Galois deformation rings. In practice, one is limited primarily by the local deformation rings at the primes dividing the residue characteristic of the field of coefficients (the $l = p$ case). We will discuss some of what is known about these rings and some applications to modularity.

(3) Potential diagonalizability. In this lecture we will discuss a technique introduced by Harris that allows one, in certain circumstances, to prove modularity lifting theorems even when the appropriate local deformation rings are very complicated. Hida families will play an important role here.
(4) Recent developments. Depending on time remaining, we may discuss re- cent work of Thorne on modularity lifting in the residually redicible case (building on work of Skinner and Wiles). Again, Hida feature prominently in this work.

p-adic L-functions for $\mathrm{GL}(2)$ **– Mladen Dimitrov.** The use of modular symbols to attach p-adic L-functions to Hecke eigenforms goes back to the work of Manin et al in the 70s. In the 90s, Stevens proposed a new approach based on his theory of overconvergent modular symbols, which was successfully used to construct p-adic L- functions on the Eigenvariety. The first two lectures will present a selection of those results. In the third lecture we will introduce some special cycles on Hilbert modular varieties, while in the last lecture we will use the overconvergent cohomology of those varieties to attach p-adic L-functions to (families of) automorphic forms on GL(2).

References

[1] D. Barrera, *Cohomologie surconvergente des variétés modulaires de Hilbert et fonctions L p-adiques,* Thesis, Université Lille 1, 2013.
[2] J. Bellache, *Critical p-adic L-functions,* Invent. Math., 189 (2012), pp. 1-60.
[3] M. Dimitrov, *Automorphic symbols, p-adic L-functions and ordinary cohomology of Hilbert modular varieties,* Amer. J. Math., 135 (2013), pp. 1-39.
[4] B. Mazur, J. Tate, and J. Teitelbaum, *On p-adic analogues of the conjectures of Birch and Swinnerton-Dyer,* Invent. Math., 84 (1986), pp. 1-48.
[5] G. Stevens, *Rigid analytic modular symbols,* preprint 1994.

Arithmetic of adjoint L-values – Haruzo Hida. In the series of lectures, we discuss

(1) Construction of one variable adjoint p-adic L-function in the elliptic modular case
(2) Extension of the result to Hilbert modular case,
(3) A proof of the one variable adjoint main conjecture,
(4) Overview of the two variable L-functions and adjoint main conjecture.

The first lecture covers material in §5.2-3 of [1]. The second lecture extends the result to Hilbert modular cases via "$\mathbf{R} = \mathbb{T}$"-theorem in [2]. Some topics related to base-change could be covered. In the third lecture, we prove the adjoint main conjecture for the weight variable. The last lecture is an overview of recent progress by other mathematicians and myself.

References

[1] H. Hida. *Modular Forms and Galois Cohomology*, Cambridge Studies in Advanced Mathematics **69**, Cambridge University Press, Cambridge, England, 2000.
[2] H. Hida. *Hilbert modular forms and Iwasawa theory,* Oxford Mathematical Monographs. The Clarendon Press, Oxford University Press, Oxford, 2006.

p-adic L-functions for $\mathrm{GL}(n)$ **– Debargha Banerjee and A. Raghuram.** In the early 70's Mazur and Swinnerton-Dyer constructed p-adic L-functions attached to an elliptic curve. This was generalized to attaching p-adic L-functions for cusp forms on GL(2) by Manin and others. Later this construction was generalized in two different directions: for cusp forms on $\mathrm{GL}(n)\times\mathrm{GL}(n-1)$ by C.-G.Schmidt and his collaborators and students, and in another direction for cusp forms on $\mathrm{GL}(2n)$ which admit a Shalika model by Ash and Ginzburg. This mini-course of four lectures will be a survey of such constructions of p-adic L-functions.

References

[1] Mazur, B., Swinnerton-Dyer, P.: *Arithmetic of Weil curves.* Invent. Math. 25, 1–61 (1974).

[2] Manin, Y.: *Non-archimedean integration and Jacquet-Langlands p-adic L-functions.* Russian Math. Surveys 31, 5–57 (1976).

[3] Kazhdan, D., Mazur, B., and Schmidt, C.-G.: *Relative modular symbols and Rankin-Selberg convolutions.* J. Reine Angew. Math., 519, 97-141 (2000).

[4] Ash, A., and Ginzburg, D.: *p-adic L-functions for* GL($2n$). Invent. Math. 116, 27–73 (1994).

Advanced Discussion Meeting

- Jeanine van Order – *Average values of automorphic L-functions via spectral decomposition of shifted convolution sums*

  I will explain a technique for estimating average values Rankin-Selberg L-functions for GL(2) via spectral decompositions of certain shifted convolution sums. Such a technique can be used to determine the nonvanishing of certain moments of central values of interest to the Iwasawa main conjectures (along the lines of previous works of Greenberg, Rohrlich, Vatsal, and Cornut), allowing e.g. for an analytic study of the heights of CM points on Shimura curves. The technique applies rather more generally than this though. For instance, it also suggests some interesting avenues for the study of central values of automorphic L-functions of certain higher-rank groups, which I will describe if time permits.

- Haruzo Hida – *Growth of Hecke fields*

  We study growth of Hecke fields on a **thin** p-adic analytic family of Hilbert modular forms. For a non CM family, the field generated by most of $T(l)$-eigenvalue over the family has **unbounded** degree over $\mathbb{Q}(\mu_{p^\infty})$. Our purpose is then to ask many open questions out of this results.

- Baskar Balasubramanyam – *Adjoint L-values and congruences of automorphic forms*

  In 1981, Hida proved that a prime dividing the special value of the adjoint L-function of f at $s = 1$ is a congruence prime for f. This result was later generalized by Ghate, Dimitrov, Urban and Namikawa to cover all automorphic forms with respect to GL(2) over any number field F. I will discuss a generalization of these results to automorphic forms with respect to GL(n) over any number field field F. This is joint work with A. Raghuram.

- Olivier Fouquet – *Congruences and special values of L-functions*

  Except the classical case of Dirichlets class formuler, results on special values of L-functions of motives have been obtained by the consideration of p-adic families of such objects. Following Mazur, it is thus natural to ask whether congruent motives have congruent special values of L-functions. Using results of Saha on the monodromy of p-adic families, we show that this the case for very general motives. As an application, we prove new cases of the Equivariant Tamagawa Number Conjecture for eigencuspforms with odd order of vanishing at the central point.

- Chandrakant Sharma – *Non-commutative Iwasawa theory for deformation rings*

  The universal deformation rings under base change along subfields of the cyclotomic $\mathbb{Z}_p$-extension have been studied by Hida. We extend this and study the universal deformation rings under base change along subfields of certain pro-p, p-adic Lie extensions.

- Giovanni Rosso – *Derivative of the symmetric square p-adic L-function via pull-back formula*

  Let f be a weight k modular form Steinberg at p. Under certain hypotheses on the conductor of f, we give a formula for the derivative at $s = k-1$ of the symmetric square p-adic L-function of f, thus proving a conjecture of Benois on trivial zeros. Crucial is the construction of the p-adic L-function by Böcherer et Schmidt which we will recall.

- Riccardo Brasca – *Families of overconvergent modular forms over Shimura varieties without ordinary locus*
  Overconvergent modular forms are sections of some sheaves over strict neighborhoods of the ordinary locus of the relevant Shimura variety. This notion is hence meaningless if this ordinary locus is empty. We can replace the ordinary locus by the so called μ-ordinary locus introduced by Wedhorn, that is always not empty. In this talk we will explain some results towards the generalization of the theory of families of p-adic overconvergent modular forms to the case of Shimura varieties without ordinary locus.

- Denis Benois – *Selmer complexes, p-adic height pairings and the Main Conjecture*
  In this talk, I will discuss the following related topics: Selmer complexes for non-ordinary p-adic representations, construction of the p-adic height pairing and the Main Conjecture for semistable representations.

- Mahesh Kakde – *Higher rank congruences and geometric Iwasawa theory*
  In the first half of the talk I will show how previous results on K_1 of Iwasawa algebra of one dimensional pro-p p-adic Lie group can be extended to Iwasawa algebra of arbitrary compact p-adic Lie group. Then I will use this description to give a proof of noncommutative main conjecture for abelian varieties defined over finite fields. This is joint work with David Burns.

- David Geraghty – *Patching and the p-adic local Langlands Correspondence*
  The p-adic local Langlands correspondence is well understood for GL_1 of a p-adic field and for $\mathrm{GL}_2(\mathbb{Q}_p)$. In general, for GL_n of a p-adic field, the situation appears to be much more complicated. In this talk I will describe an approach to the problem that uses global methods, namely, completed cohomology and the Taylor–Wiles method. The talk is based on joint work with Caraiani, Emerton, Gee, Paskunas and Shin.

- Ming-Lun Hsieh – *Heegner point Euler system and p-adic L-functions*
  This is a joint work with F. Castella. We prove an explicit reciprocity law for Heegner point Euler systems. We prove an identity between the image of p-adic deformation of classes arising from Heegner points under the dual exponential map and certain Rankin-Selberg central L-values. Applications to some Bloch-Kato conjectures in the rank zero case will be discussed.

- Devika Sharma – *Locally indecomposable modular Galois representations with full residual image*
  In 2011, Ghate and Vatsal gave non-trivial examples of level 1 non-CM Hida families with reducible or dihedral residual Galois representation. In this talk, we consider certain 3-adic non-CM Hida families with full residual Galois representation that split at 3 and give conditions under which every classical point in these families of weight ≥ 2 is locally indecomposable. As a consequence we generate the first non-trivial examples of p-ordinary non-CM locally indecomposable classical modular forms with full Galois representation. This is joint work with Eknath Ghate.

- Fabian Januszewski – *Cohomological induction over $\mathbb{Q}$ and rational structures on automorphic representations*
  It is well known that the actions of the Hecke algebra on the spaces of cusp forms are defined over number fields. This generalizes to cohomological automorphic representations over number fields. In this talk we discuss the analogous statement at infinity, i.e. we show that, in a sense we'll make precise, the representations at infinity are defined over number fields as well, and we thus obtain rational structures on global representations. In order to show the existence of these rational models we introduce an analogue of Zuckerman's cohomological induction functors over $\mathbb{Q}$, and also construct a character theory in this rational setting. It turns out that the field of rationality of a cohomological representation agrees with the field of rationality of its (algebraic) character. In the

light of recent progress on the representability of Gamma factors by cohomological vectors, this should naturally lead to representation-theoretic interpretations of the periods controlling the rationality of special values.

- Eknath Ghate – *Reductions of Local Galois Representations via the LLC*
  The mod p Local Langlands Correspondence (LLC) for GL(2) over $\mathbb{Q}_p$ was originally used by Breuil and most recently by Buzzard-Gee to understand the reductions of local Galois representations attached to modular forms away from the level. The complexity of the problem increases with the weight and with the slope. The problem has been solved completely for weights at most $2p+1$ or for slopes in $(0,1)$. In joint work with A. Ganguli we show how to tackle this problem for slopes in $(1,2)$ and for weights (roughly) less than p^2. We shall talk about this work, and the possibility of extending it to higher weights.
- Tadashi Ochiai – *On the Iwasawa Main Conjecture for CM Hilbert cuspforms*
  We review the basic setting and problem of the cyclotomic Iwasawa Main Conjecture for ordinary Hilbert modular cuspforms. Then we remark on what can be expected when the Hilbert modular has complex multiplication. This is joint work with Takashi Hara.
- Mladen Dimitrov – *Albanese of Picard modular surfaces, and rational points*
  This is a report on a work in progress in collaboration with Dinakar Ramakrishnan. A celebrated result of Mazur proves that open modular curves of large enough level do not have rational points. Our aim is to establish a weak analogue for the Picard modular surfaces, which are Shimura varieties for a unitary group in three variables defined over a CM field. We establish the finiteness of rational points when the level is deep enough, depending on number theoretic invariants of the CM field. The proof applies and combines certain key results of Faltings with the work of Rogawski.
- Sudhanshu Shekhar – *Iwasawa theory and residual Galois representations*
  Iwasawa theory predicts some congruence relations between p-adic L-values of elliptic curves, ordinary at a given odd prime p, and having equivalent residual Galois representation. In this talk, we shall discuss recent results on congruences between p-adic L-values of congruent elliptic curves defined over certain non abelian extensions of the field of rational numbers.
- Ashay Burungale – *On the non-triviality of Heegner points modulo p*
  Let l and p be distinct odd primes unramified in an imaginary quadratic extension $K/\mathbb{Q}$. We outline the proof of the non-triviality of the p-adic formal group logarithm of Heegner points modulo p associated to the Rankin-Selberg convolution of an elliptic modular form of weight two and a theta series over the $\mathbb{Z}_l$-anticyclotomic extension of K. We also make remarks regarding the analogous non-triviality of generalised Heegner cycles.
- Jacques Tilouine – *Big image of Galois representations and congruence ideals*
  In a joint work in progress with Hida, we generalize to GSp_4 a previous result for GL_2 due to Hida. We define a Galois level for general families and relate it to a congruence ideal.