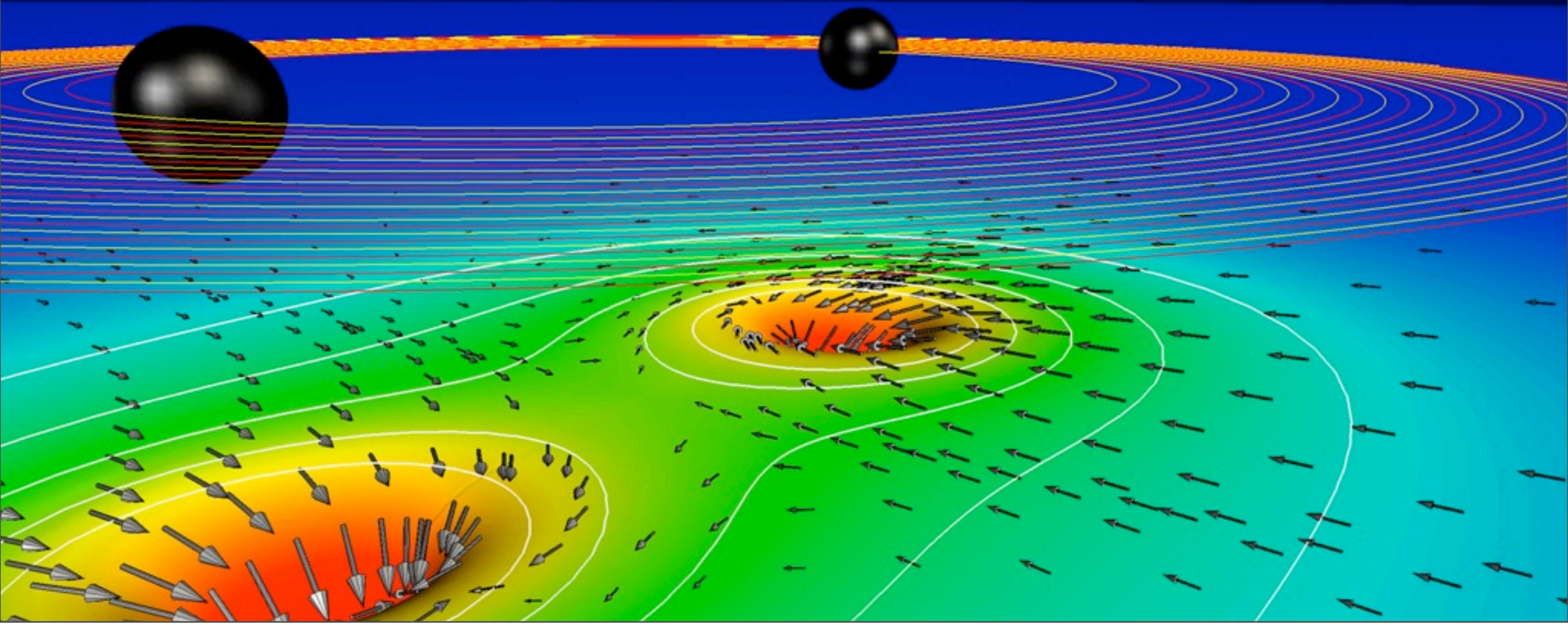
Numerical simulations of binary black holes

Harald Pfeiffer

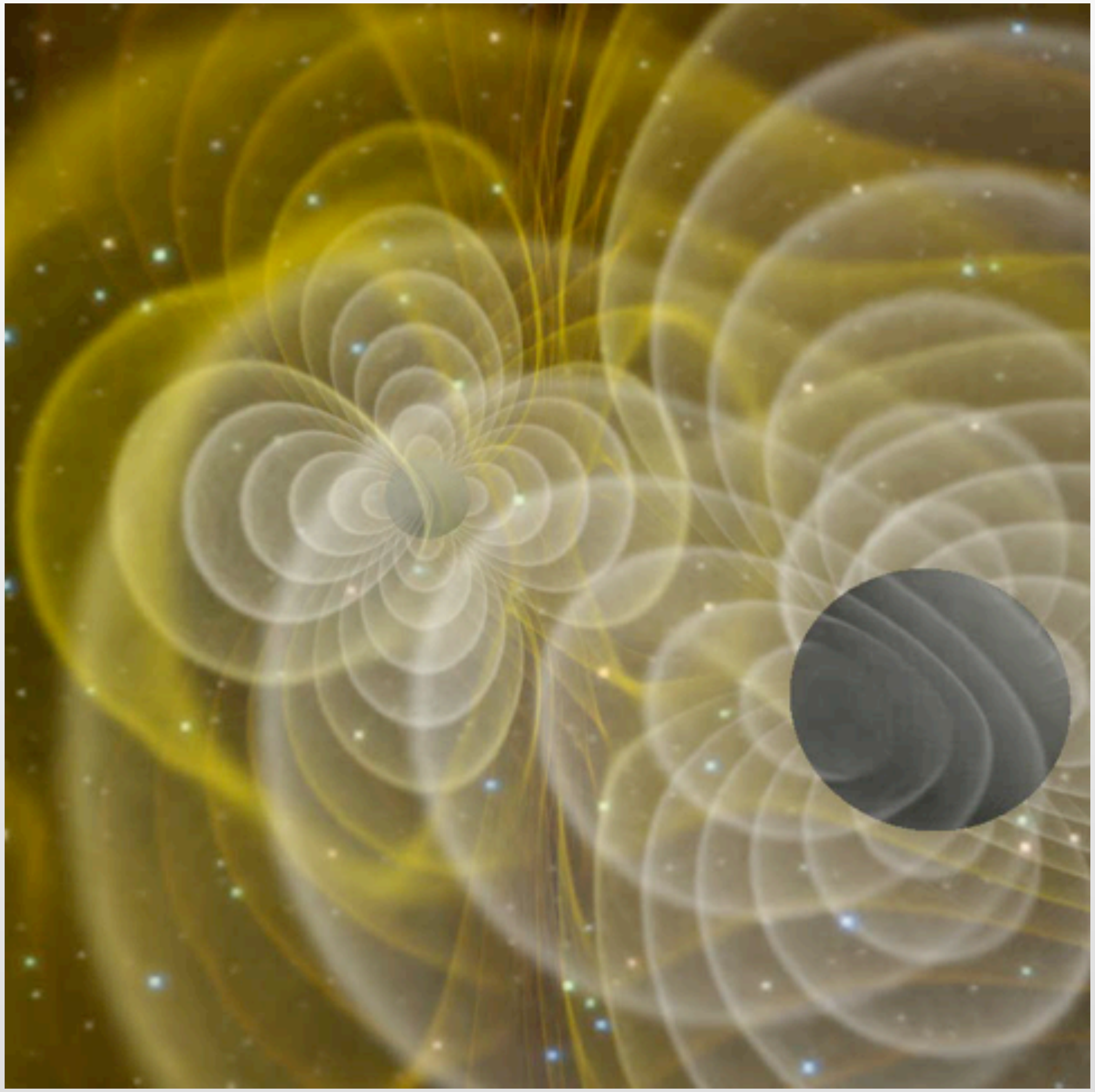
Canadian Institute for Theoretical Astrophysics

ICTS Colloquium

ICTS/TIFR Bengaluru Jul 4, 2013



Binary Black Holes



(Courtesy J. Centrella, Goddard)

$$L_{\rm max} = 10^{23} L_\odot \sim L_{\rm universe}$$

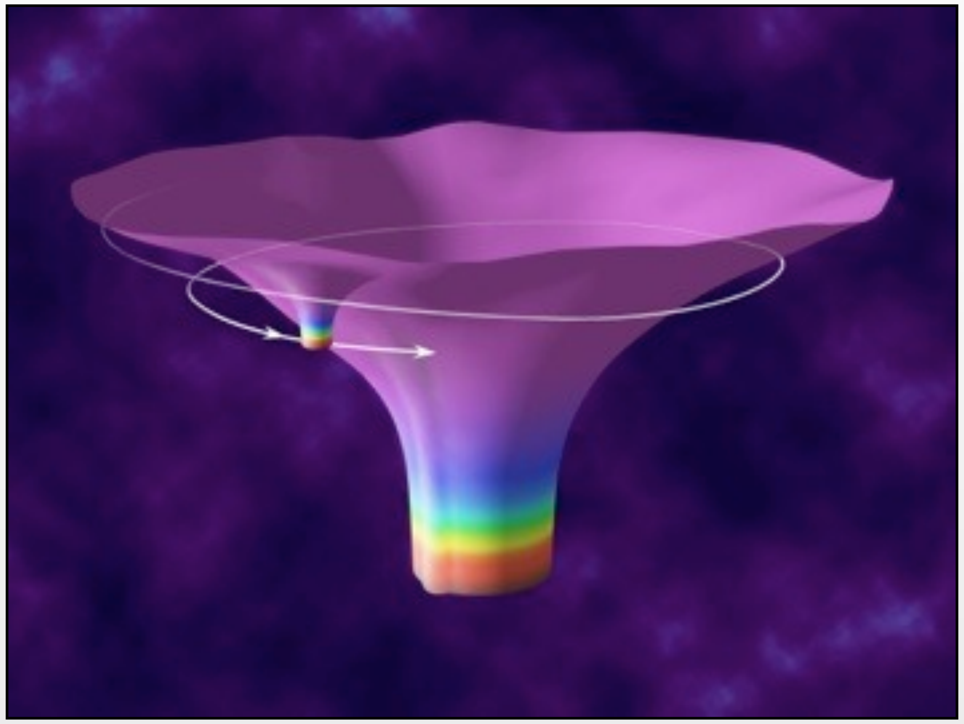
- ❖ at distance 10Gpc

$$\Phi_{\rm max} \sim 10^4 \Phi_{\rm Moon}$$

- ❖ No electro-magnetic emission
  - Only gravitational radiation

Black hole

- ❖ Made entirely of warped space-time
  - Curvature of space
  - Slowing of flow of time
  - Dragging of space around BH



Courtesy Kip Thorne

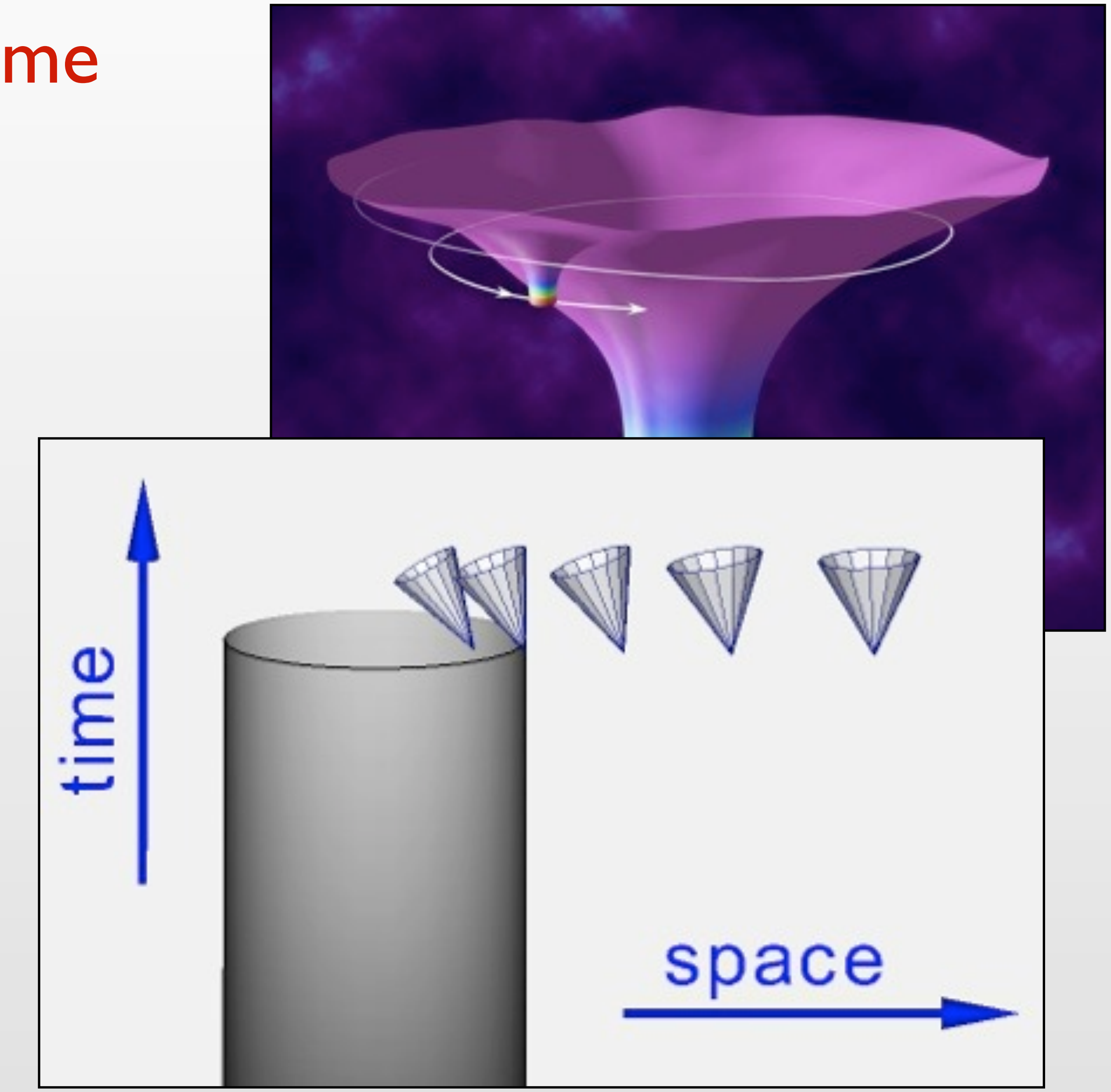
Black hole

❖ Made entirely of warped space-time

- Curvature of space
- Slowing of flow of time
- Dragging of space around BH

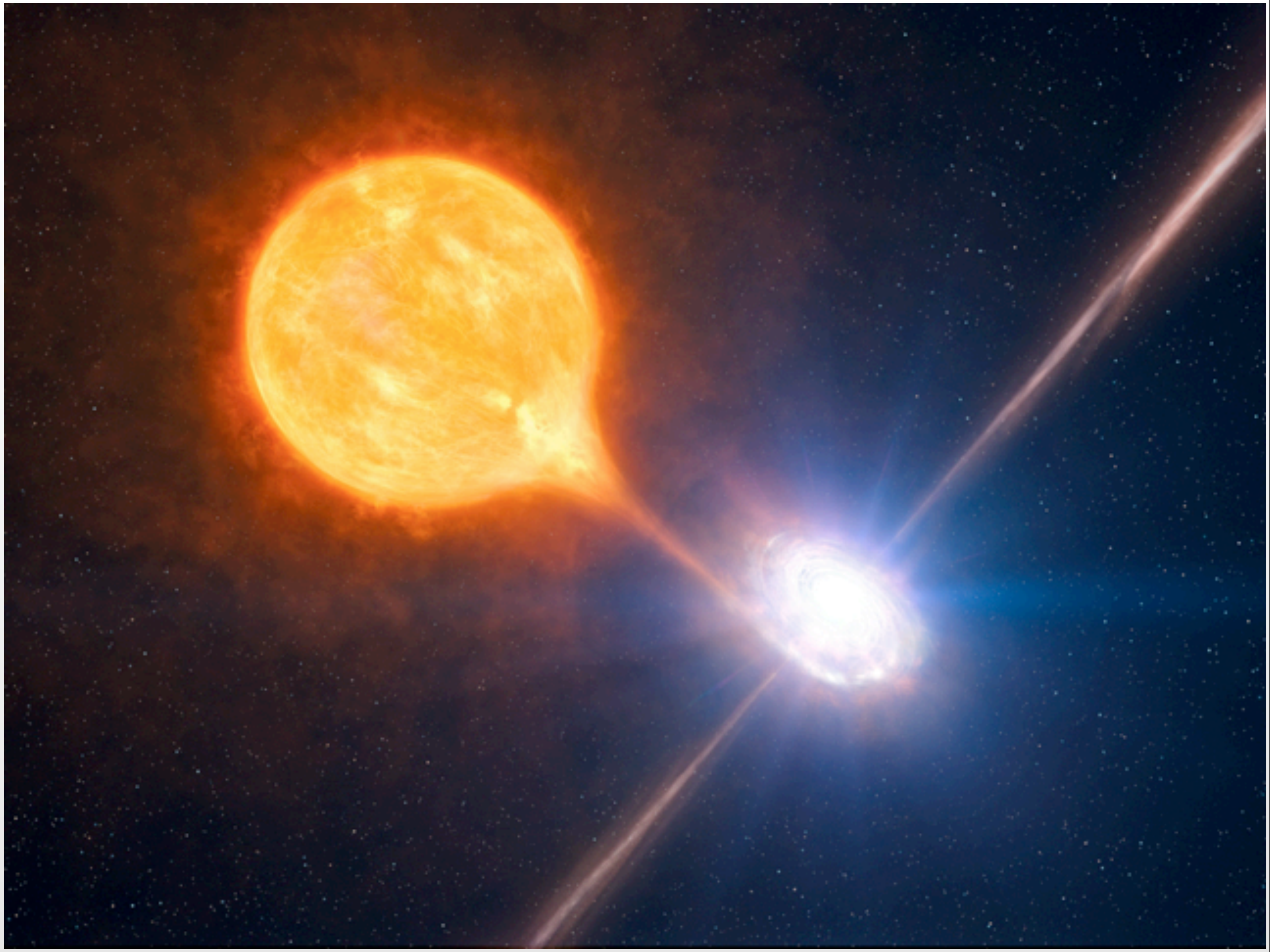
❖ Causal structure changes

- Tipping of light-cones
- Event horizon



$$A_{\mathrm{EH}} = 4\pi r_S^2, \quad r_S = \frac{2GM}{c^2} = 3\frac{M}{M_\odot}\mathrm{km}$$

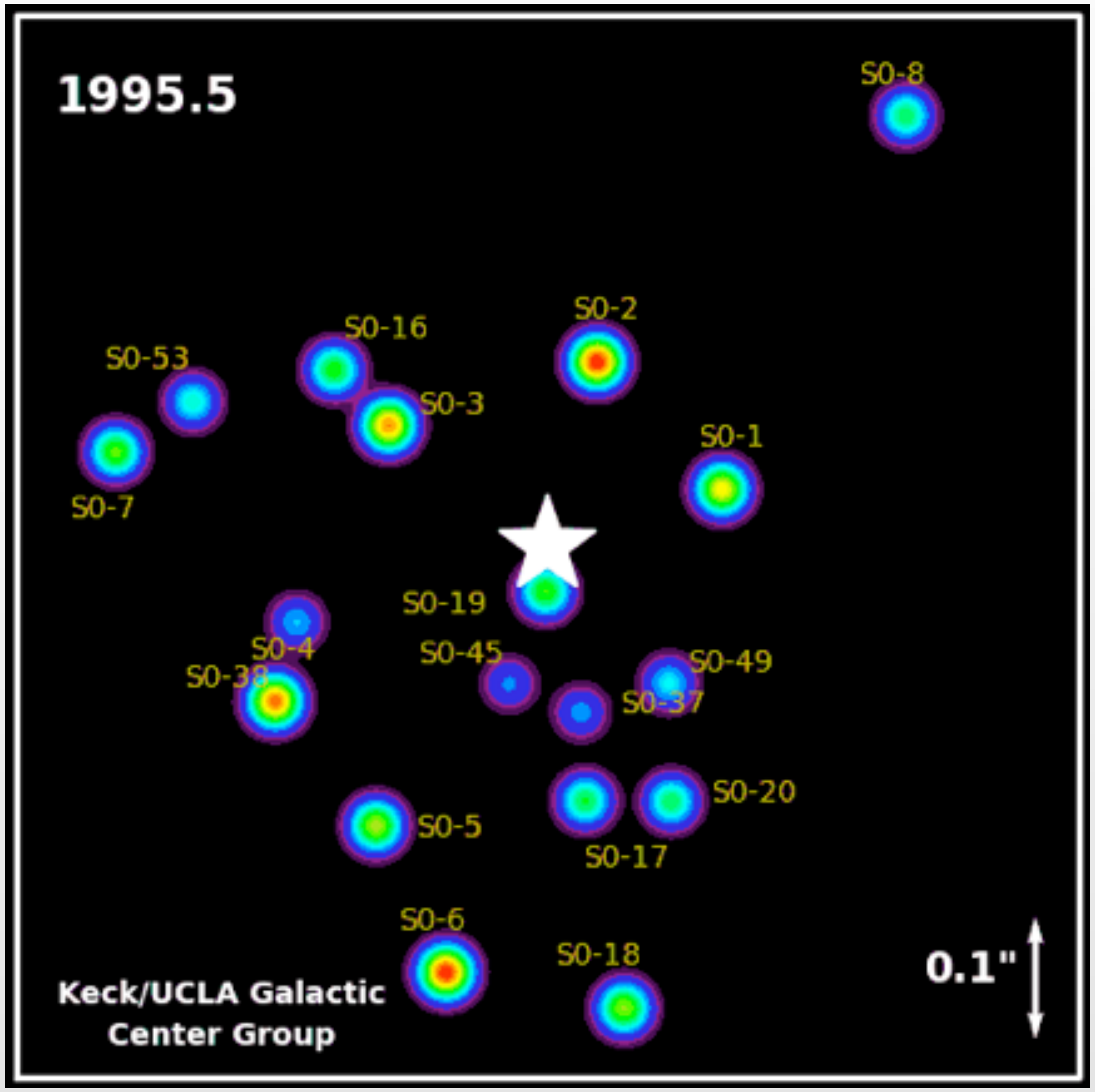
X-ray binaries



ESO/L. Calçada/M.Kornmesser

- $M_{\text{partner}} = 5 - 30 M_{\text{Sun}}$
- Orbit too tight for star, object to massive for Neutron star. **Stellar mass** Black Hole.

Center of the Milky Way



Ghez et al.

ESO/S. Gillessen et al.

- ❖ $4.1\times10^6 M_{sun}$ within size of solar system

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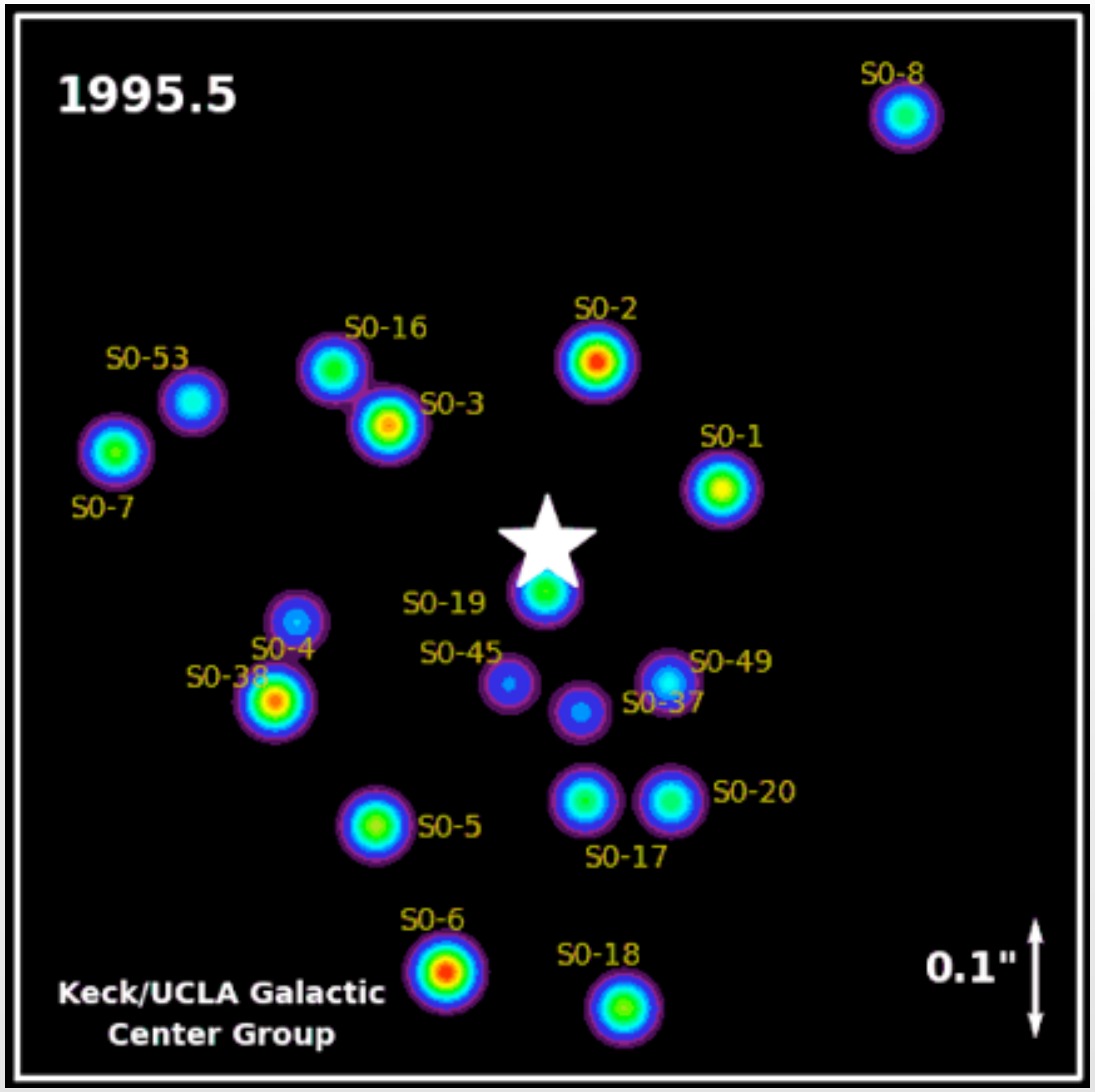

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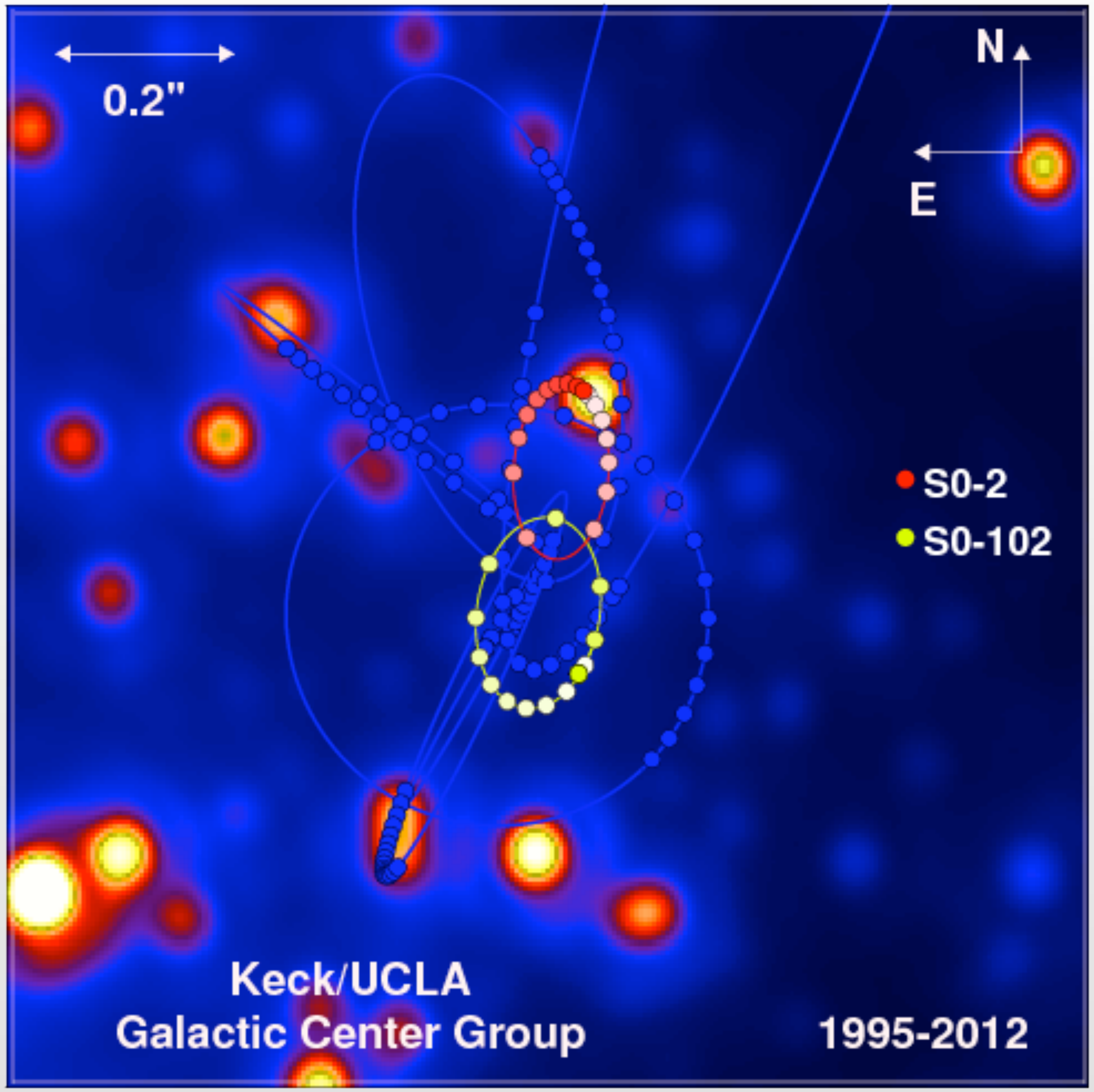
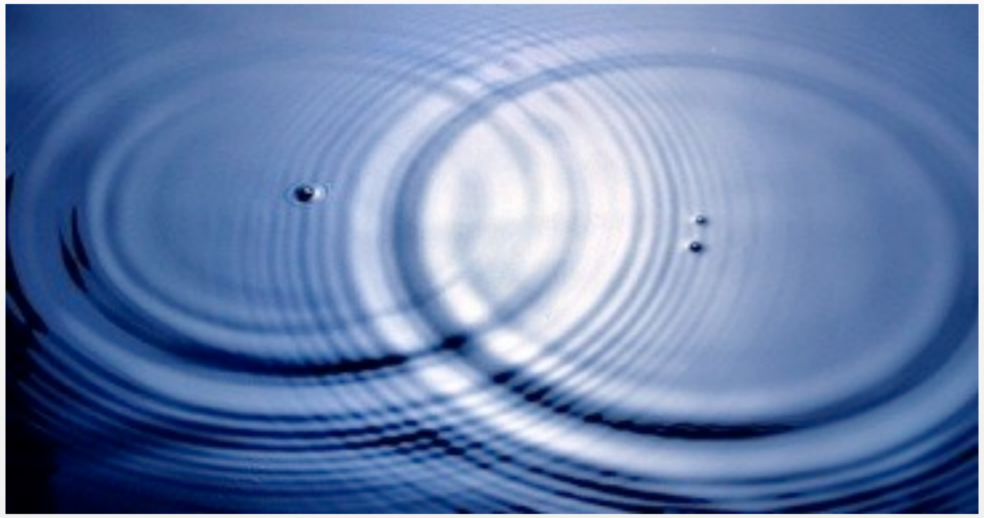
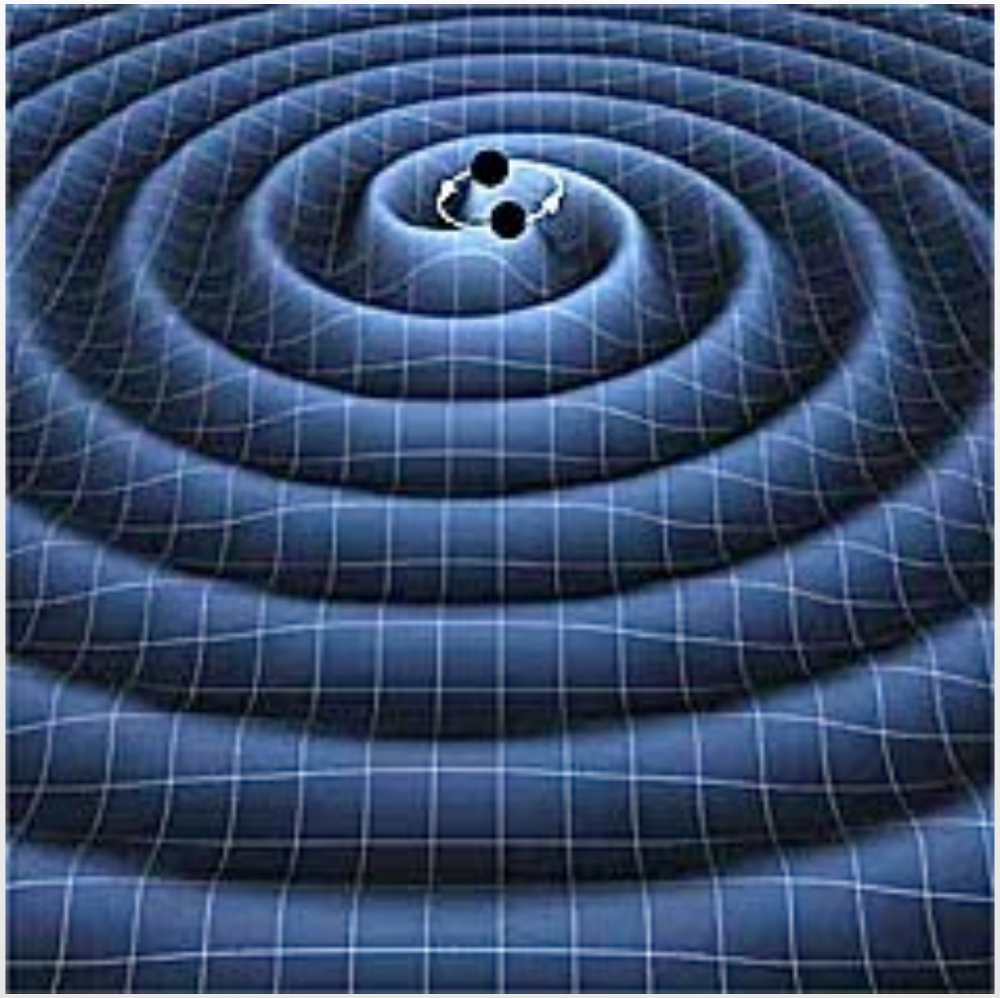


- $4.1\times 10^6 M_{sun}$ within size of solar system

S0-102 P =11.5yr Meyer ea 12

Gravitational waves





- ❖ Perturbations of space-time itself, traveling at speed of light

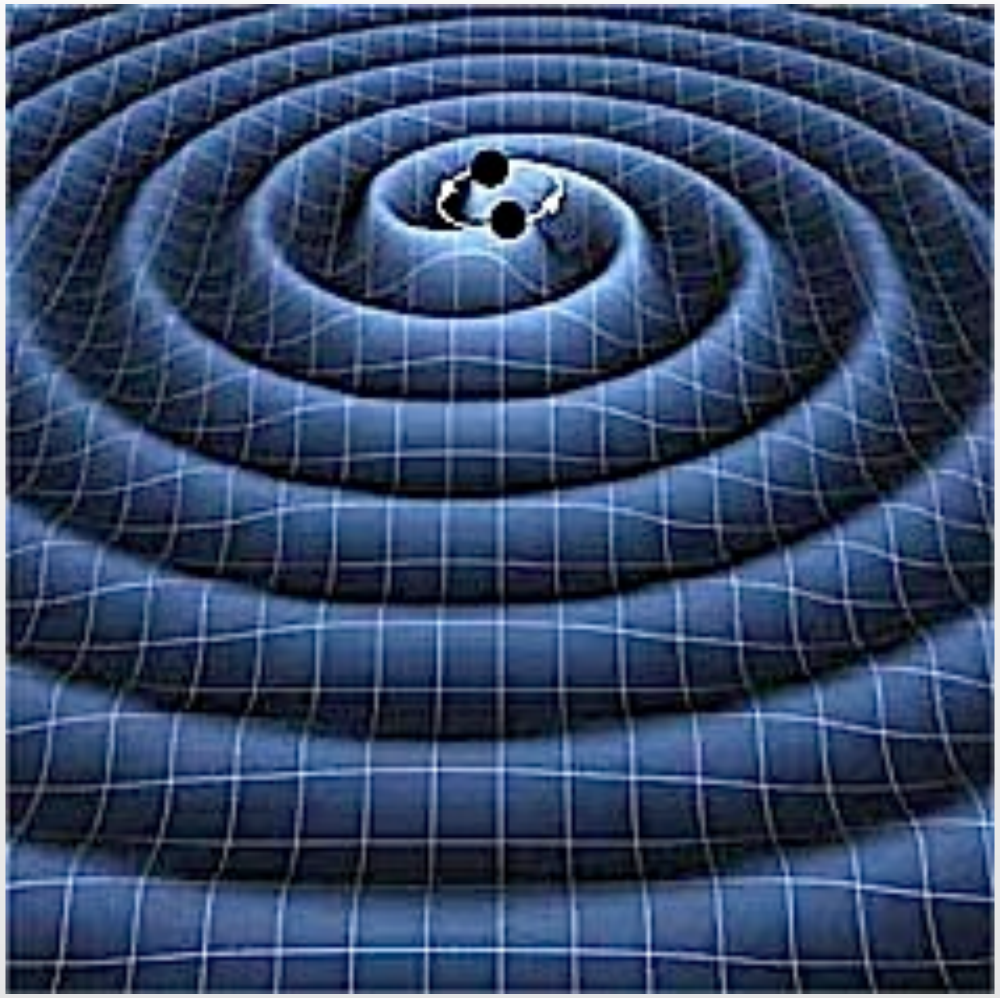
Gravitational waves

- ❖ Generated by time-varying quadrupole moments

$$Q_{ij} = \sum_{\mathrm{A}} x^i_A x^j_A M_A$$

$$h_{ij} \sim \frac{2G}{c^4 r}\ddot{Q}_{ij} \sim \frac{G}{c^4 r}E_{\mathrm{kin,bulk}}$$

  - Large masses. Fast, asymmetric motion



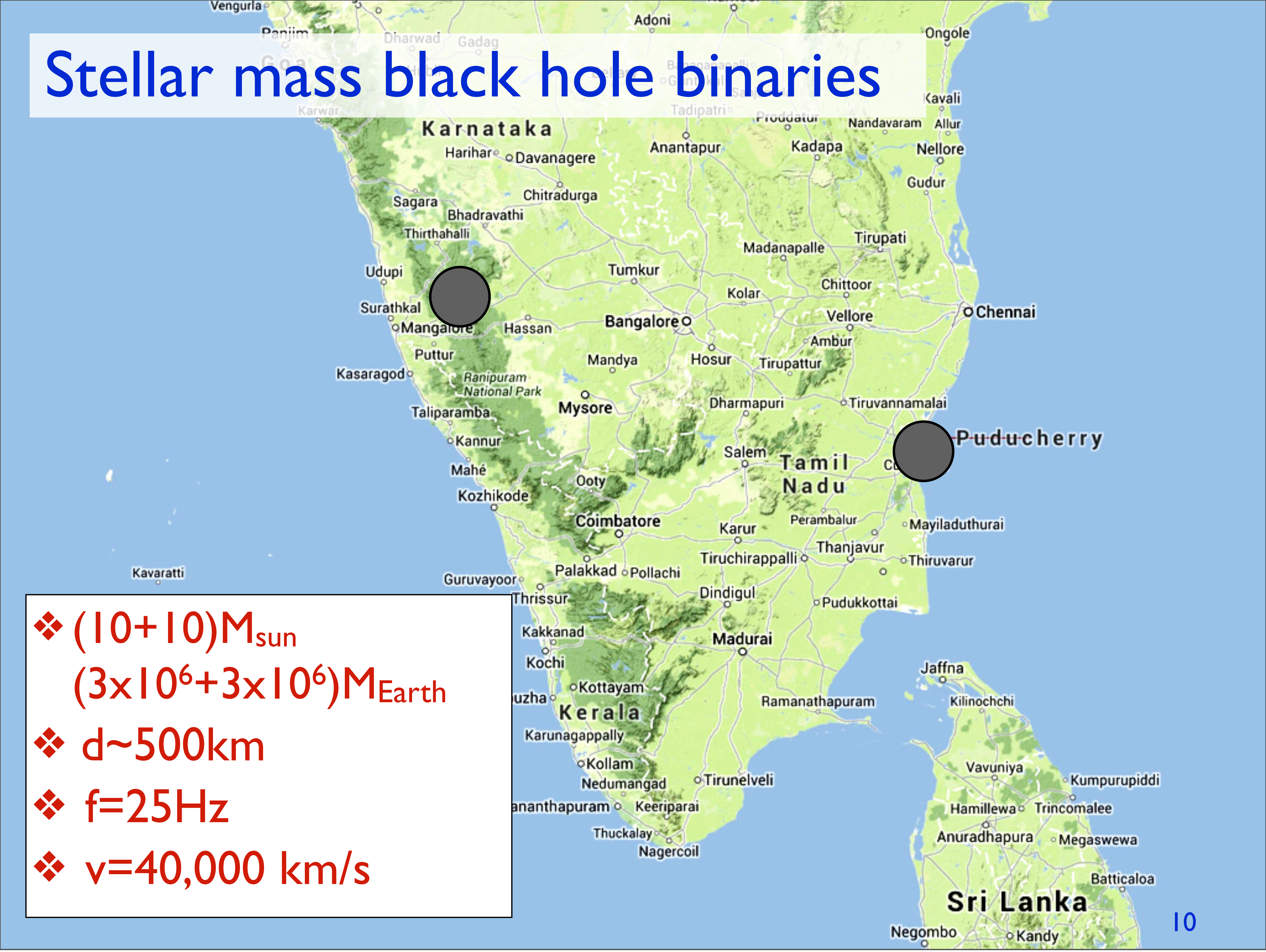
- ❖ Black Hole binaries

$$h_{ij} \sim 4\times 10^{-22}\left(\frac{r}{100\mathrm{Mpc}}\right)^{-1}\left(\frac{v}{0.3c}\right)^2\left(\frac{M}{10M_\odot}\right)$$

Ground-based GW detectors

© 2012 Microsoft Corporation Image courtesy of NASA,

Stellar mass black hole binaries



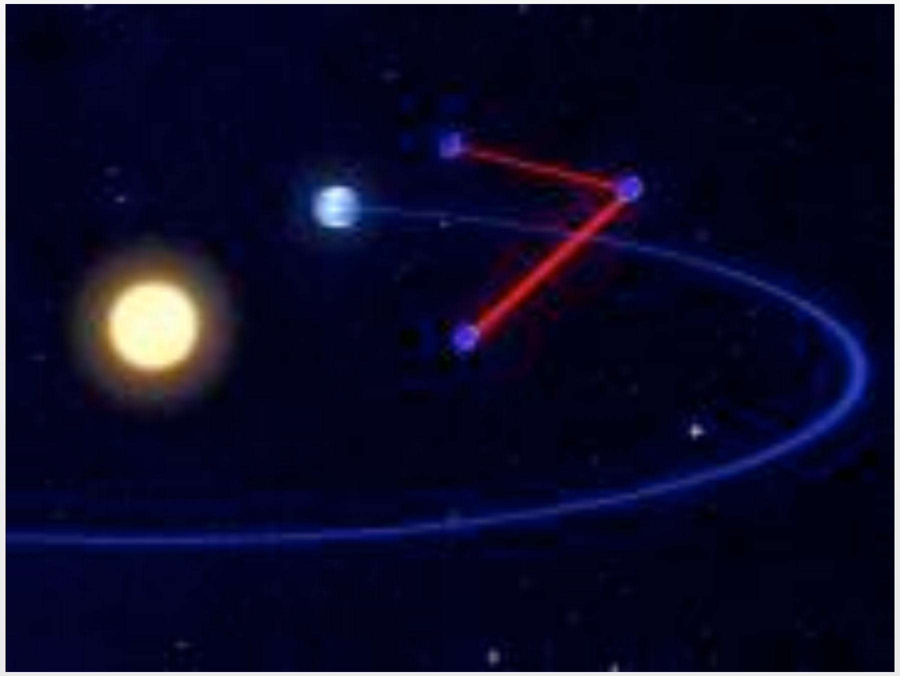
- ❖ $(10+10)M_{sun}$
  $(3\times10^6+3\times10^6)M_{Earth}$
- ❖ d~500km
- ❖ f=25Hz
- ❖ v=40,000 km/s

Supermassive black hole binaries

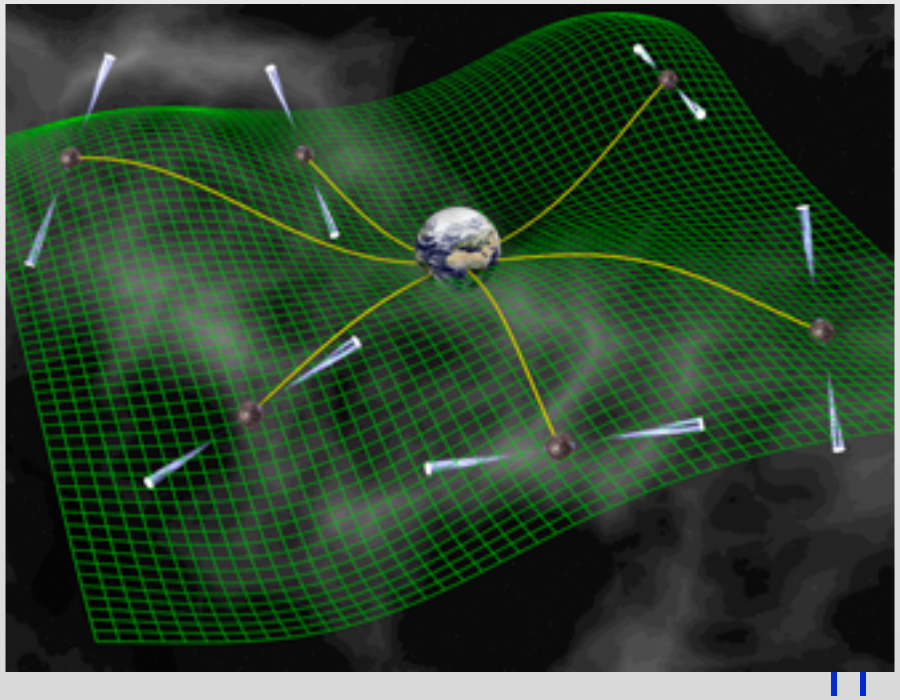
Colliding Galaxies Arp157

ESO

eLISA



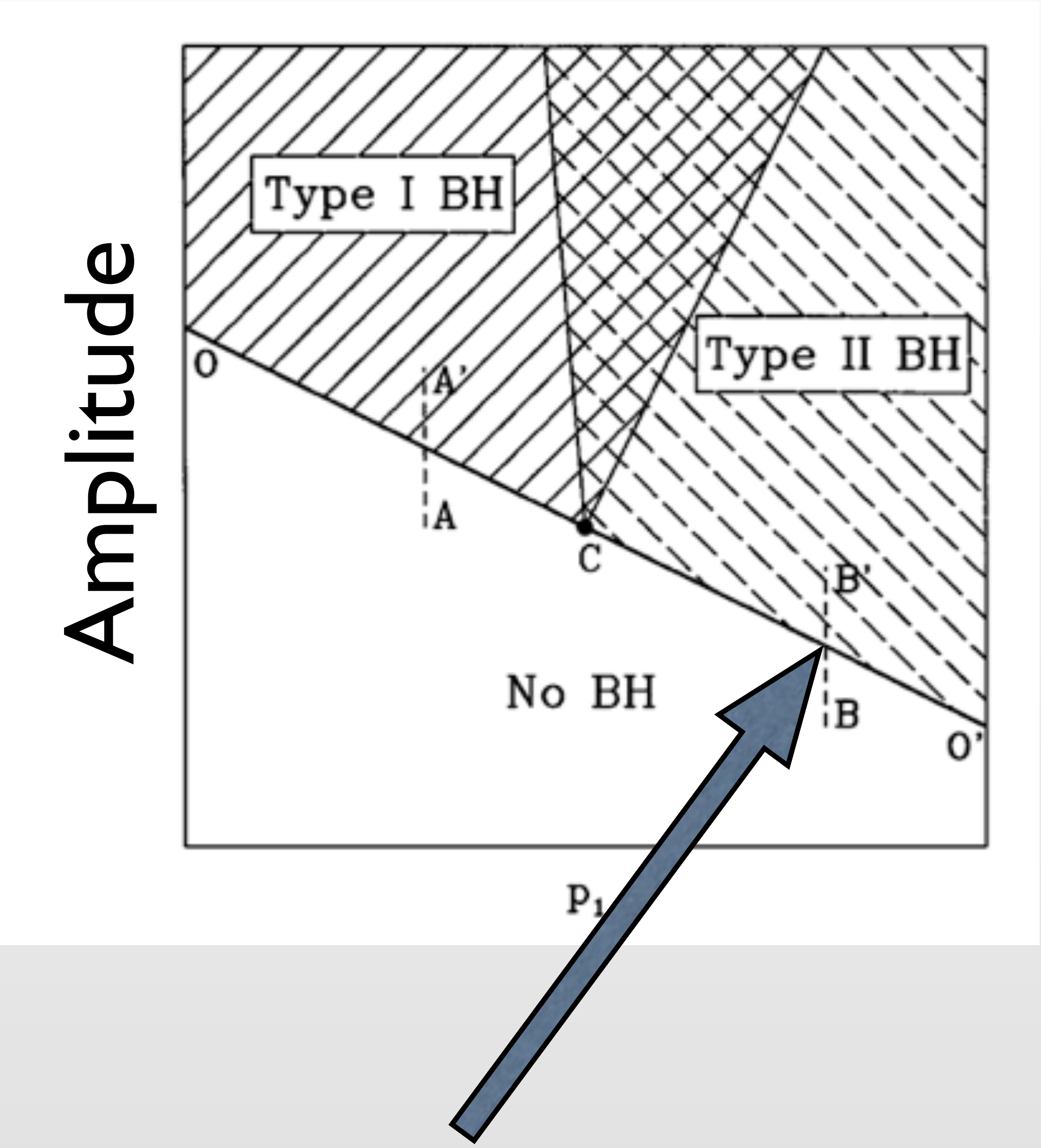
Pulsar timing arrays



Numerical Relativity

Motivation

- ❖ Astrophysics: What happens when
  - ... stars collapse?
  - ... compact objects collide?
- ❖ Elucidate Properties of GR
  - critical collapse
  - higher-dimensional gravity
- ❖ **Aid GW detectors**



Naked Singularity

Choptuik,Chmaj,Bizon 96

LIGO's many numerical relativity needs

❖ Signal detection

- Template bank for searched parameter-space

  state-of-the-art: aligned spin binaries

- signal characteristics to inform χ^2 vetoes

❖ Detection efficiency (event rates)

- Some waveforms elsewhere in parameter space

  e.g. precessing systems; eccentric systems

❖ Parameter estimation

- Especially accurate waveform models in all parameters being estimated

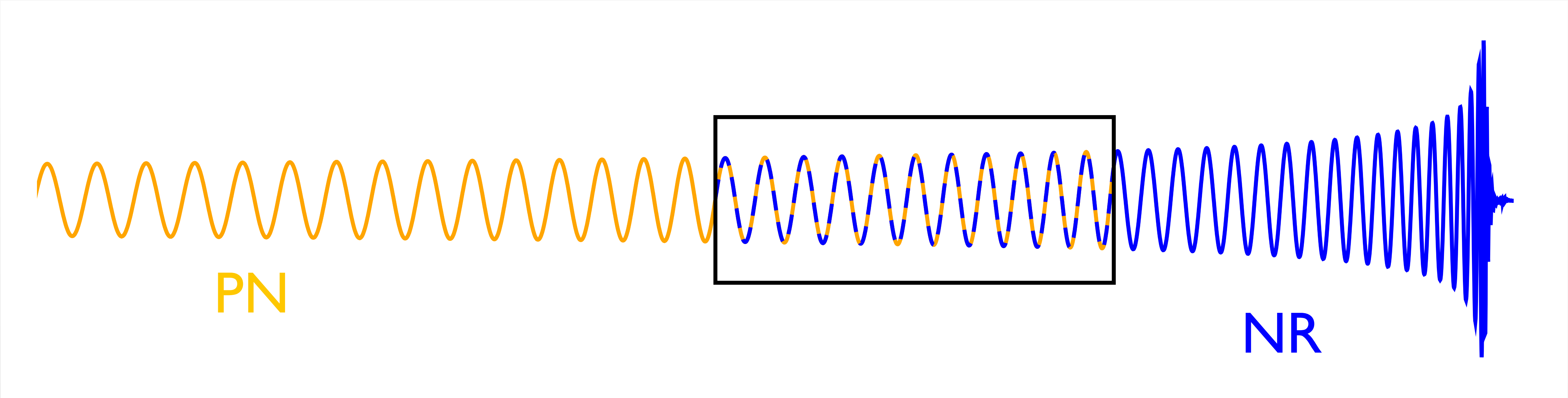
$$M_1, M_2, \vec{S}_1, \vec{S}_2, e, \ldots, \mathrm{RA}, \mathrm{dec}$$

❖ Properties of **EM & ν counterparts**

- What should telescopes look for?

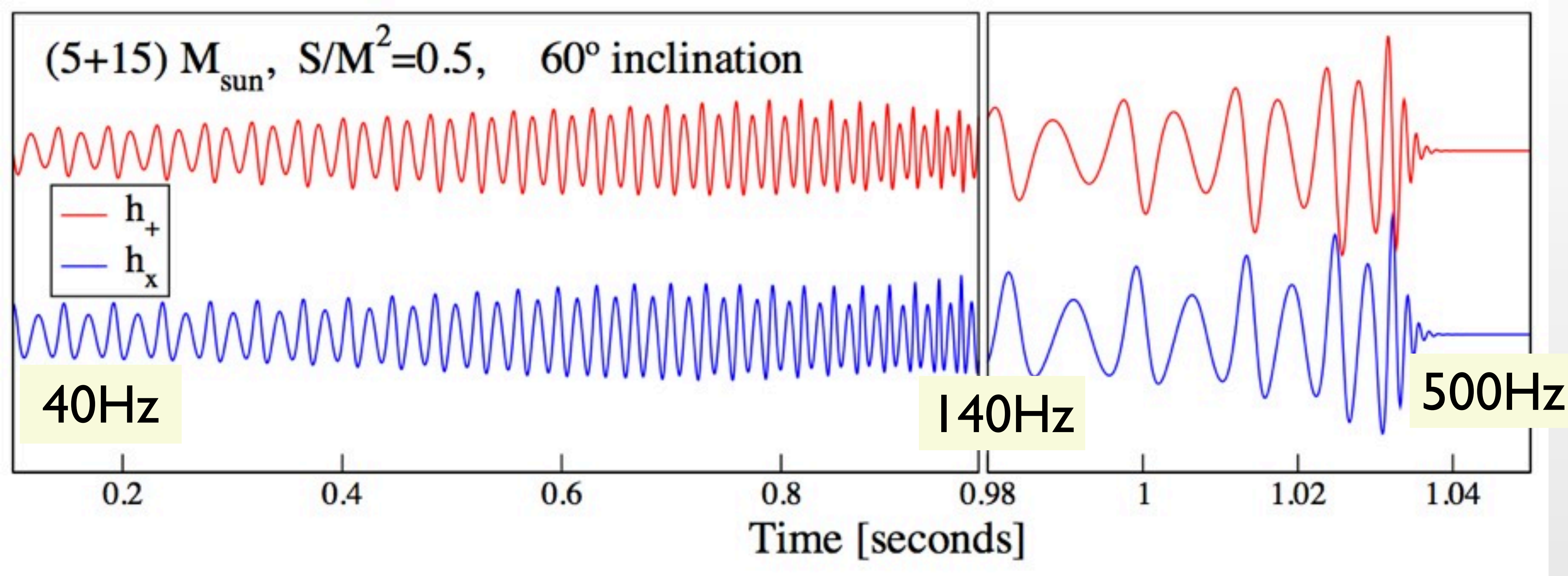
Primary input of NR:
Accurate waveforms

Tools for computing waveforms



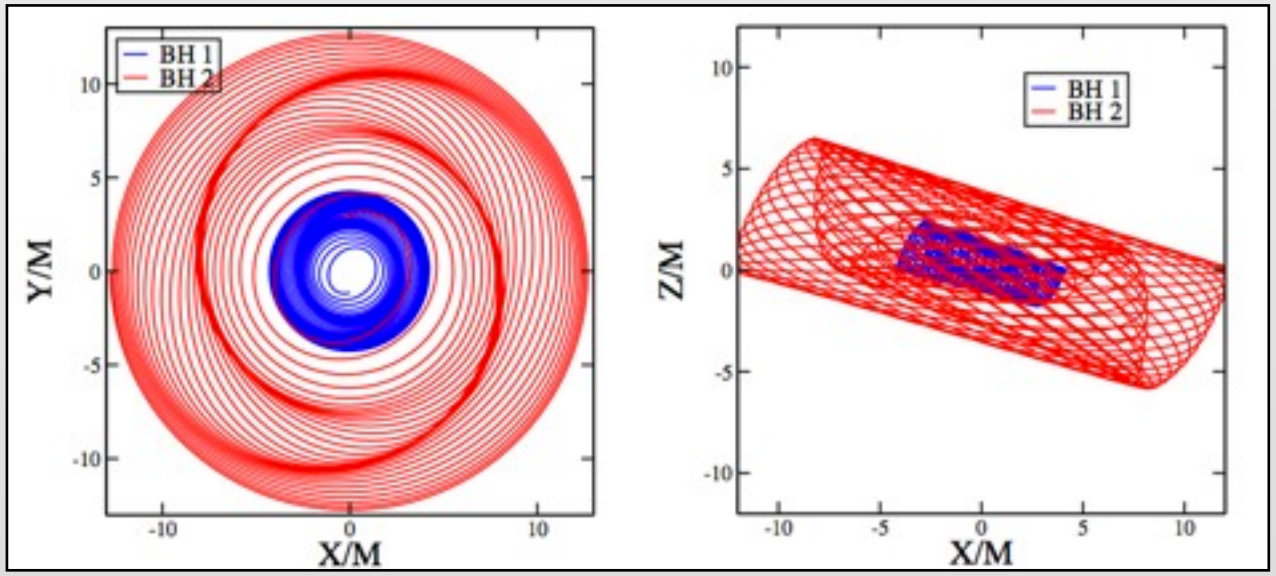
- ❖ Early inspiral
  - Post-Newtonian calculations
- ❖ Late inspiral & Merger
  - Computer simulations
- ❖ Ringdown
  - Perturbation theory
  - Computer simulations

Aanatomy of a waveform



Mroue, ... HP 1204.6077

- ❖ Modulated amplitude
- ❖ Higher temporal harmonics
- ❖ Dependence on inclination
- ❖ Modified phasing



CITA–ICAT

A brief history of black hole simulations

Solving Einstein Equations - Basic idea

- Goal: Space-time metric g_{ab} satisfying

$$R_{ab}[g_{ab}] = 0$$

- Split spacetime into space *and* time

- Evolution equations

$$\begin{aligned} \partial_t g_{ij} &= \dots \\ \partial_t K_{ij} &= \dots \end{aligned}$$

- Constraints

$$\begin{aligned} R[g_{ij}] + K^2 - K_{ij}K^{ij} &= 0 \\ \nabla_j \left(K^{ij} - g^{ij}K \right) &= 0 \end{aligned}$$

cf. Maxwell's equations

$$\begin{aligned} \partial_t \vec{E} &= \nabla \times \vec{B} \\ \partial_t \vec{B} &= -\nabla \times \vec{E} \\ \nabla \cdot \vec{E} &= 0 \\ \nabla \cdot \vec{B} &= 0 \end{aligned}$$

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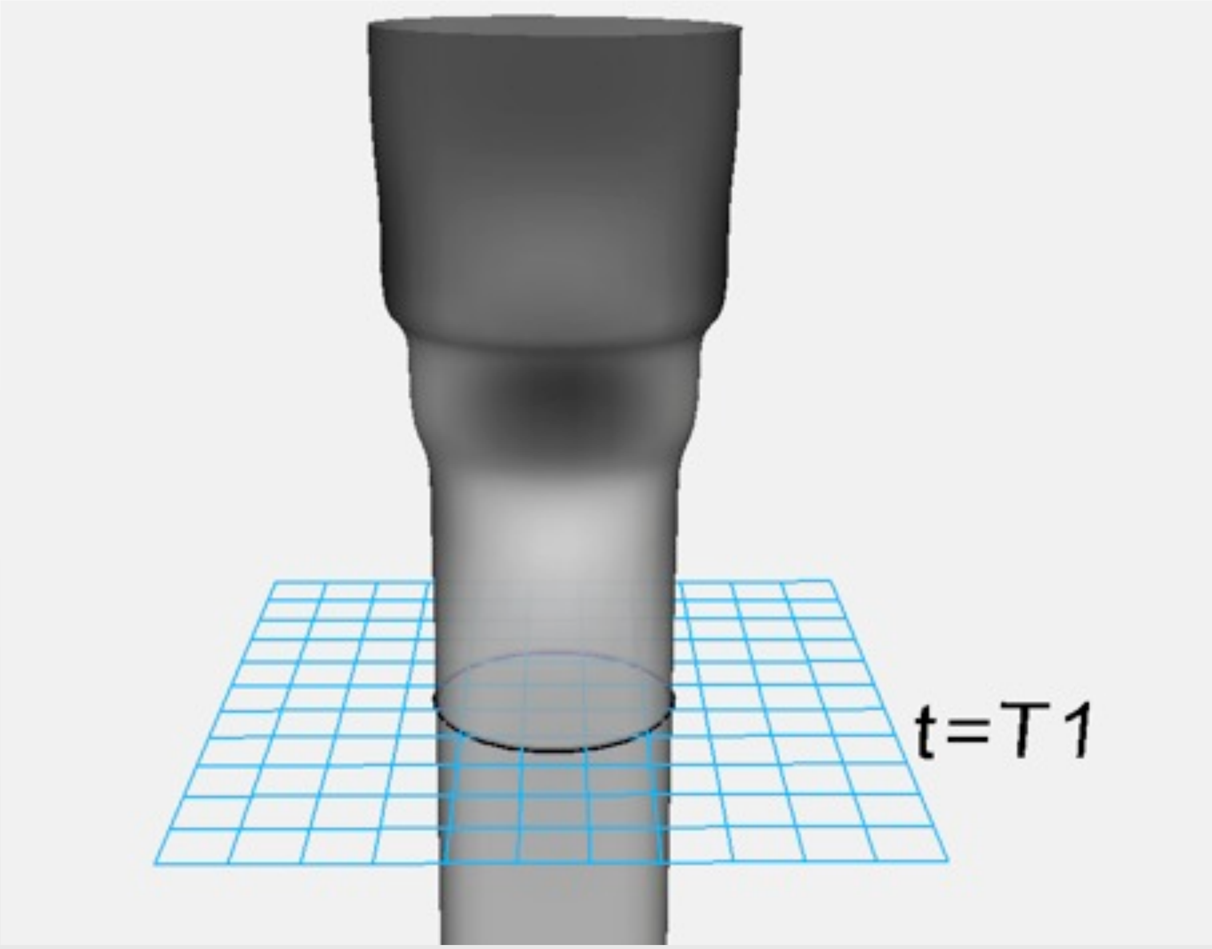

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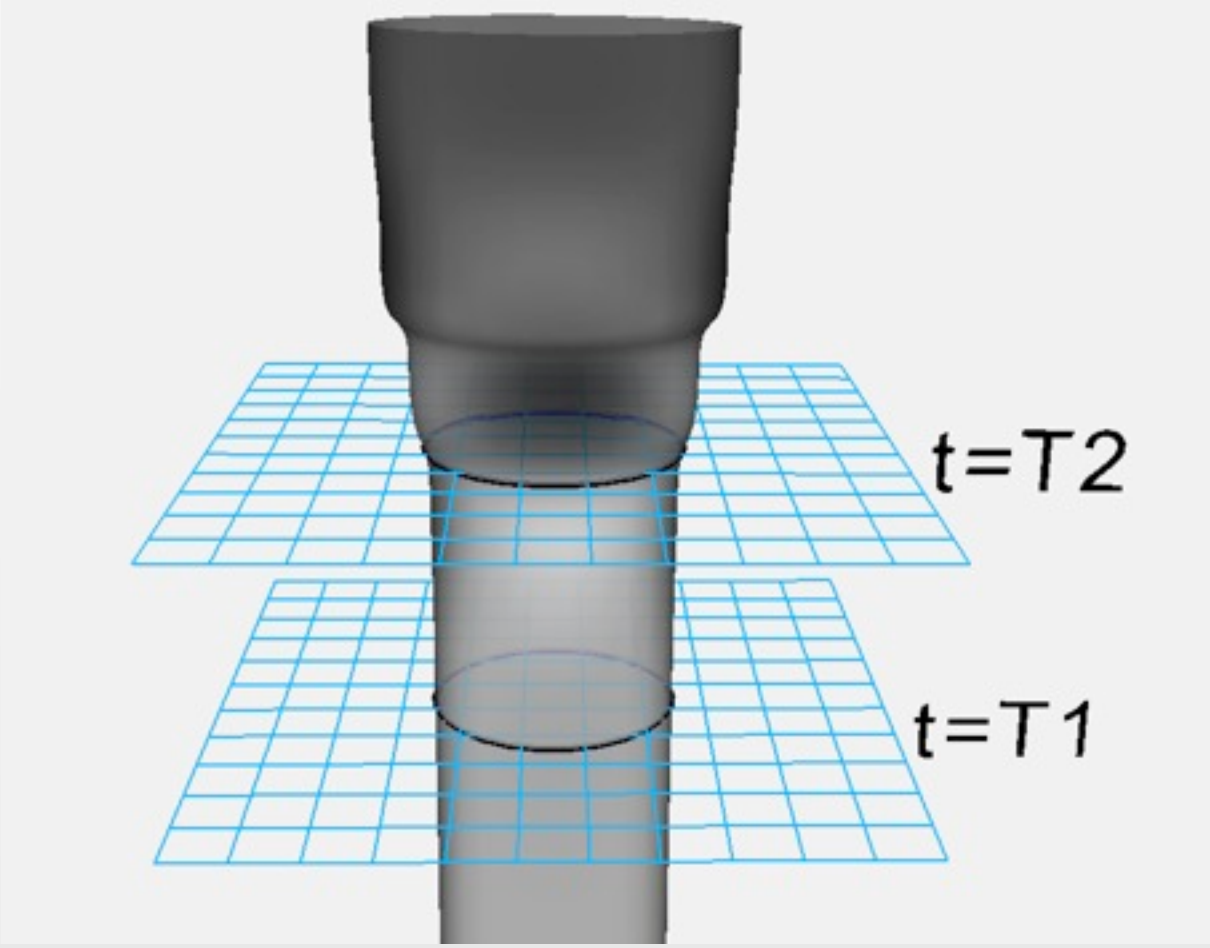

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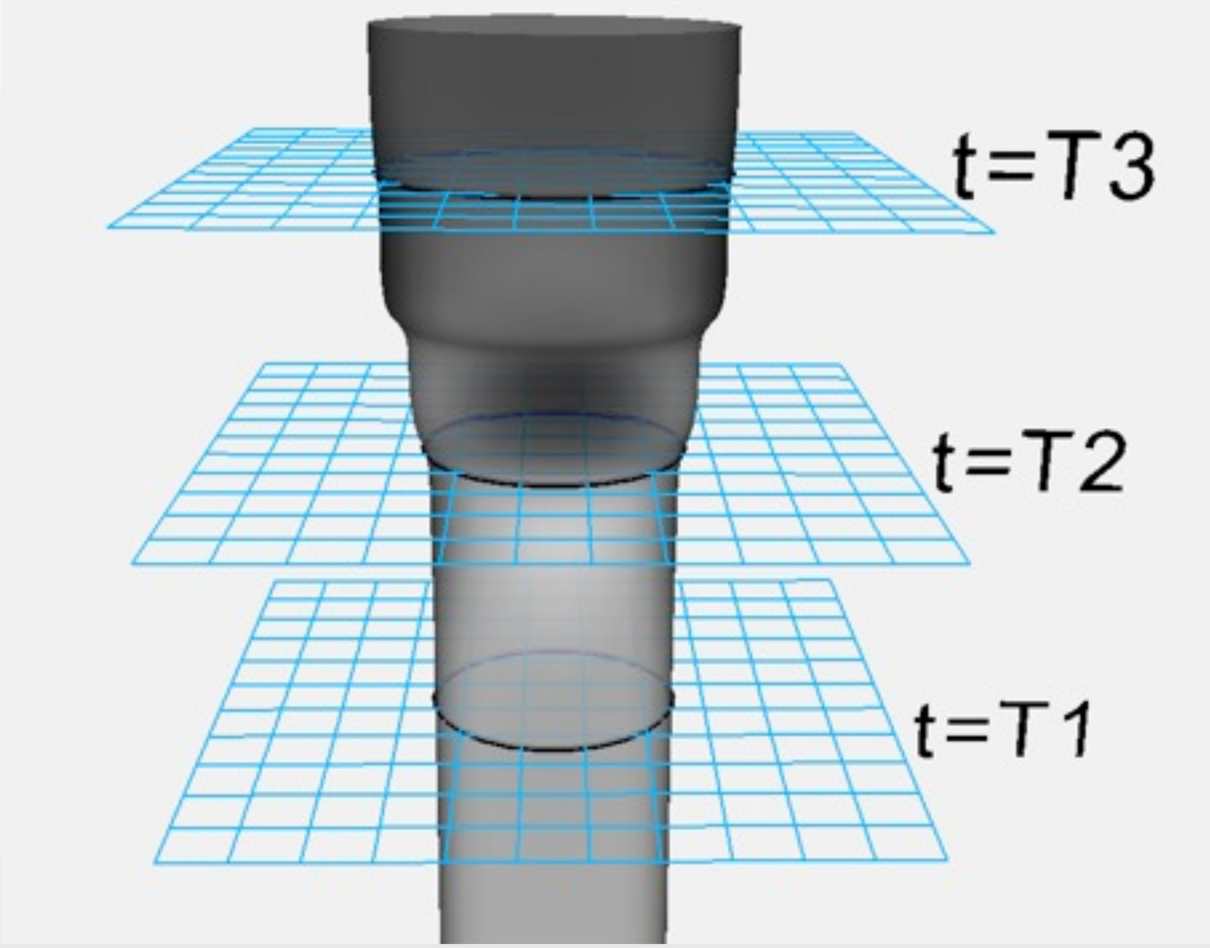

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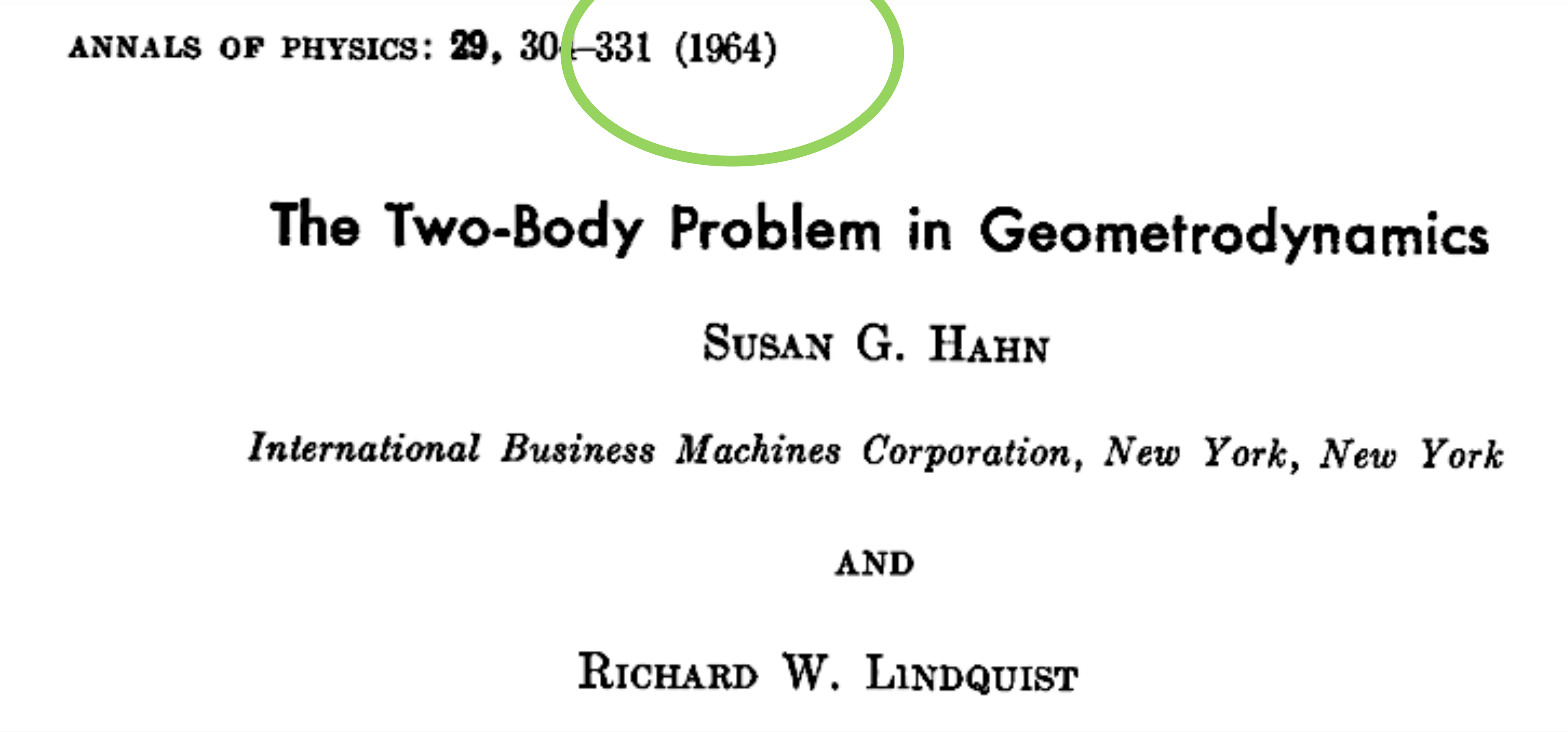
Why is this hard?

- Singularities inside Black holes

- Constraints difficult to preserve

- Coordinate freedom
  - How does one choose coordinates for a space-time one does not know yet?

- Challenging numerical issues
  - 20-50 variables
  - 10,000 FLOP / grid-point / time-step
  - Different length scales, high accuracy requirements



ANNALS OF PHYSICS: **29**, 304–331 (1964)

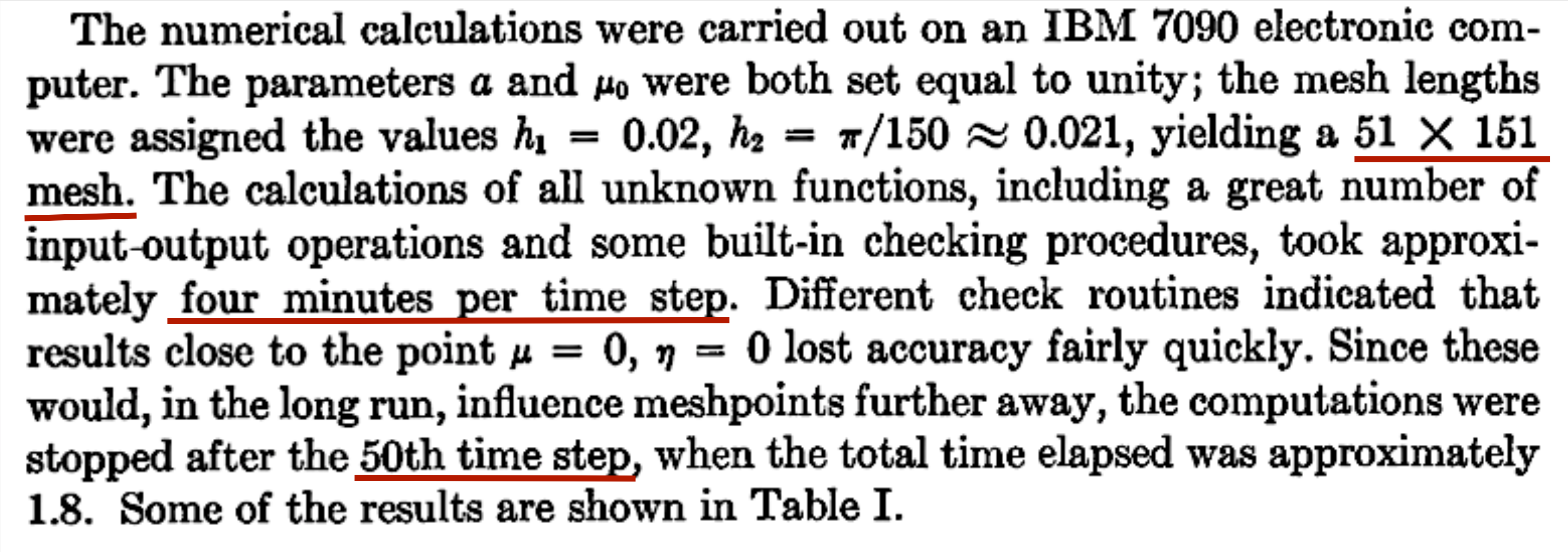
The Two-Body Problem in Geometrodynamics

Susan G. Hahn

International Business Machines Corporation, New York, New York

AND

Richard W. Lindquist



The numerical calculations were carried out on an IBM 7090 electronic computer. The parameters a and μ_0 were both set equal to unity; the mesh lengths were assigned the values $h_1 = 0.02$, $h_2 = \pi/150 \approx 0.021$, yielding a 51×151 mesh. The calculations of all unknown functions, including a great number of input-output operations and some built-in checking procedures, took approximately four minutes per time step. Different check routines indicated that results close to the point $\mu = 0$, $\eta = 0$ lost accuracy fairly quickly. Since these would, in the long run, influence meshpoints further away, the computations were stopped after the 50th time step, when the total time elapsed was approximately 1.8. Some of the results are shown in Table I.

50 Years of BBH: The beginnings

1962 *ADM*
3+1 formulation

1964
Hahn-Lindquist
2 wormholes

1984
Unruh
excision

1992,3
Choptuik;
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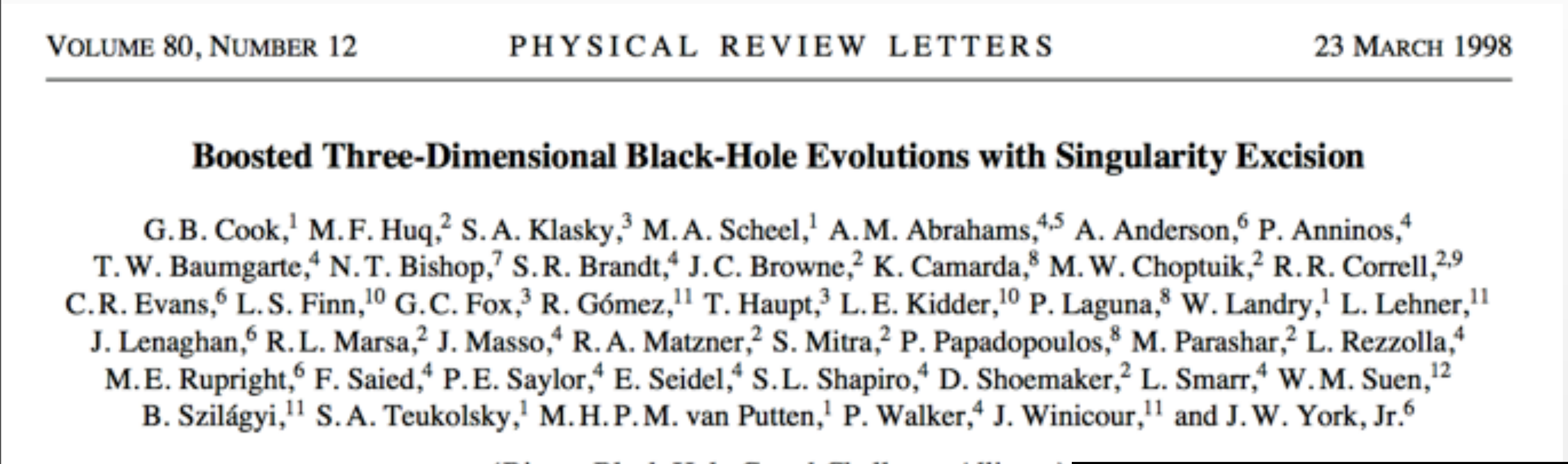
Courtesy Carlos Lousto,
updated by HP

LIGO

advanced LIGO

BBH Grand Challenge 1994-98



VOLUME 80, NUMBER 12 PHYSICAL REVIEW LETTERS 23 MARCH 1998

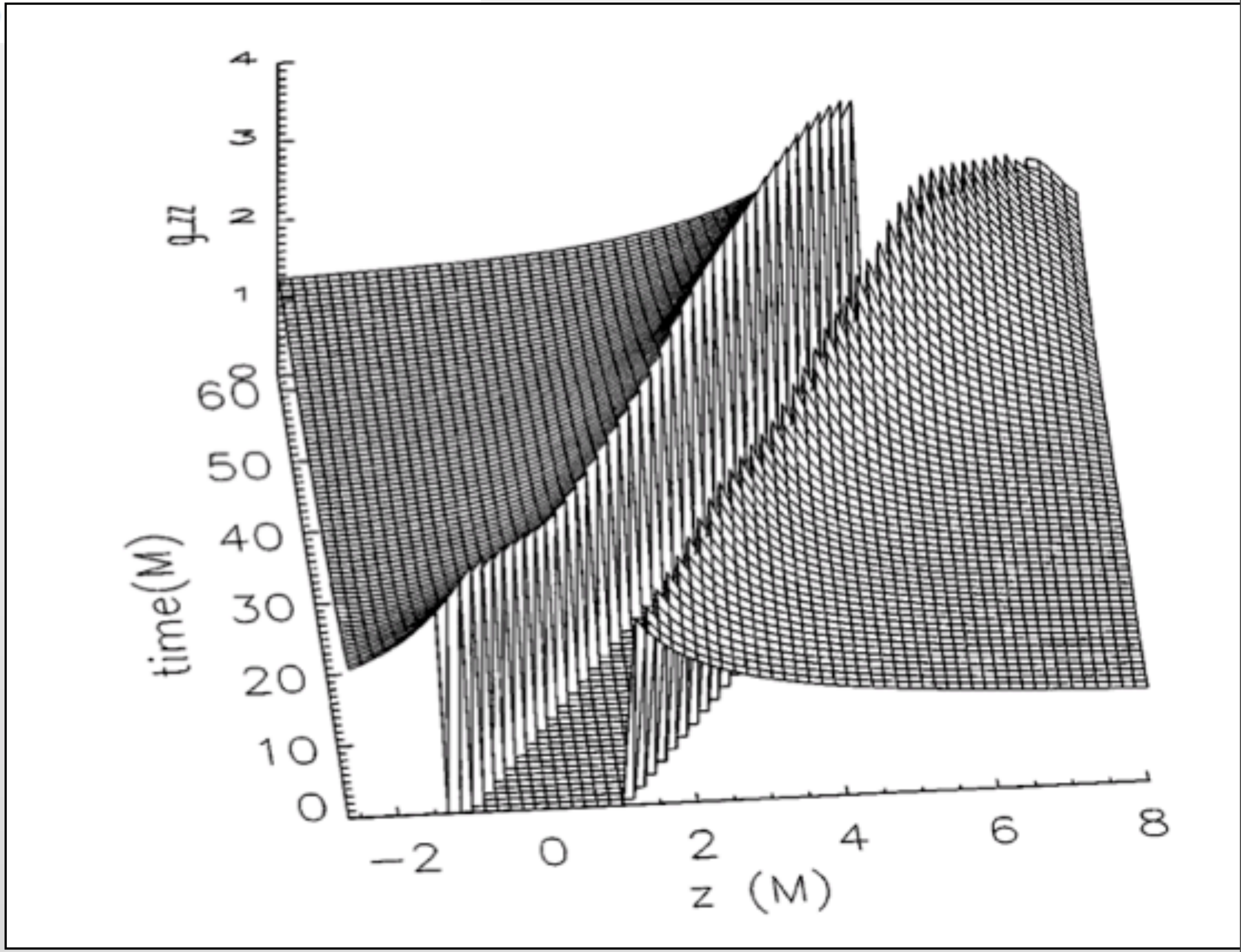
Boosted Three-Dimensional Black-Hole Evolutions with Singularity Excision

G. B. Cook,[1] M. F. Huq,[2] S. A. Klasky,[3] M. A. Scheel,[1] A. M. Abrahams,[4,5] A. Anderson,[6] P. Anninos,[4] T. W. Baumgarte,[4] N. T. Bishop,[7] S. R. Brandt,[4] J. C. Browne,[2] K. Camarda,[8] M. W. Choptuik,[2] R. R. Correll,[2,9] C. R. Evans,[6] L. S. Finn,[10] G. C. Fox,[3] R. Gómez,[11] T. Haupt,[3] L. E. Kidder,[10] P. Laguna,[8] W. Landry,[1] L. Lehner,[11] J. Lenaghan,[6] R. L. Marsa,[2] J. Masso,[4] R. A. Matzner,[2] S. Mitra,[2] P. Papadopoulos,[8] M. Parashar,[2] L. Rezzolla,[4] M. E. Rupright,[6] F. Saied,[4] P. E. Saylor,[4] E. Seidel,[4] S. L. Shapiro,[4] D. Shoemaker,[2] L. Smarr,[4] W. M. Suen,[12] B. Szilágyi,[11] S. A. Teukolsky,[1] M. H. P. M. van Putten,[1] P. Walker,[4] J. Winicour,[11] and J. W. York, Jr.[6]

(Binary Black Hole Grand Challenge Alliance)

❖ Single Black Hole

- 60M in time
- 6M motion in space



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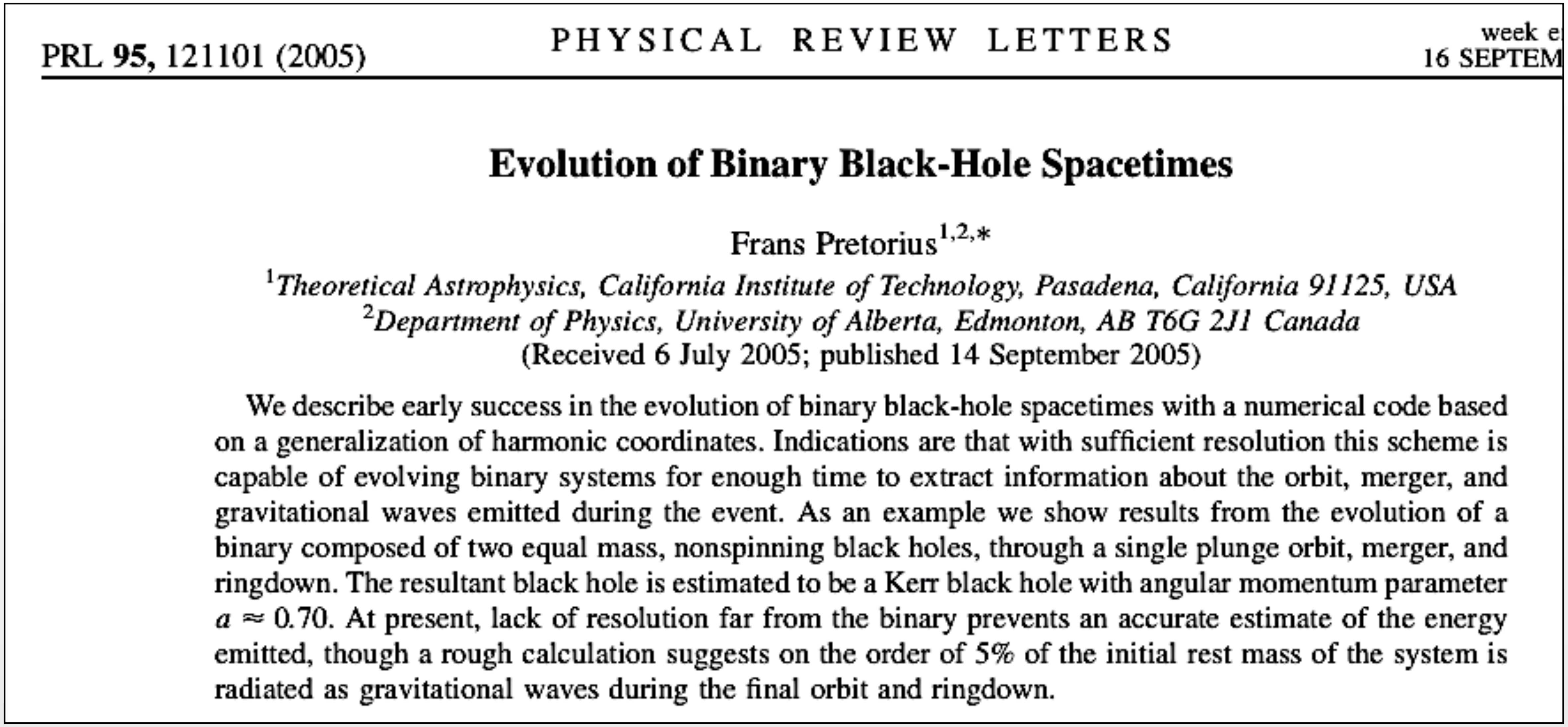


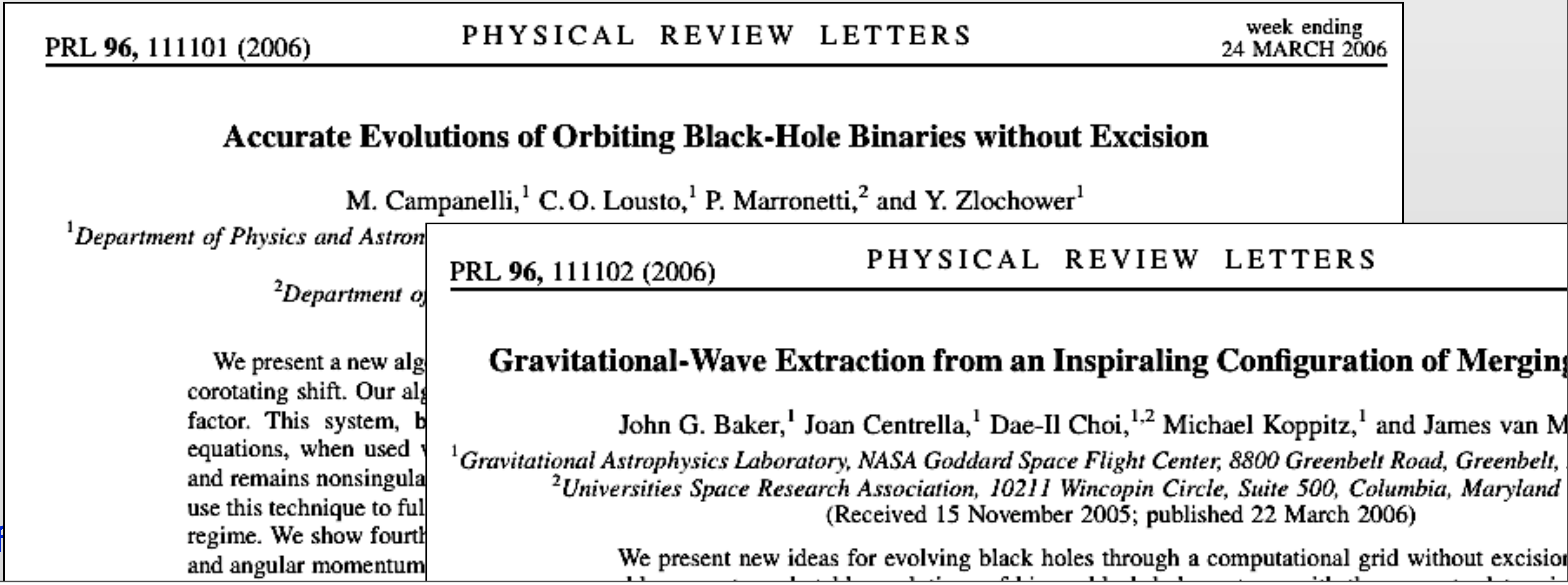
PRL **95,** 121101 (2005) PHYSICAL REVIEW LETTERS week e
16 SEPTEM

Evolution of Binary Black-Hole Spacetimes

Frans Pretorius[1,2,*]

[1]*Theoretical Astrophysics, California Institute of Technology, Pasadena, California 91125, USA*
[2]*Department of Physics, University of Alberta, Edmonton, AB T6G 2J1 Canada*
(Received 6 July 2005; published 14 September 2005)

We describe early success in the evolution of binary black-hole spacetimes with a numerical code based on a generalization of harmonic coordinates. Indications are that with sufficient resolution this scheme is capable of evolving binary systems for enough time to extract information about the orbit, merger, and gravitational waves emitted during the event. As an example we show results from the evolution of a binary composed of two equal mass, nonspinning black holes, through a single plunge orbit, merger, and ringdown. The resultant black hole is estimated to be a Kerr black hole with angular momentum parameter $a \approx 0.70$. At present, lack of resolution far from the binary prevents an accurate estimate of the energy emitted, though a rough calculation suggests on the order of 5% of the initial rest mass of the system is radiated as gravitational waves during the final orbit and ringdown.



PRL **96,** 111101 (2006) PHYSICAL REVIEW LETTERS week ending
24 MARCH 2006

Accurate Evolutions of Orbiting Black-Hole Binaries without Excision

M. Campanelli,[1] C. O. Lousto,[1] P. Marronetti,[2] and Y. Zlochower[1]

[1]*Department of Physics and Astron*

[2]*Department o*

We present a new alg
corotating shift. Our alg
factor. This system, b
equations, when used
and remains nonsingula
use this technique to ful
regime. We show fourt
and angular momentum

PRL **96,** 111102 (2006) PHYSICAL REVIEW LETTERS

Gravitational-Wave Extraction from an Inspiraling Configuration of Mergin

John G. Baker,[1] Joan Centrella,[1] Dae-Il Choi,[1,2] Michael Koppitz,[1] and James van M
[1]*Gravitational Astrophysics Laboratory, NASA Goddard Space Flight Center, 8800 Greenbelt Road, Greenbelt,*
[2]*Universities Space Research Association, 10211 Wincopin Circle, Suite 500, Columbia, Maryland*
(Received 15 November 2005; published 22 March 2006)

We present new ideas for evolving black holes through a computational grid without excisio

Harald Pfeif

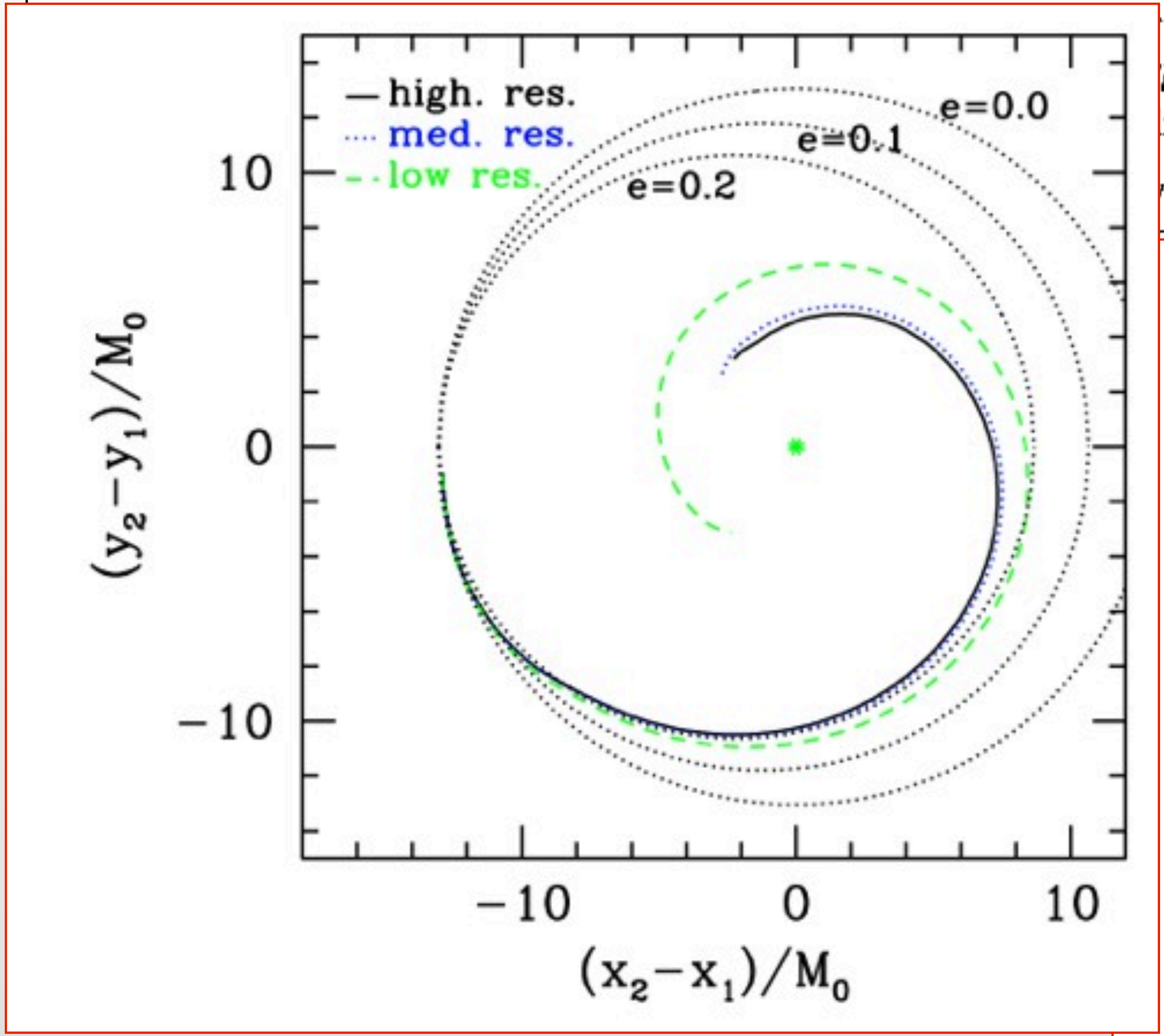
CITA–ICAT

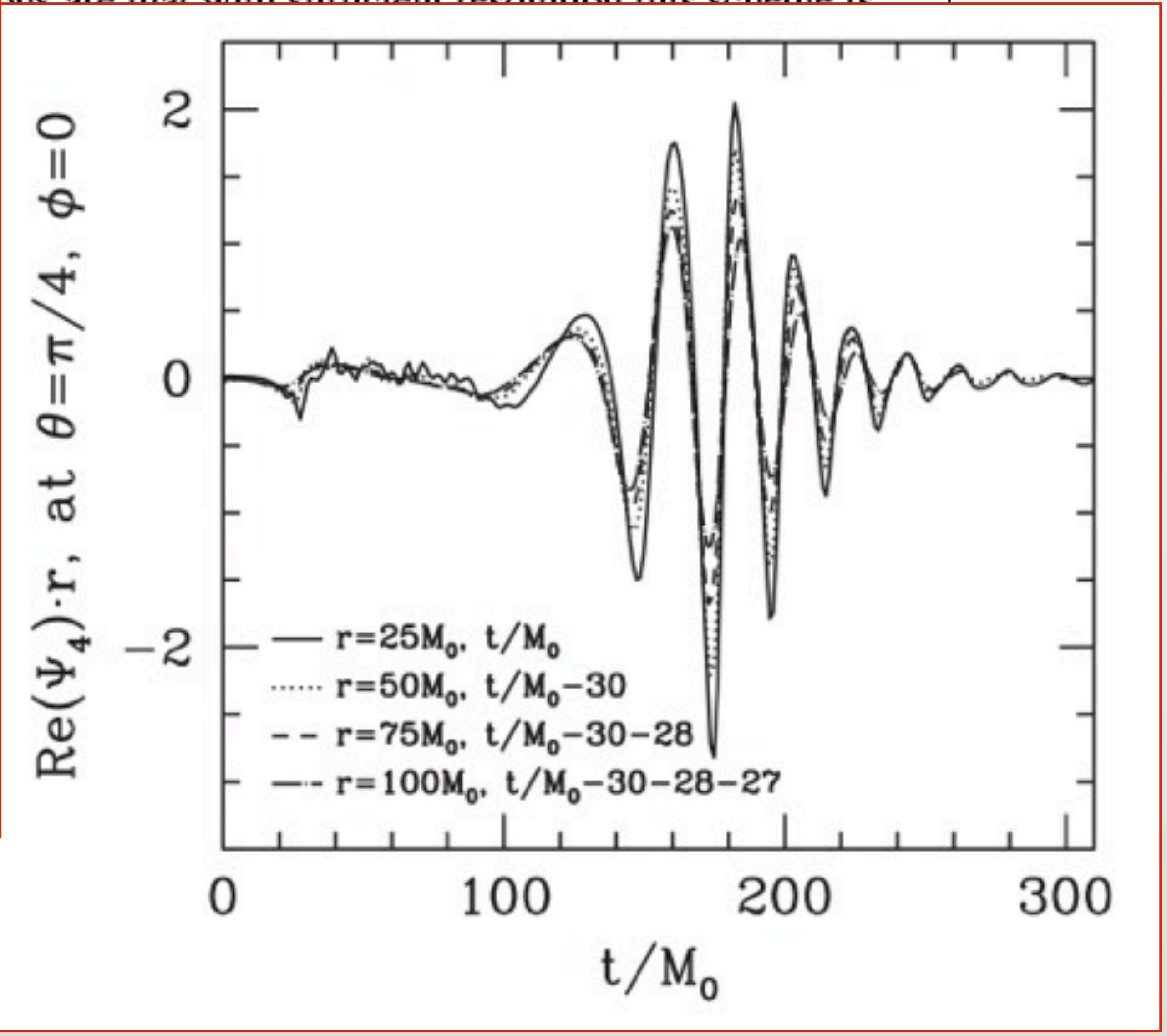
PRL 95, 121101 (2005) PHYSICAL REVIEW LETTERS week e
16 SEPTEM

Evolution of Binary Black-Hole Spacetimes

Frans Pretorius[1,2,*]
Technology, Pasadena, California 91125, USA
berta, Edmonton, AB T6G 2J1 Canada
ished 14 September 2005)

black-hole spacetimes with a numerical code based





50 Years of BBH: Modern age

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1962 *ADM*
3+1 formulation

1964
Hahn-Lindquist
2 wormholes

1984
Unruh
excision

1992,3
Choptuik;
Abrahams+Evans
critical phenomena

1997
Brandt-
Brügmann
puncture data

1994-98
BBH Grand Challenge

1999-00
AEI/PSU
grazing collisions

2000-04
AEI/UTB-NASA
revive crashing
codes (Lazarus)

~2000 *Choptuik;*
Schnetter;Brügmann
mesh refinement

2005
Gundlach ea
constraint damping

2005 *Pretorius*
inspiral-merger-
ringdown (IMR)
w/ harmonic

2005-06
UTB/NASA
IMR with BSSN +
moving punctures

2006-08
Cornell/Caltech
IMR w/ spectral

2007-
Ajith, AEI, Jena
phenom GW models

2009-
UMD, SXS
EOB GW models

2011
Lousto ea
q=100

2011
Schmidt ea;
Boyle ea
Radiation aligned
frame

1964 ~1998 ~2005 Time

1975-77
Smarr-Eppley
head-on
collision

1979 *York*
kinematics and
dynamics of GR

1989-95
Bona-Masso
modified ADM,
(hyperbolicity)

1994 *Cook*
Bowen-York
initial data

1994-95
NCSA-WashU
improved
head-on collision

1999
BSSN
evolution
system

1999 *York*
conformal thin
sandwich ID

1999-2005 *JW York,*
Cornell, Caltech, LSU
hyperbolic formulations

2000-02
Alcubierre
gauge conditions

2004
Brügmann ea
one orbit

2003-08
Cook, Pfeiffer ea
improved ID

2000 *Ashtekar*
isolated horizons

2006,07
Baker et al
Gonzalez et al
non-spinning BBH
kicks

2007-11
RIT, Jena, AEI,...
BBH superkicks

2008
all of NR
NINJA

2007 *SXS*
PN-NR
comparison

2009-11
Bishop, ...
Cauchy
characteristic
extraction

2011
Lovelace ea
S/M^2=0.97

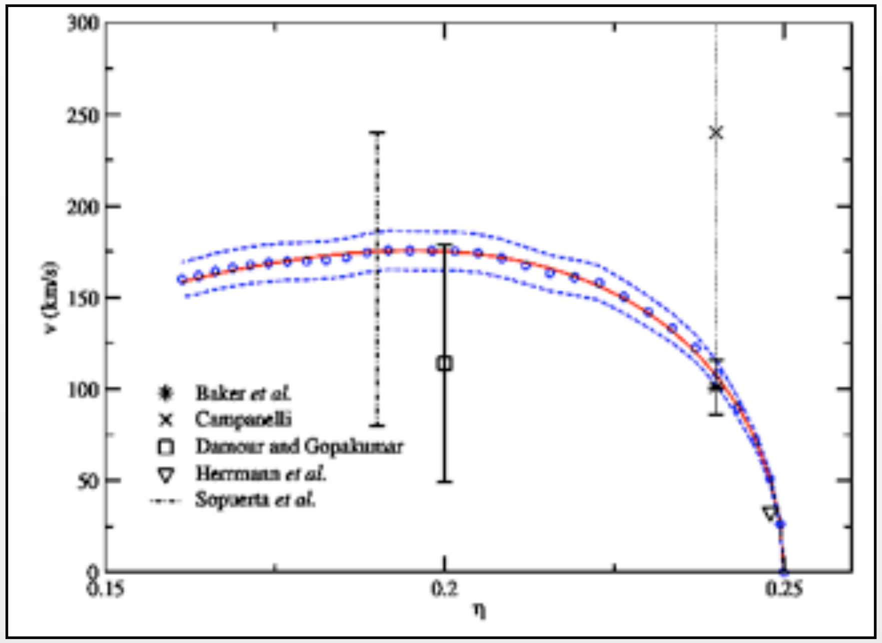
2013
GaTech, SXS
Precessing
parameter
studies

Courtesy Carlos Lousto,
updated by HP

LIGO

advanced LIGO

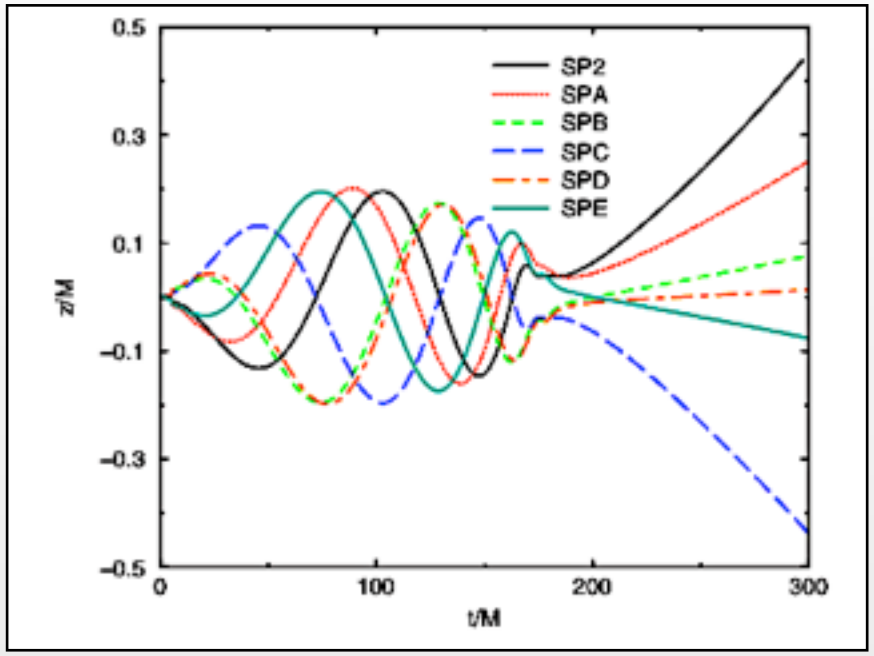
Early days of BH-BH sims



Spin=0 BH-BH kicks

Gonzalez, Sperhake, Brügmann, Hannam, Husa 07

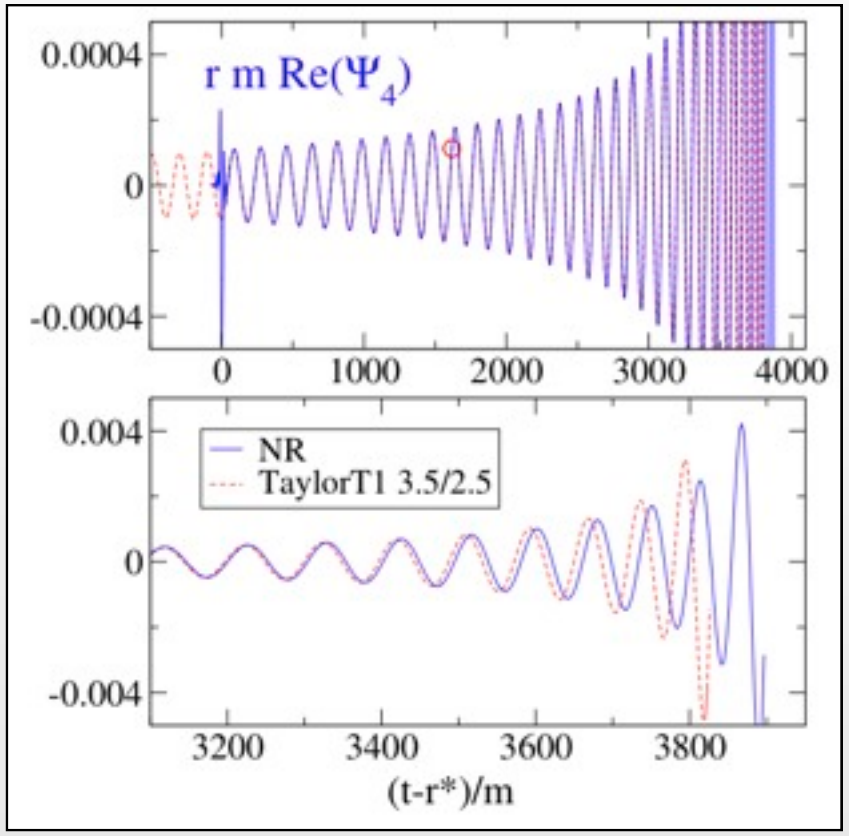
v_{max}=130km/sec



BH-BH superkicks

Campanelli ea 07

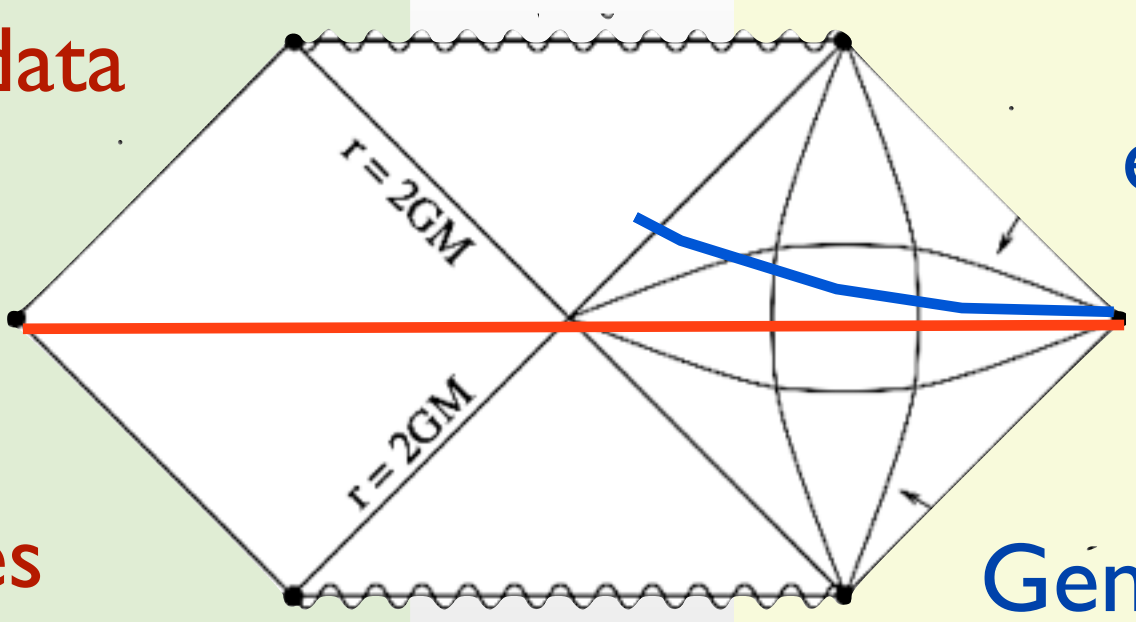
v_{max}~3500km/sec



(in-)validating PN

Boyle...HP ea 07

The two approaches to BH-BH



Puncture initial-data

(Brandt&Brügmann 97)

BSSN w/
moving punctures

(Campanelli ea 06, Baker ea 06)

$$\begin{aligned} g_{ij} &= e^{4\phi}\tilde{g}_{ij}, \\ \tilde{\Gamma}^i &= \tilde{g}^{jk}\tilde{\Gamma}^i_{jk} \\ \partial_t\phi &= \dots \\ \partial_t\tilde{g}_{ij} &\approx -\tilde{A}_{ij} \\ \partial_t\tilde{A}_{ij} &\approx -\Delta\tilde{g}_{ij} \\ \partial_t\tilde{\Gamma}^i &= \partial_t\left(\tilde{g}^{jk}\tilde{\Gamma}^i_{jk}\right) \end{aligned}$$

Finite differences w/ AMR

(RIT, AEI, GATech, Goddard, Jena, Palma, Cardiff, Perimeter)

Quasi-equilibrium
excision initial-data

(Cook 02, Cook&HP 04)

Generalized Harmonic
w/ constraint damping

(Gundlach ea 05, Pretorius 05)

$$\Box g_{ab} = -2\nabla_{(a}H_{b)} + \gamma_0\left[t_{(a}C_{b)} - \frac{1}{2}g_{ab}t^cC_c\right] + \text{lower}$$

Multi-domain spectral
methods SpEC

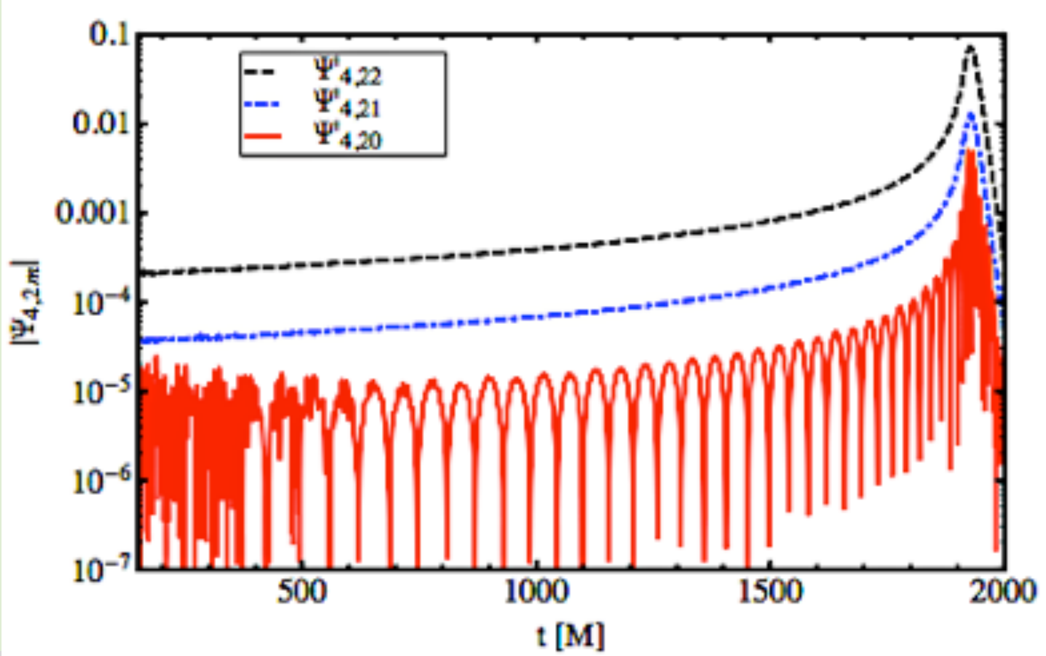
SXS collaboration (Cornell-Caltech-CITA-Washington State Univ-California State Univ Fullerton)

The two approaches to BH-BH

Finite differences w/ AMR
(RIT, AEI, Georgia Tech, Jena, Palma, Cardiff, Perimeter)

Conventional wisdom:
-- Robust, "easy"
-- Many short simulations
-- Lower accuracy, higher cost

More recent:
-- longer and more accurate sims

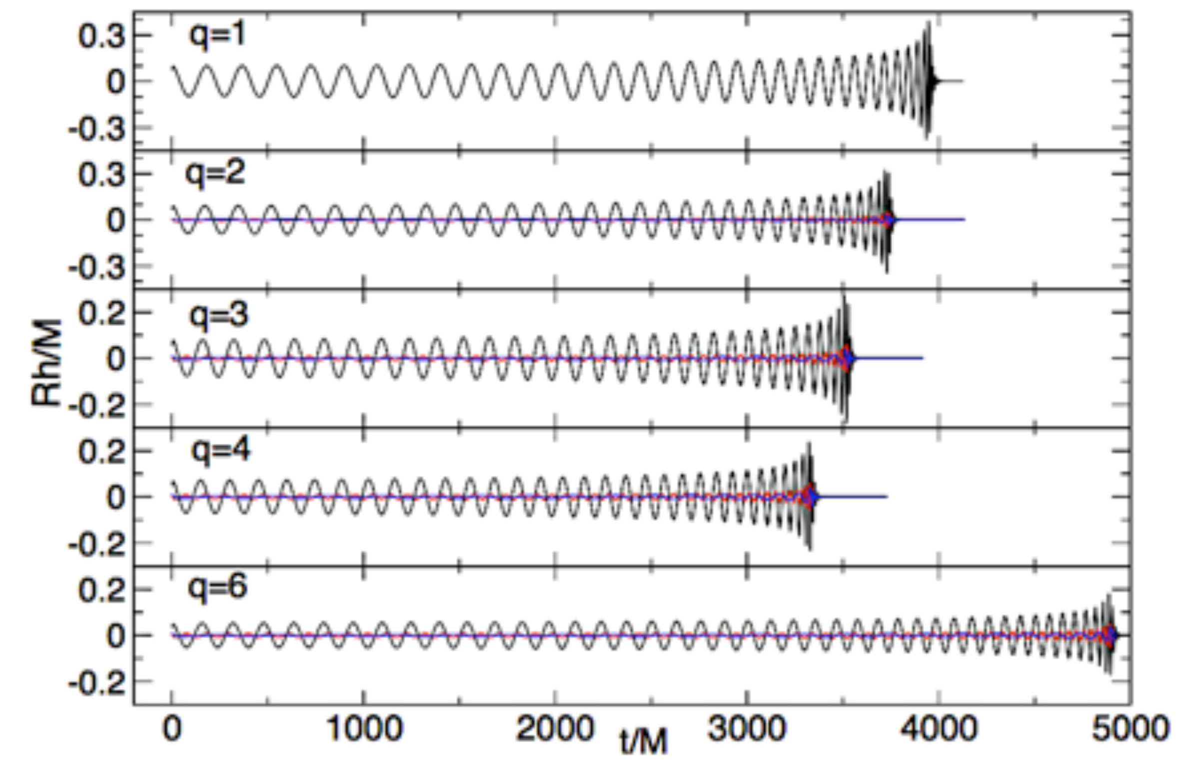


Schmidt ea 1012.2879

Multi-domain spectral methods
SpEC (Cornell-Caltech-CITA-WSU-CSUF)

Conventional wisdom:
-- Less robust, "difficult"
-- Few long simulations
-- Higher accuracy, lower cost

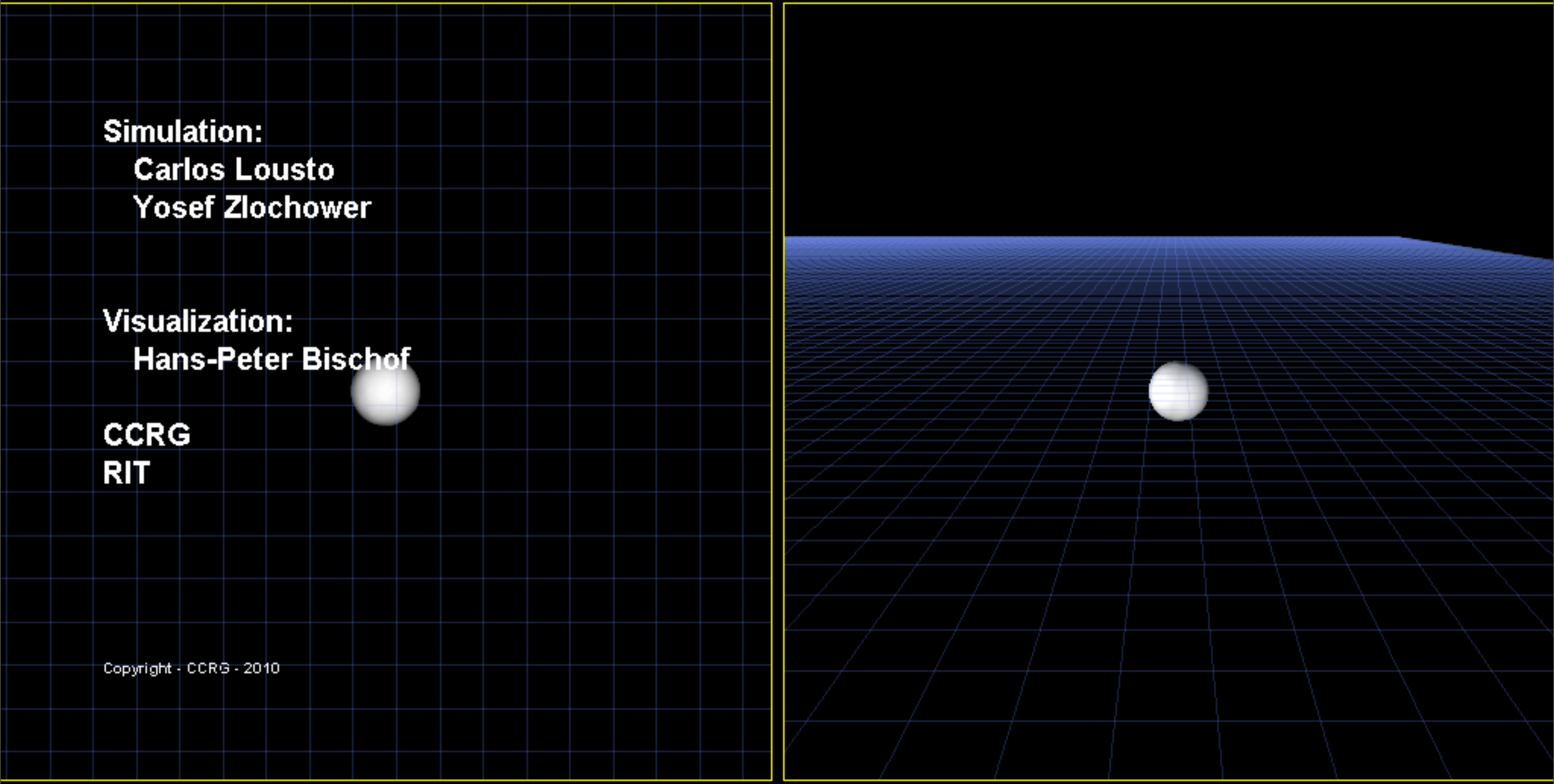
More recent:
-- mergers becoming routine



Buchman ea 1206.3015

Some recent BH-BH technical advances

Mass-ratio 1:100



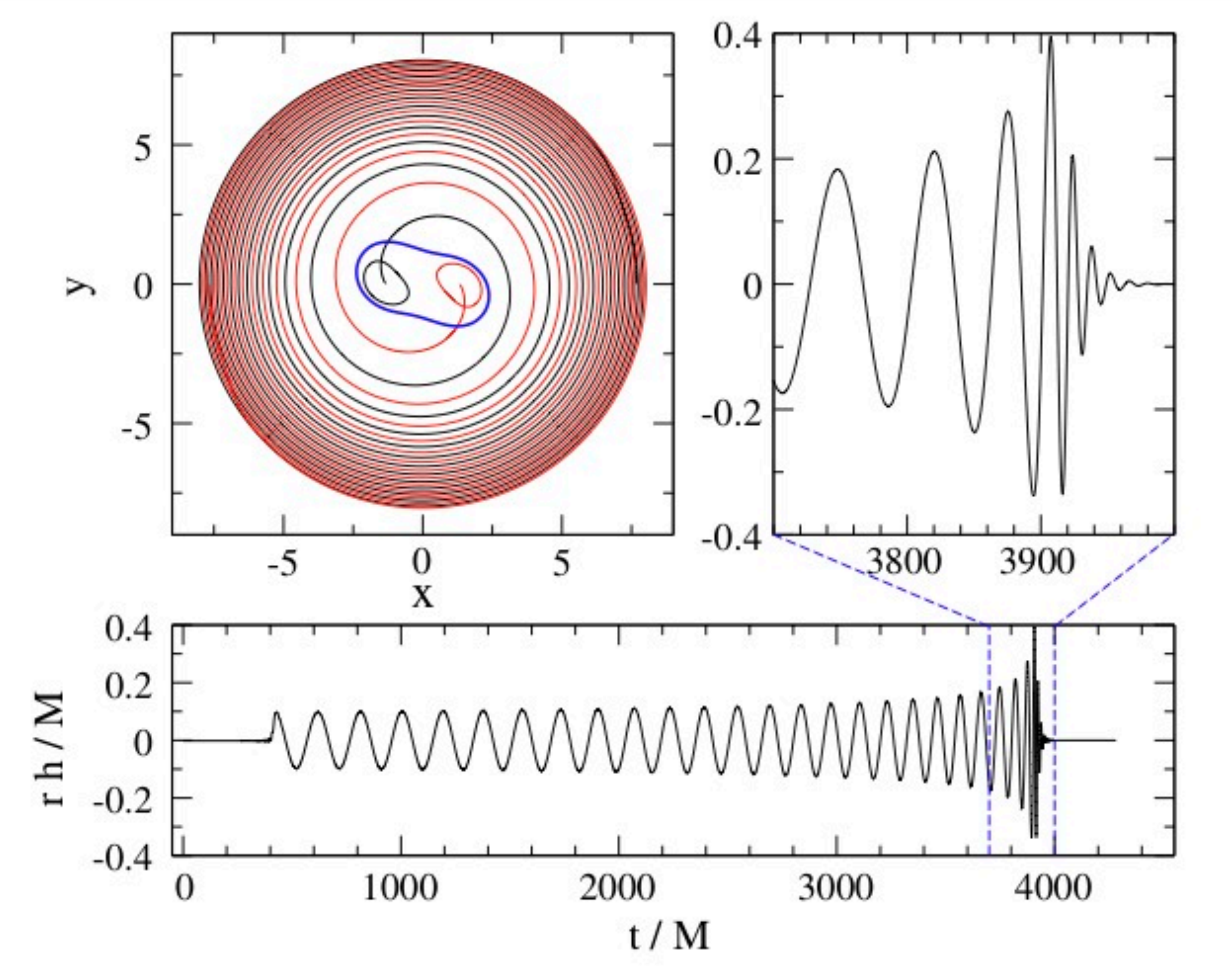
Simulation:
Carlos Lousto
Yosef Zlochower

Visualization:
Hans-Peter Bischof

CCRG
RIT

Two orbits, starting @ ISCO
Lousto, Zlochower 11

Spins above the Bowen-York limit



Lovelace, Scheel, Szilagyi 11,
Lovelace ea 11

- Puncture-data limit: **$S/M^2<0.93$**
- First complete BBH simulation above 0.93 limit!
  - Equal mass, equal spins anti-parallel to orbital L

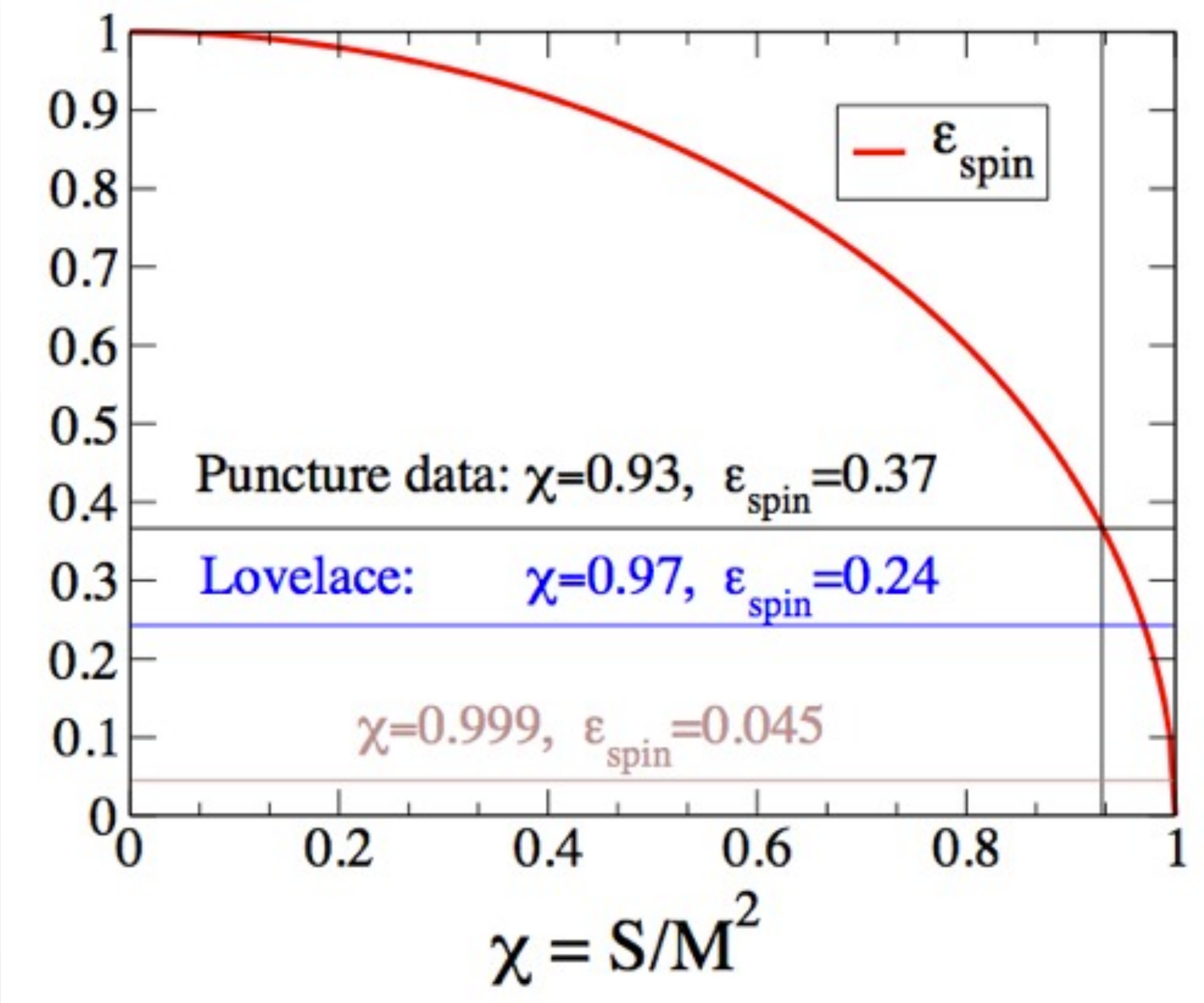
Importance of $S/M^2 > 0.93$

- Observational evidence for BH's with $S/M^2 \sim 0.998$

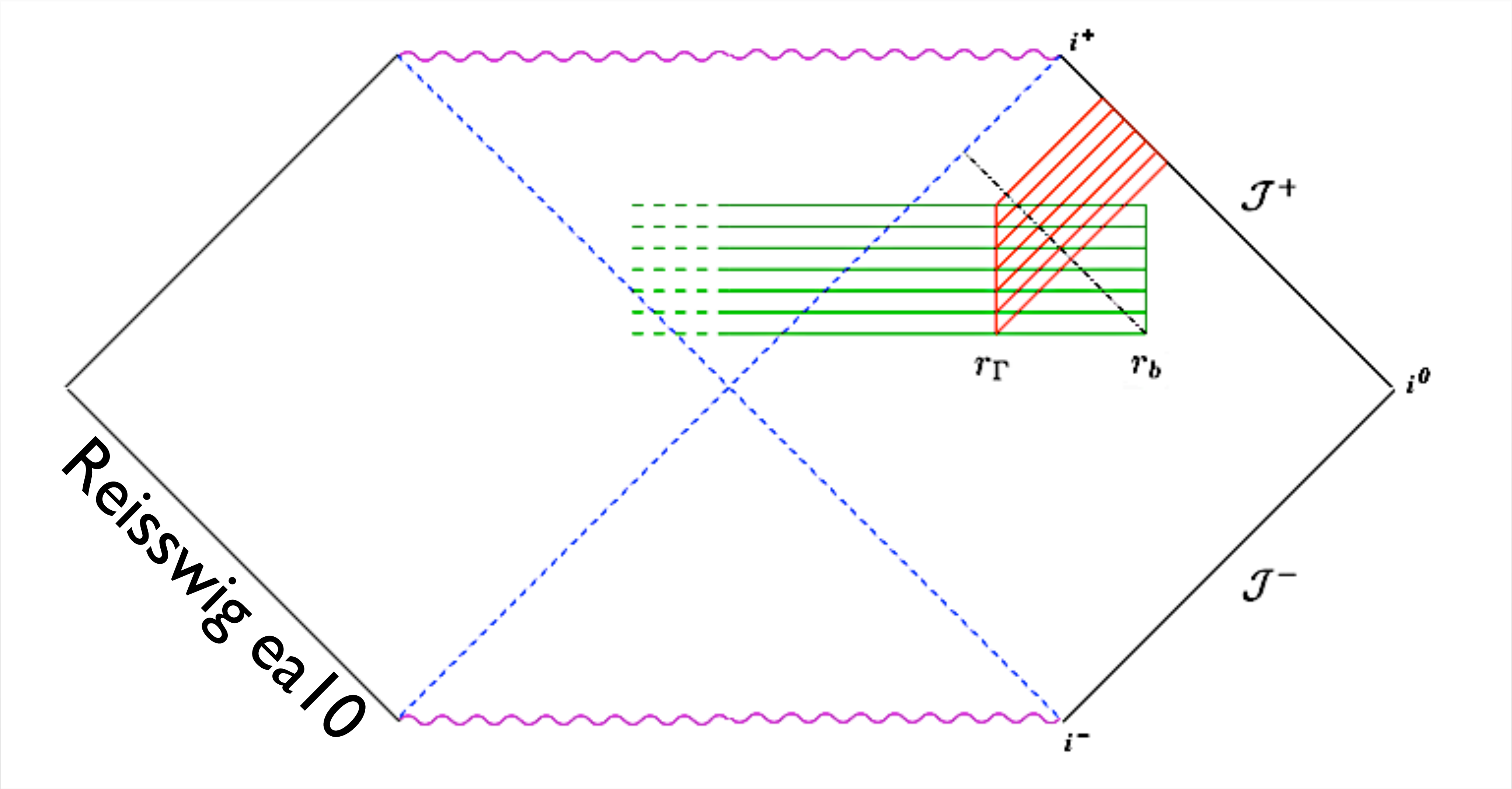
- Expansion parameter around extremality

$$\varepsilon_{\rm spin} \equiv \sqrt{1-\chi^2}$$

  - 0.93 is far from extremal!



Cauchy-characteristic Extraction



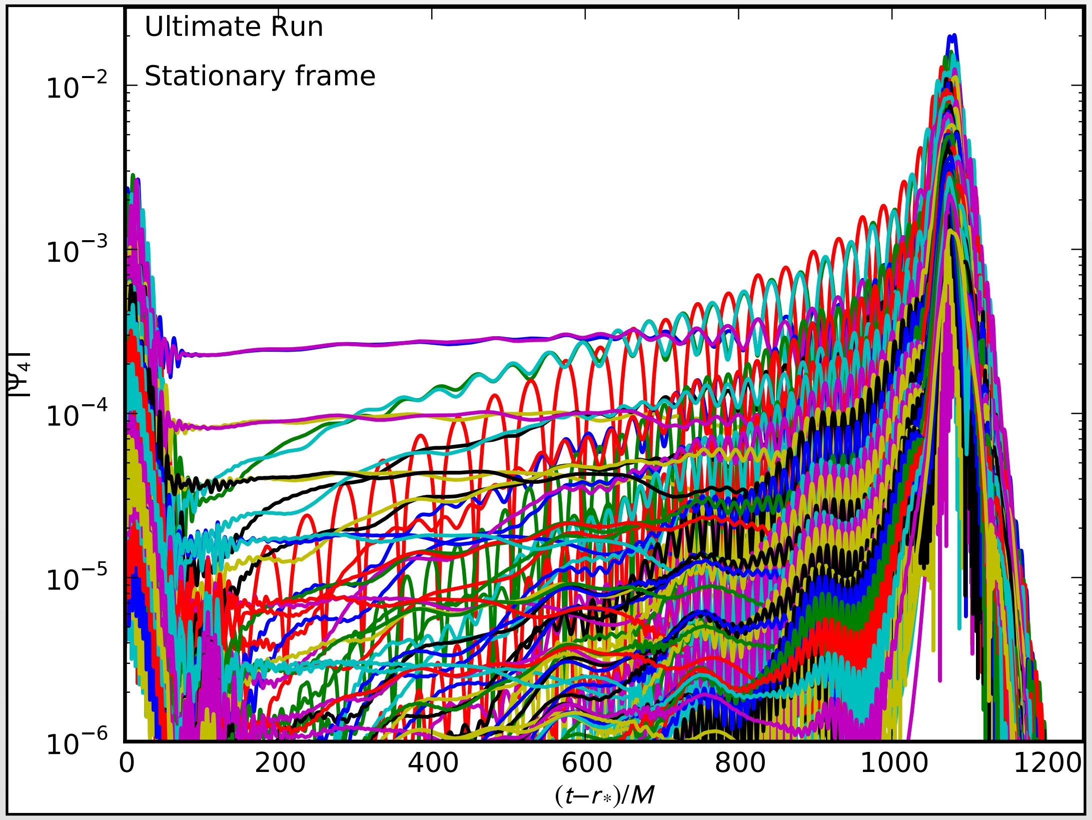
- h(t) **at** Scri+
- Post-processing tool for any Cauchy evolution (open source)

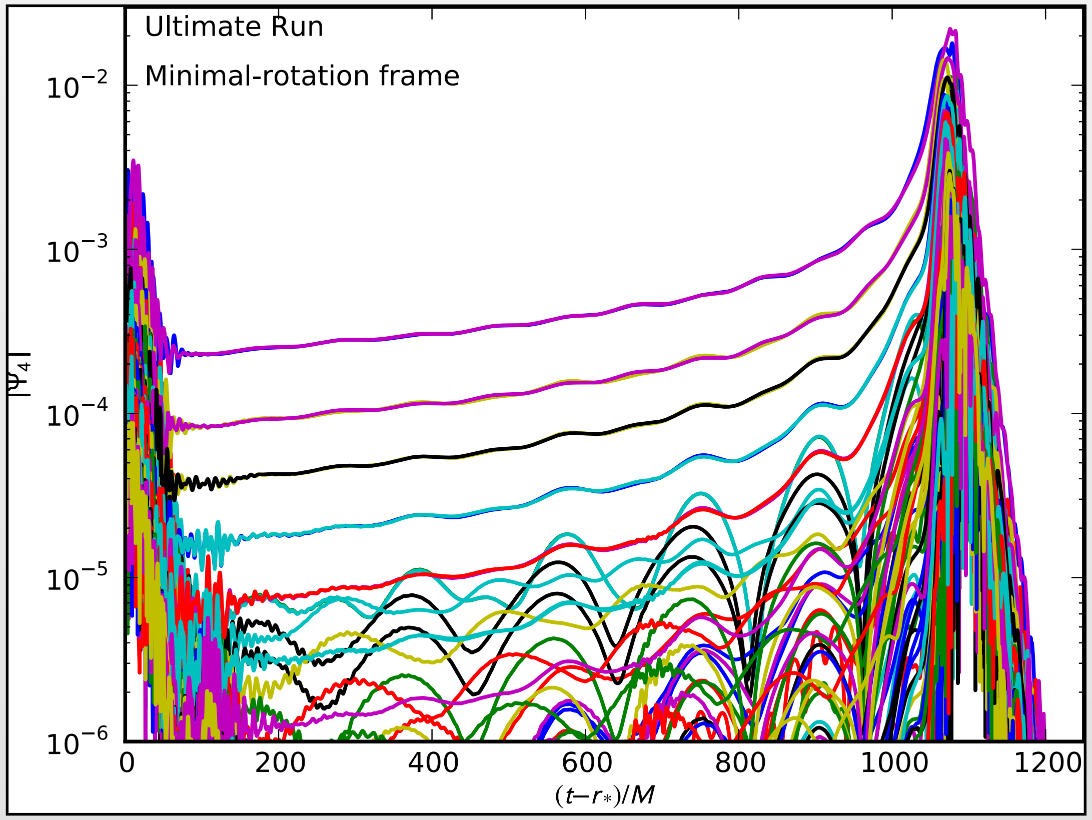
Reisswig ea 09, Reisswig ea 10, Babiuc ea 1011.4223, Babiuc ea 1106.4841

Radiation–aligned minimally–rotating frame

CITA–ICAT

- ❖ Decompose radiation in a good frame, not an inertial frame
- ❖ Schmidt ea 2011, O'Shaugnessy ea 2011, Boyle ea 2011:
  - Polar axis of Ylm-decomposition along dominant emission direction





q=6, □A=0.9, □B=0.3, 8 orbits

Figures courtesy Mike Boyle & Larry Kidder

SXS collaboration

***S**imulation of e**X**treme **S**pacetimes*

- Solve Einstein's equations accurately enough for LIGO's needs
- Cornell, Caltech, CITA, Fullerton, Oberlin, WSU
- Work presented here involves:

Numerics: L. Buchman[1], T. Chu[2], L. Kidder[3], S. Lau[4], G. Lovelace[5],
A. Mroue[2], S. Ossokine[2], R. Owen[6], M. Scheel[1],
B. Szilagyi[1], N. Taylor[1], S. Teukolsky[2]
Analysis: M. Boyle[3], D. Brown[7], A. Buonanno[8], I. MacDonald[2],
S. Nissanke[1], Y. Pan[8], A. Taracchini[8]

1 Caltech, 2 CITA, 3 Cornell, 4 Albuquerque, 5 Fullerton,
6 Oberlin, 7 Syracuse, 8 Maryland

Techniques I: Generalized Harmonic

- Einstein's equations

$$0 = R_{ab}[g_{ab}] = -\frac{1}{2}\Box g_{ab} + \nabla_{(a}\Gamma_{b)} + \text{lower order terms}, \qquad \Gamma_a = -g_{ab}\Box x^b.$$

- Generalized harmonic coordinates $g_{ab}\Box x^b \equiv H_a(x^a, g_{ab})$

  (Friedrich 1985, Pretorius 2005; $H = 0$ used since 1920's)

$$\Box g_{ab} = \text{lower order terms.}$$

$\Rightarrow$ Constraint $C_a \equiv H_a - g_{ab}\Box x^b = 0$

- Constraint damping (Gundlach, et *al.*, Pretorius, 2005)

$$\Box g_{ab} = \gamma\left[t_{(a}C_{b)} - \frac{1}{2}g_{ab}t^c C_c\right] + \text{lower order terms}$$

$$\partial_t C_a \sim -\gamma C_a.$$

Techniques II: Spectral methods

- Expand in basis-functions, solve for coefficients

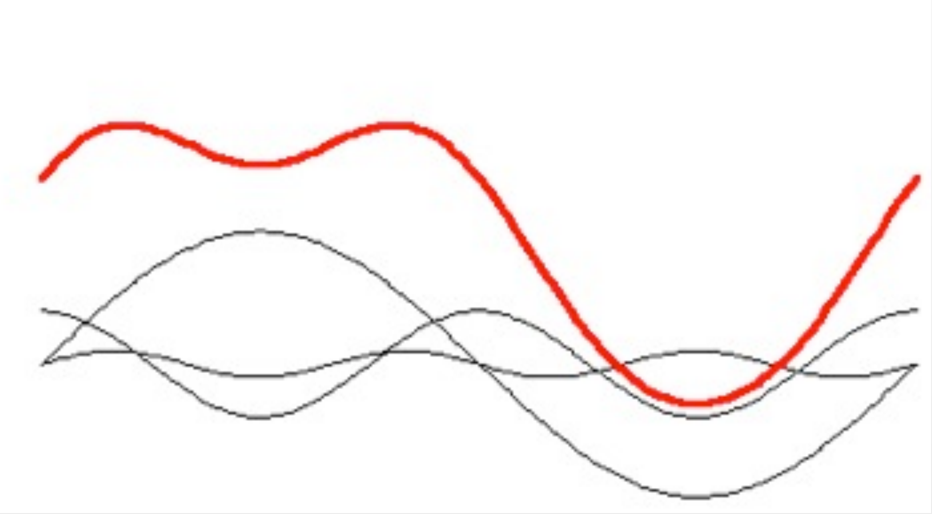
$$u(x,t) = \sum_{k=1}^{N} \tilde{u}(t)_k \Phi_k(x)$$

- Compute derivatives *exactly*

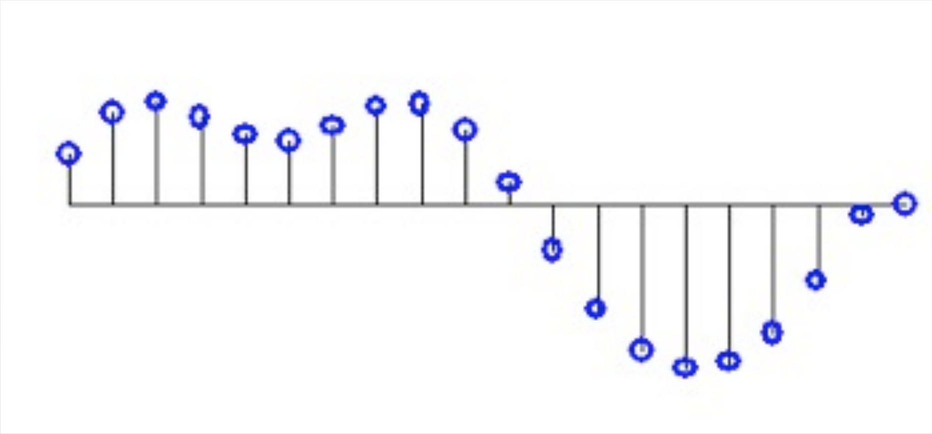
$$u'(x,t) = \sum_{k=1}^{N} \tilde{u}(t)_k \Phi'_k(x)$$

- Compute nonlinearities in physical space

Spectral

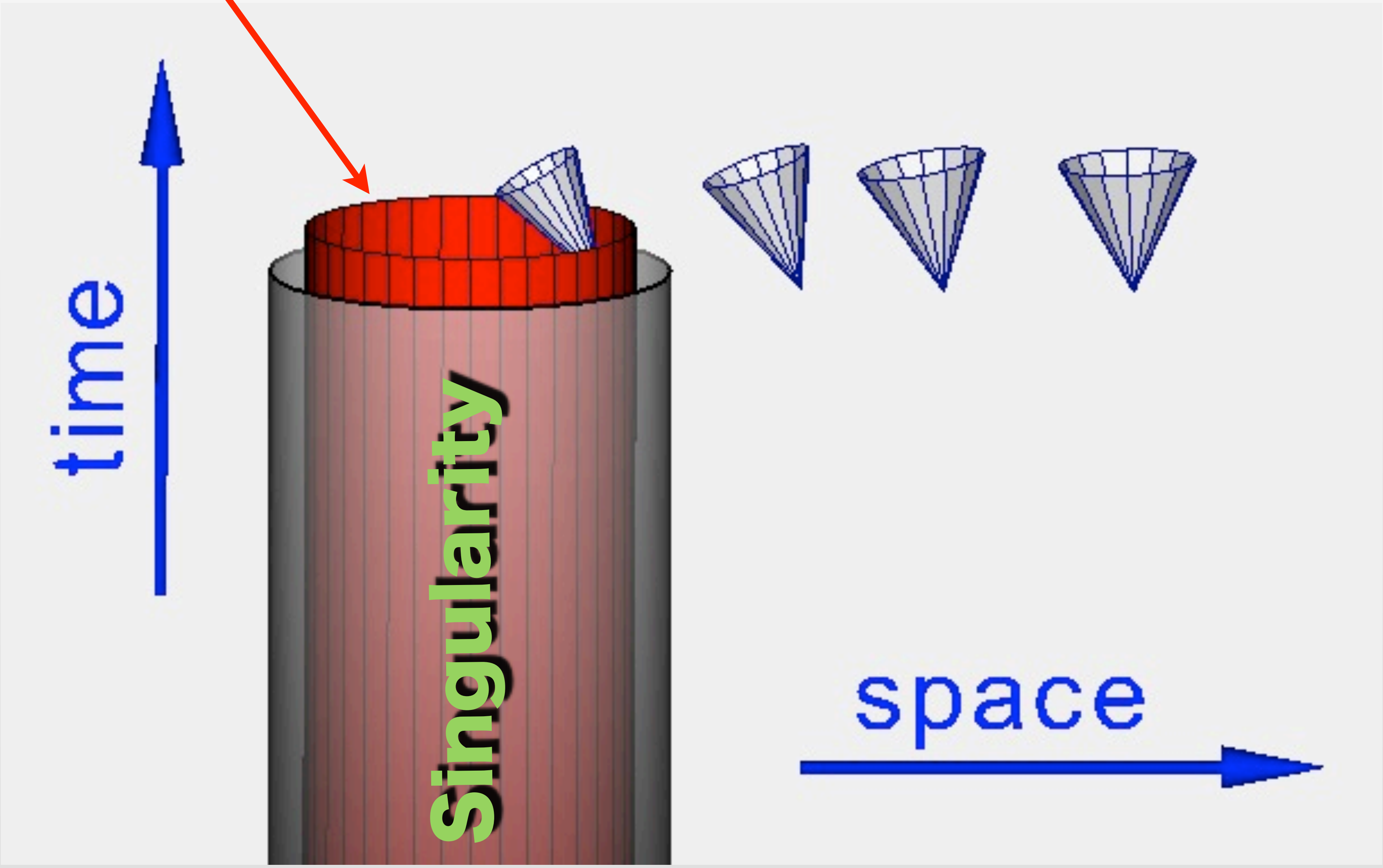


Finite differences

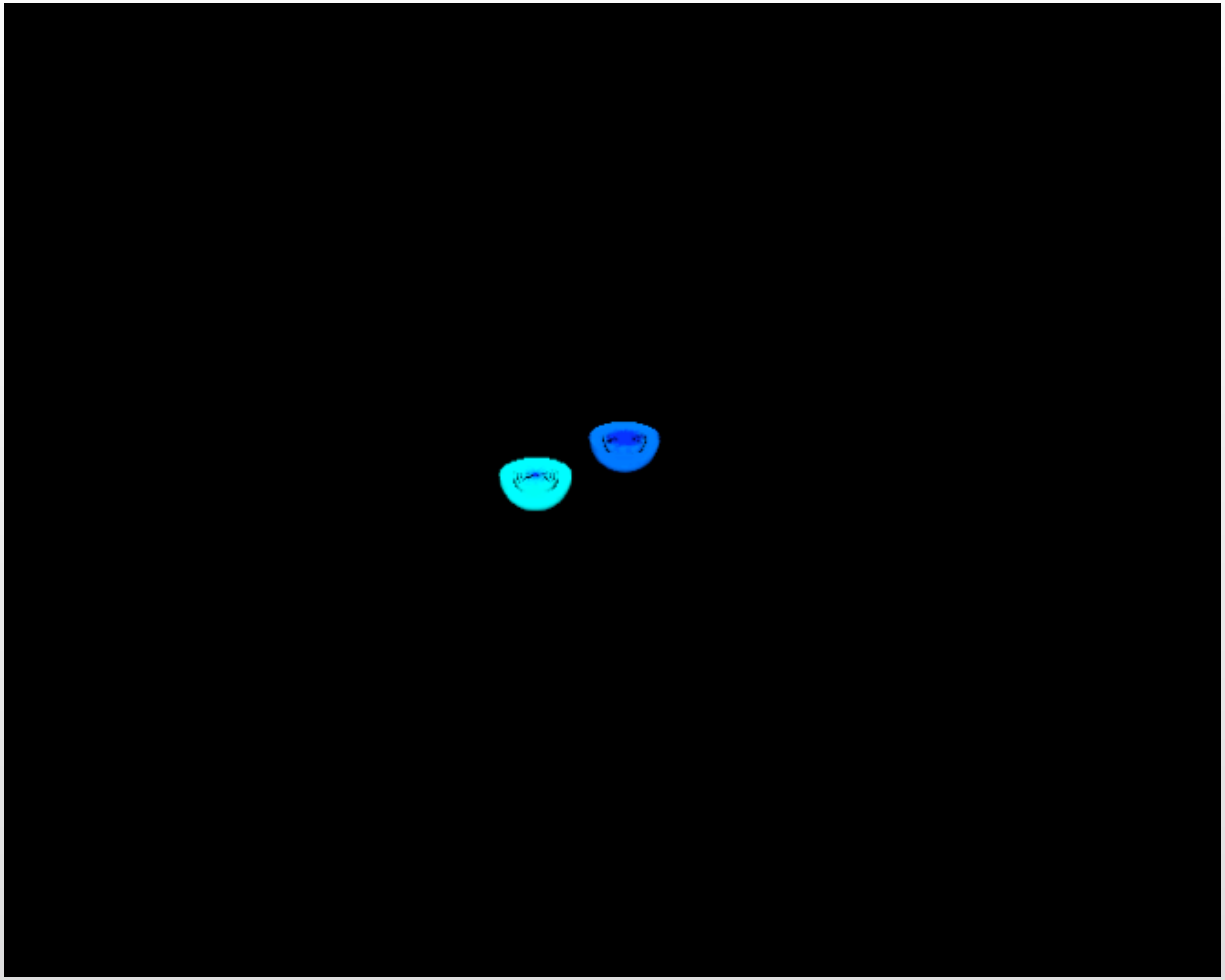


Numerics III: Black Hole Excision

Artificial boundary
inside horizon

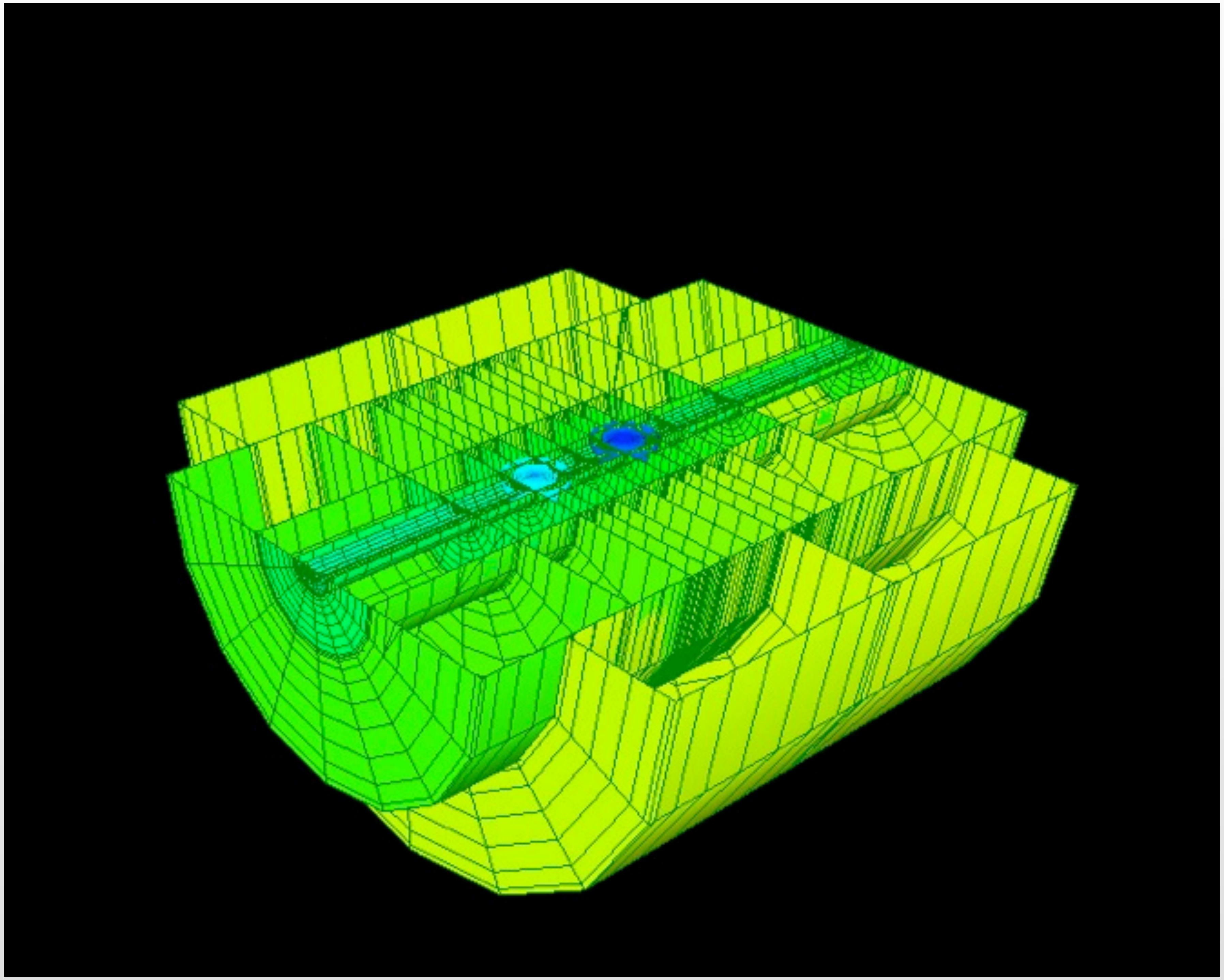


Techniques IV: Domain-decomposition



Spectral Einstein Code *SpEC* (Caltech-Cornell-CITA)
http://www.black-holes.org/SpEC.html

Techniques IV: Domain-decomposition



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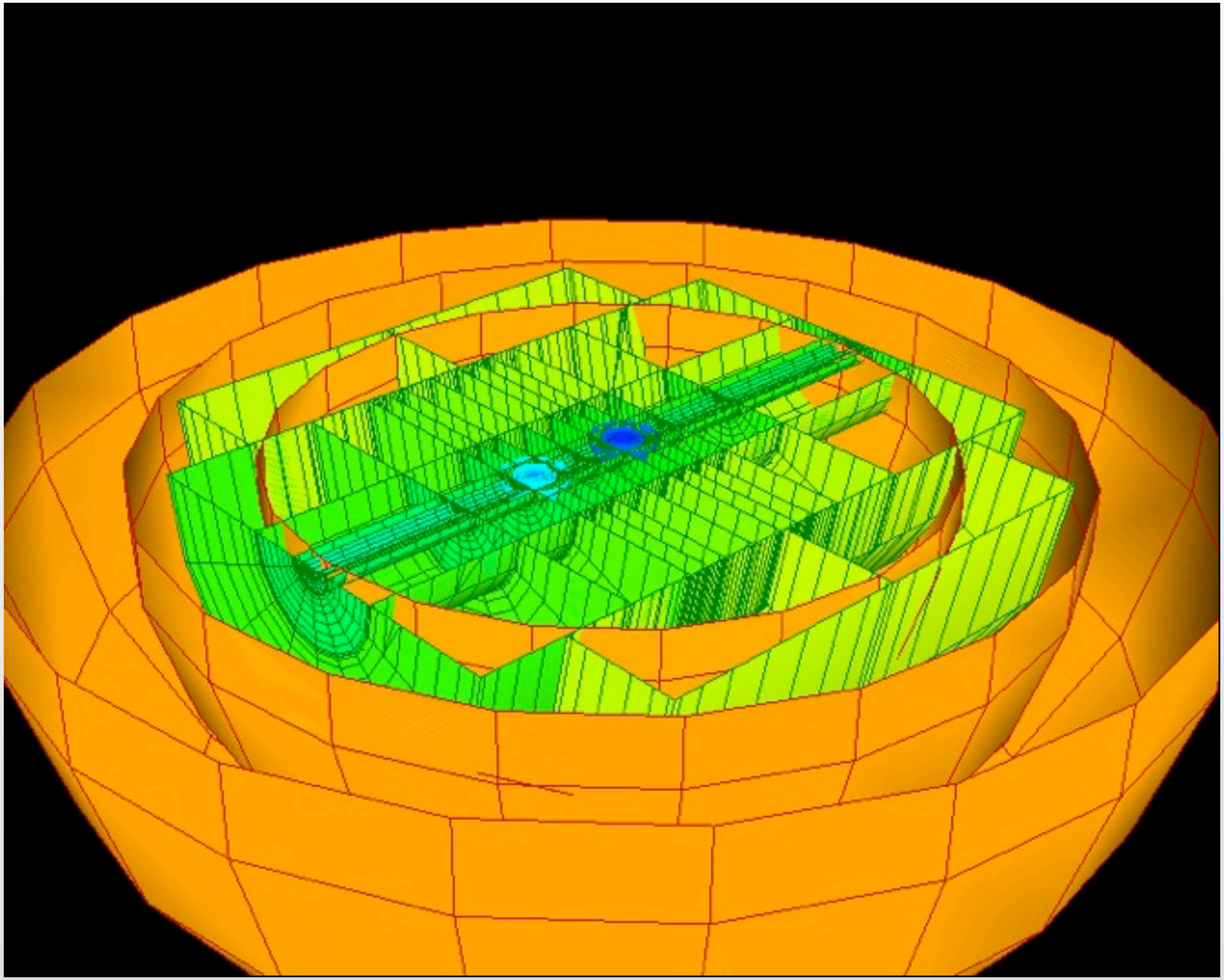

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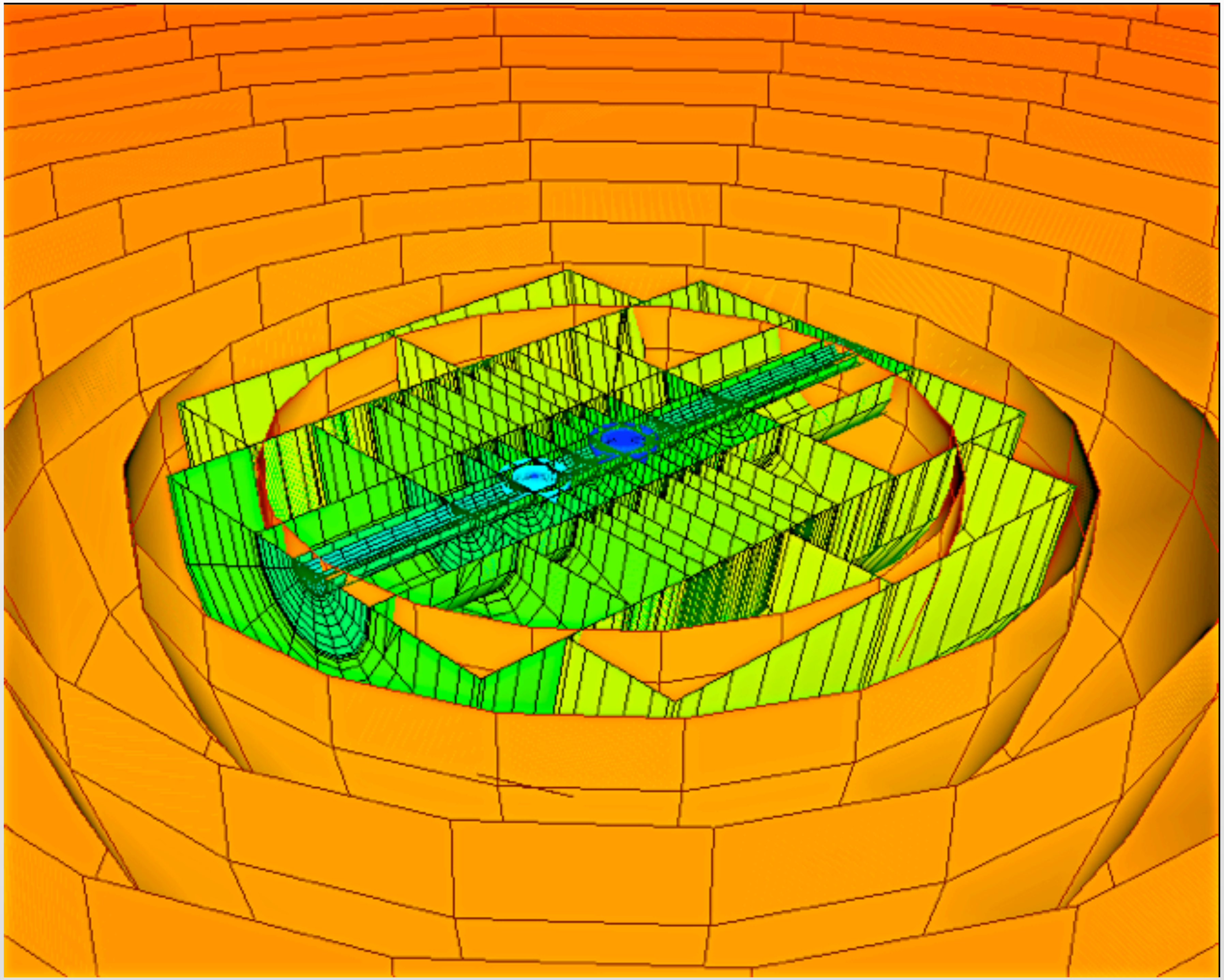


Spectral Einstein Code *SpEC* (Caltech-Cornell-CITA)
http://www.black-holes.org/SpEC.html

IV: Merger & Ringdown

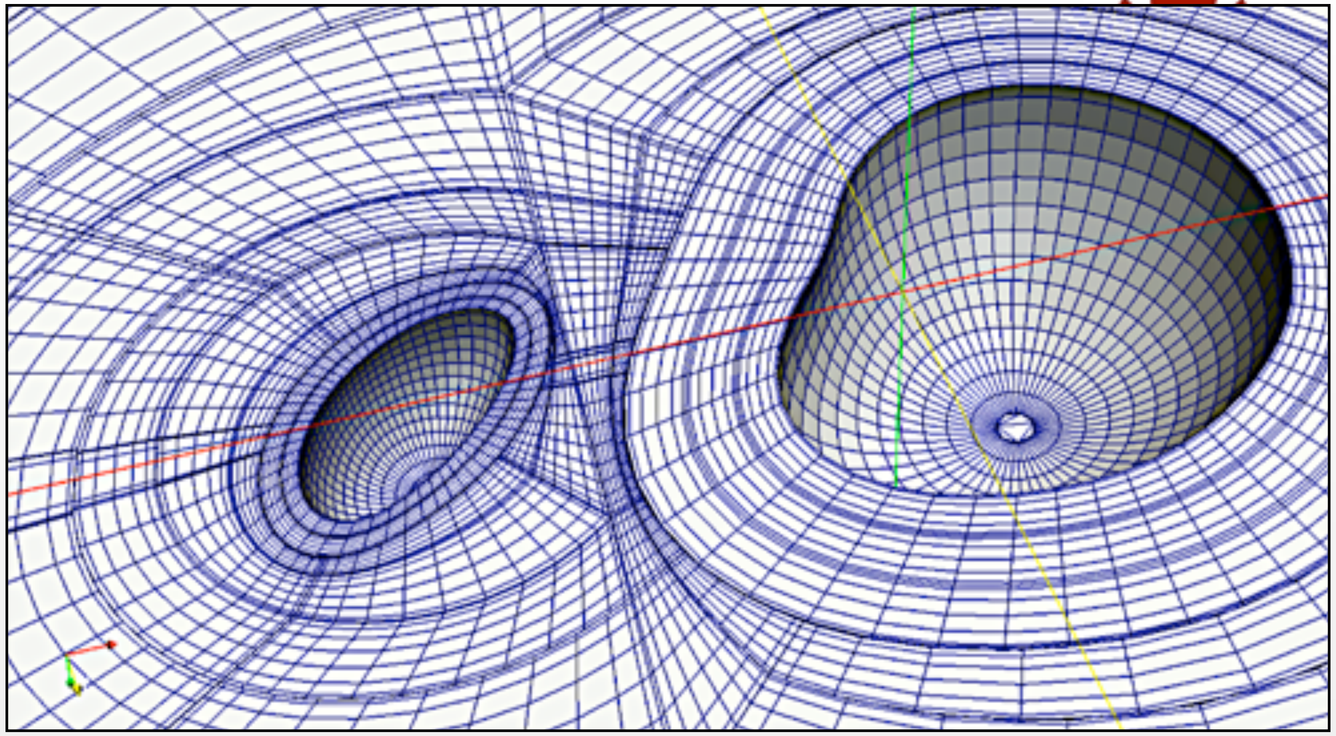
- ***Mark Scheel, Bela Szilagyi***

Szilagyi, Lindblom, Scheel 08.
Many additions since then

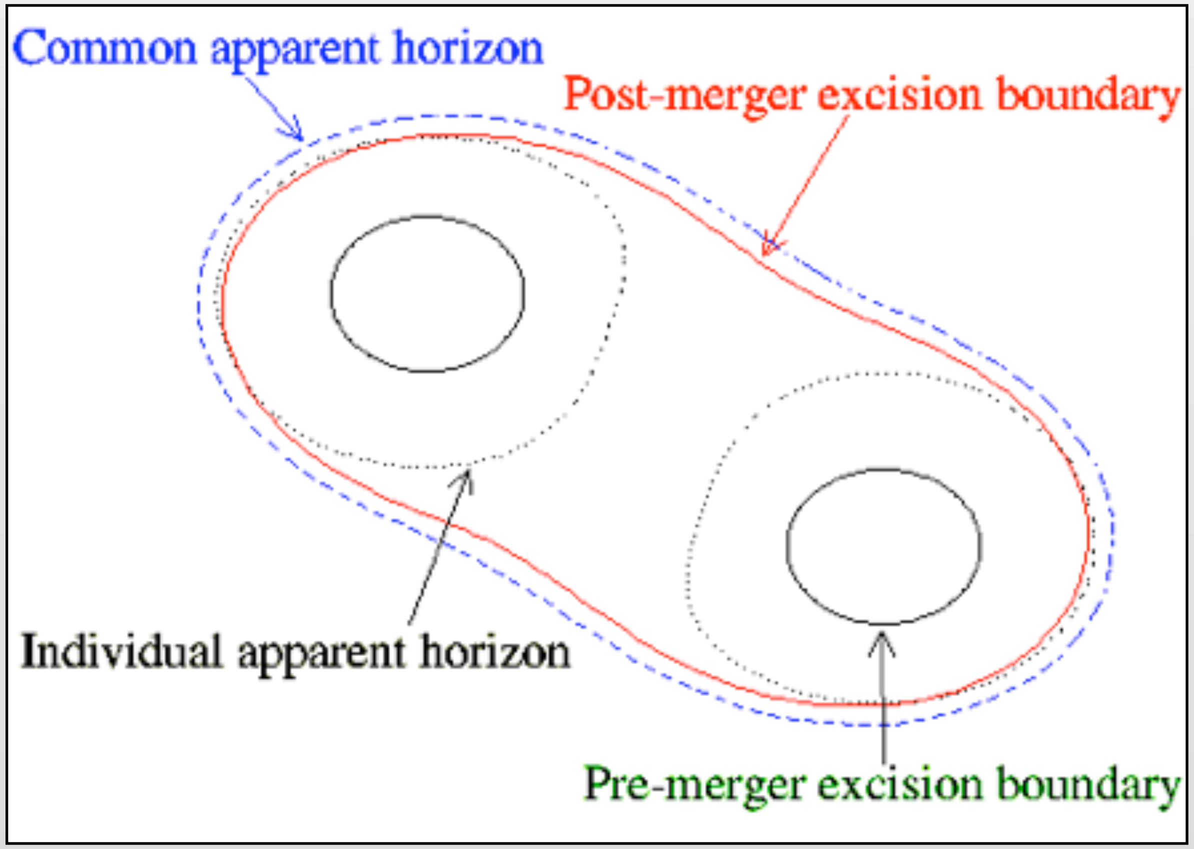
  - Hemberger ea, 1211.6079

- Close to merger
  - Switch domain-decomposition
  - Active gauge conditions
  - Adaptive Mesh Refinement

- After common horizon
  - Switch to distorted concentric shells



Bela Szilagyi



Mark Scheel

Scale

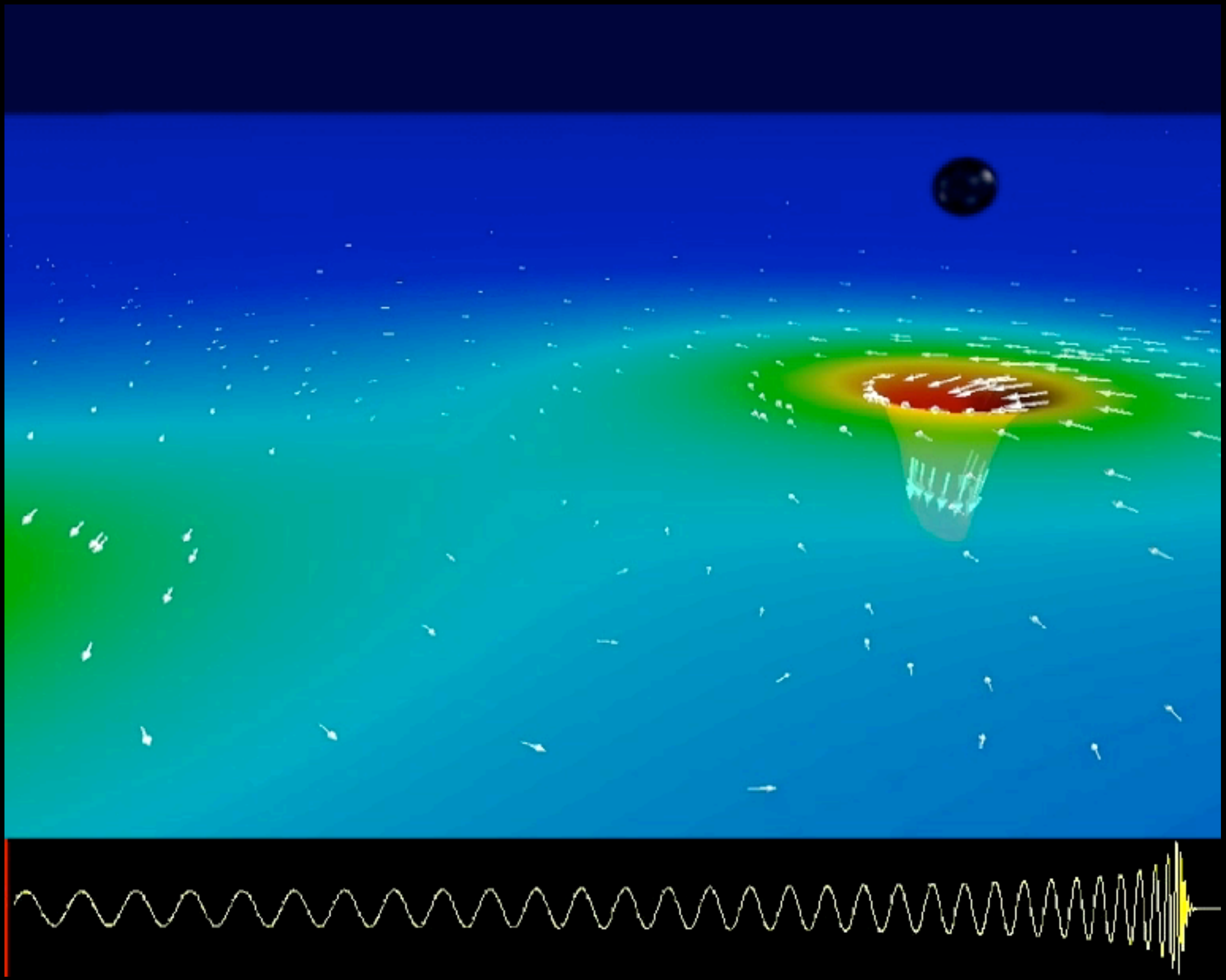
❖ Spectral Einstein Code

- 500,000 lines
- In development since 2000
- ~50 person years
- Used in ~80 publications

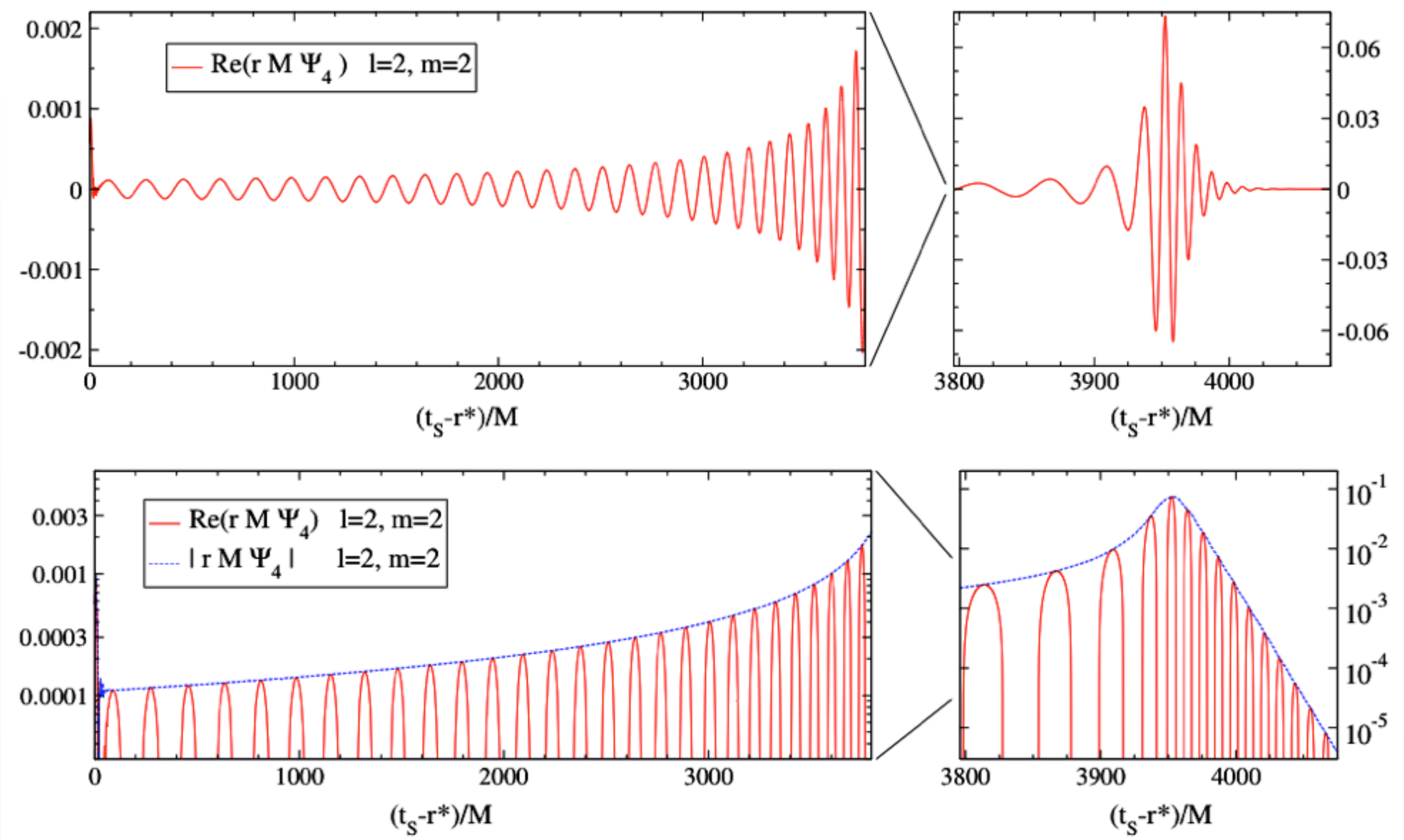
❖ Per simulation

- tens of CPU-*years* (100,000 CPU-hours)
- months of wall-clock time

CITA–ICAT

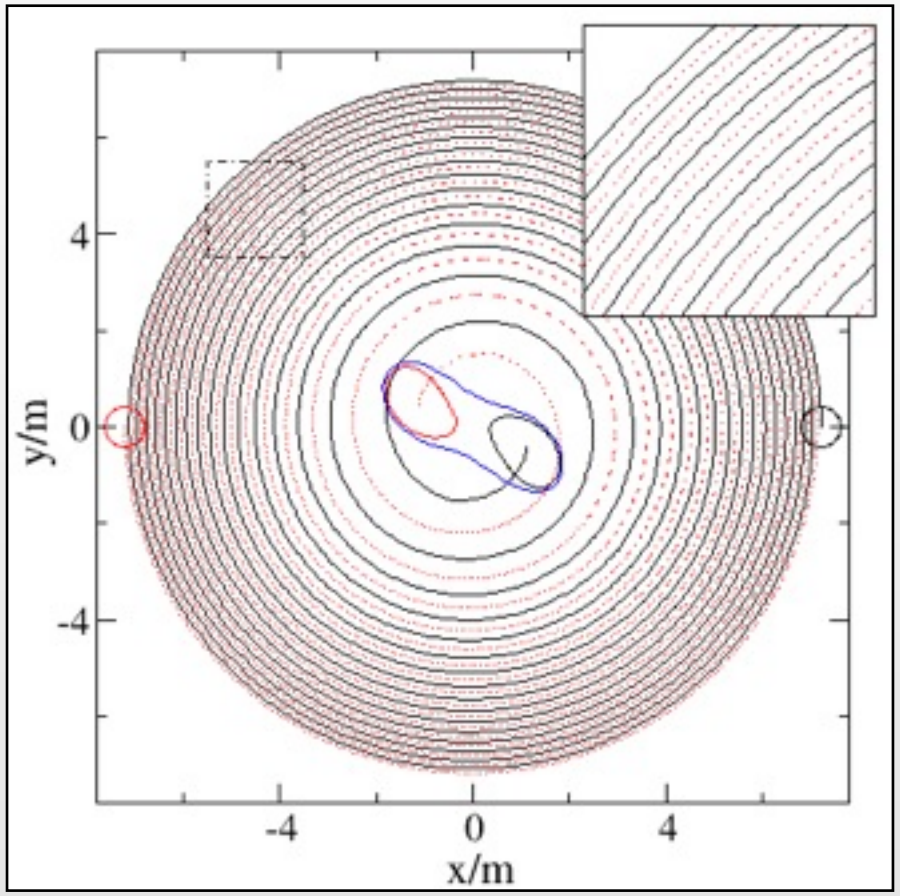


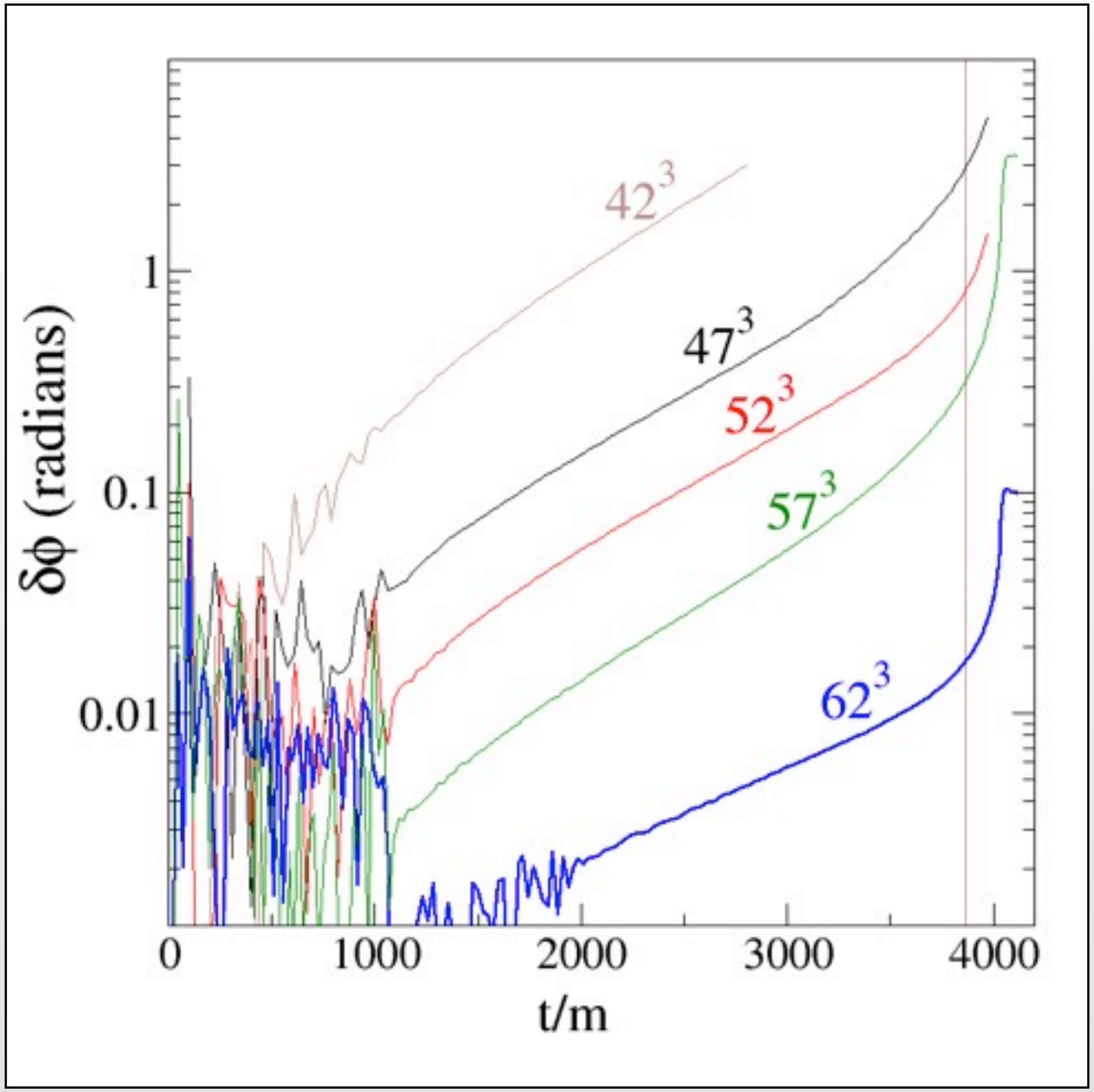
A waveform, at last!



Boyle ea 07, Scheel ea 09

Phase accuracy

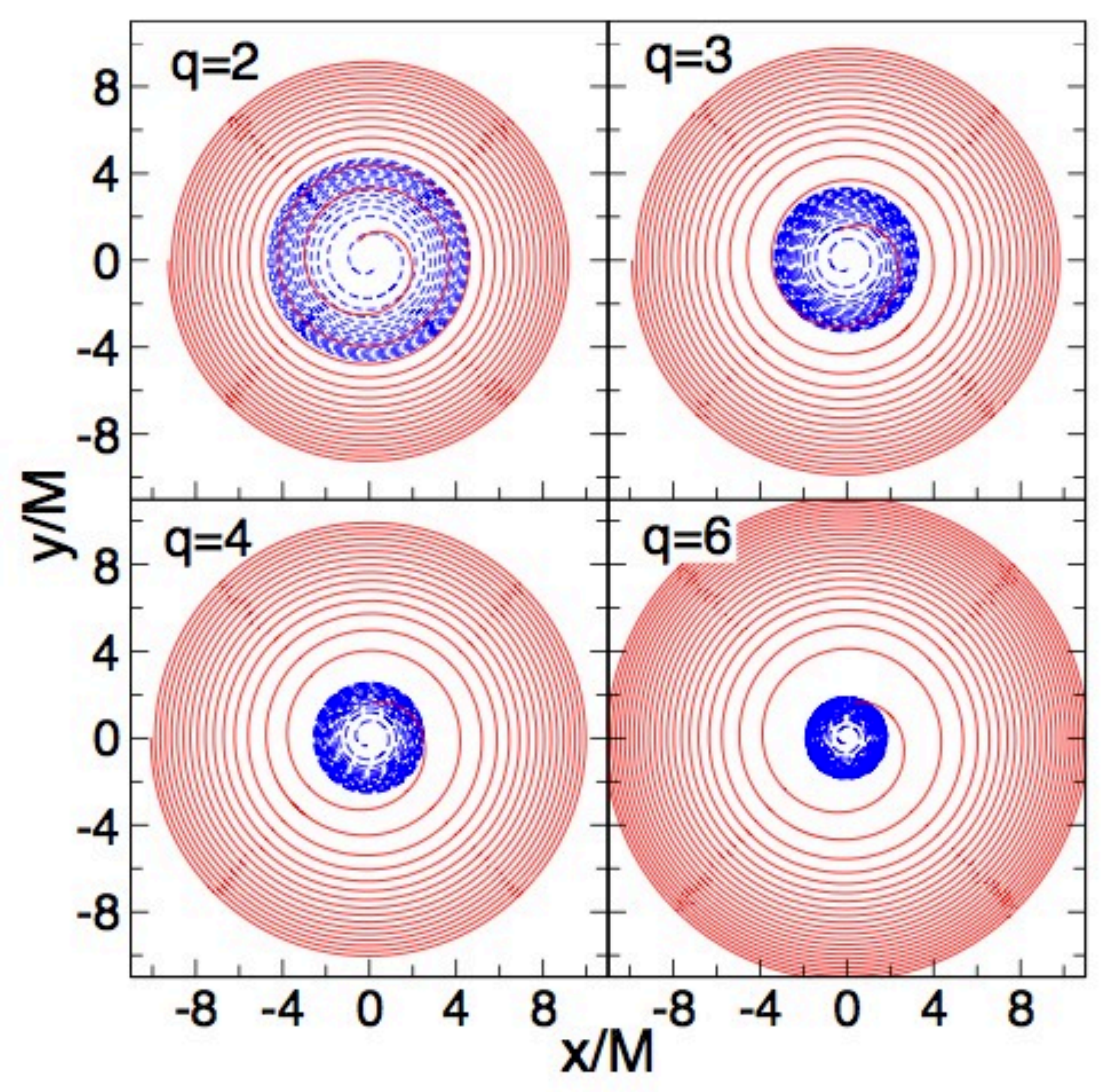


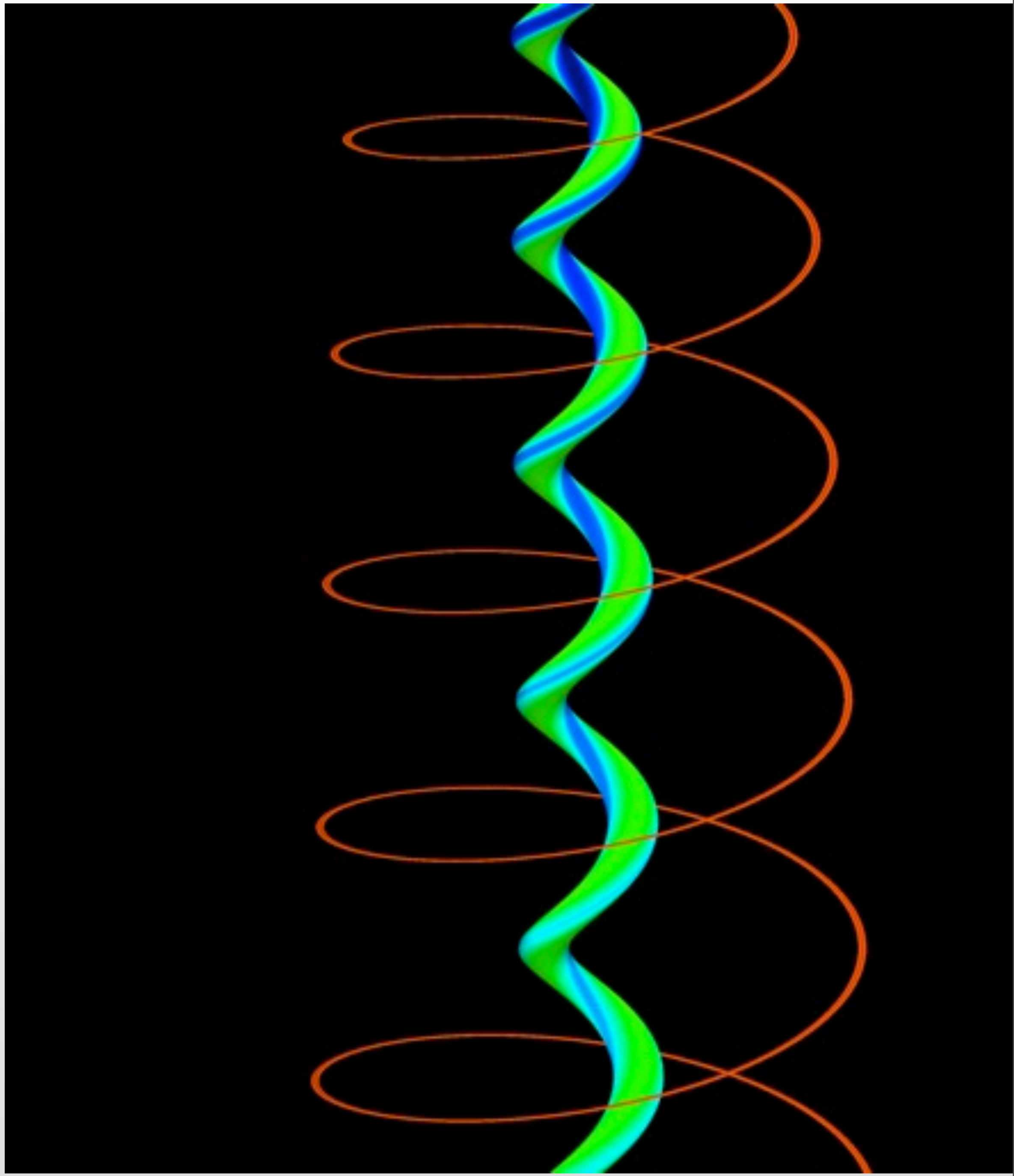


- Rapid convergence due to spectral methods
- Allows long & numerous simulations

Boyle ea 07, Scheel ea 09

Unequal masses, no spin

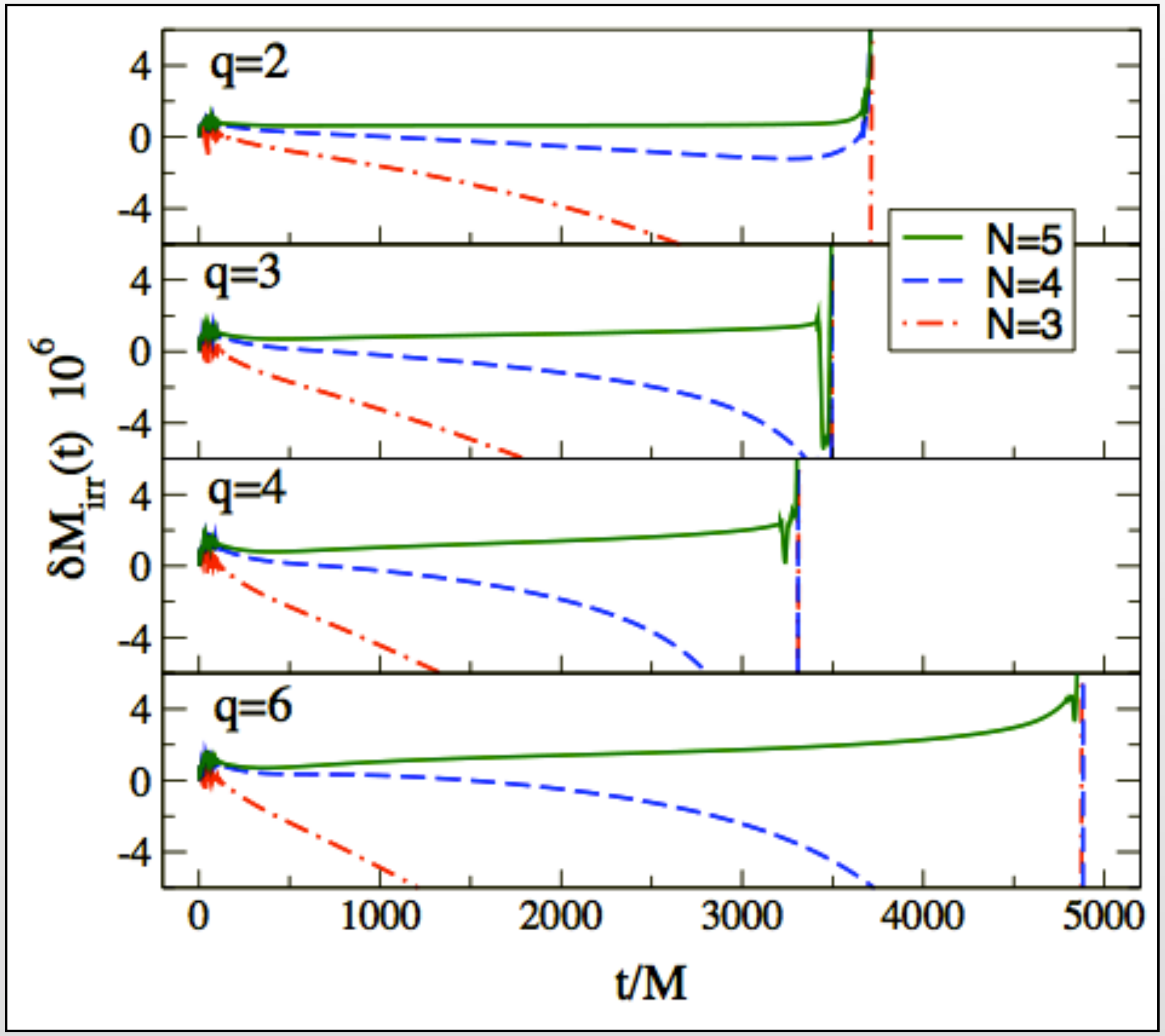


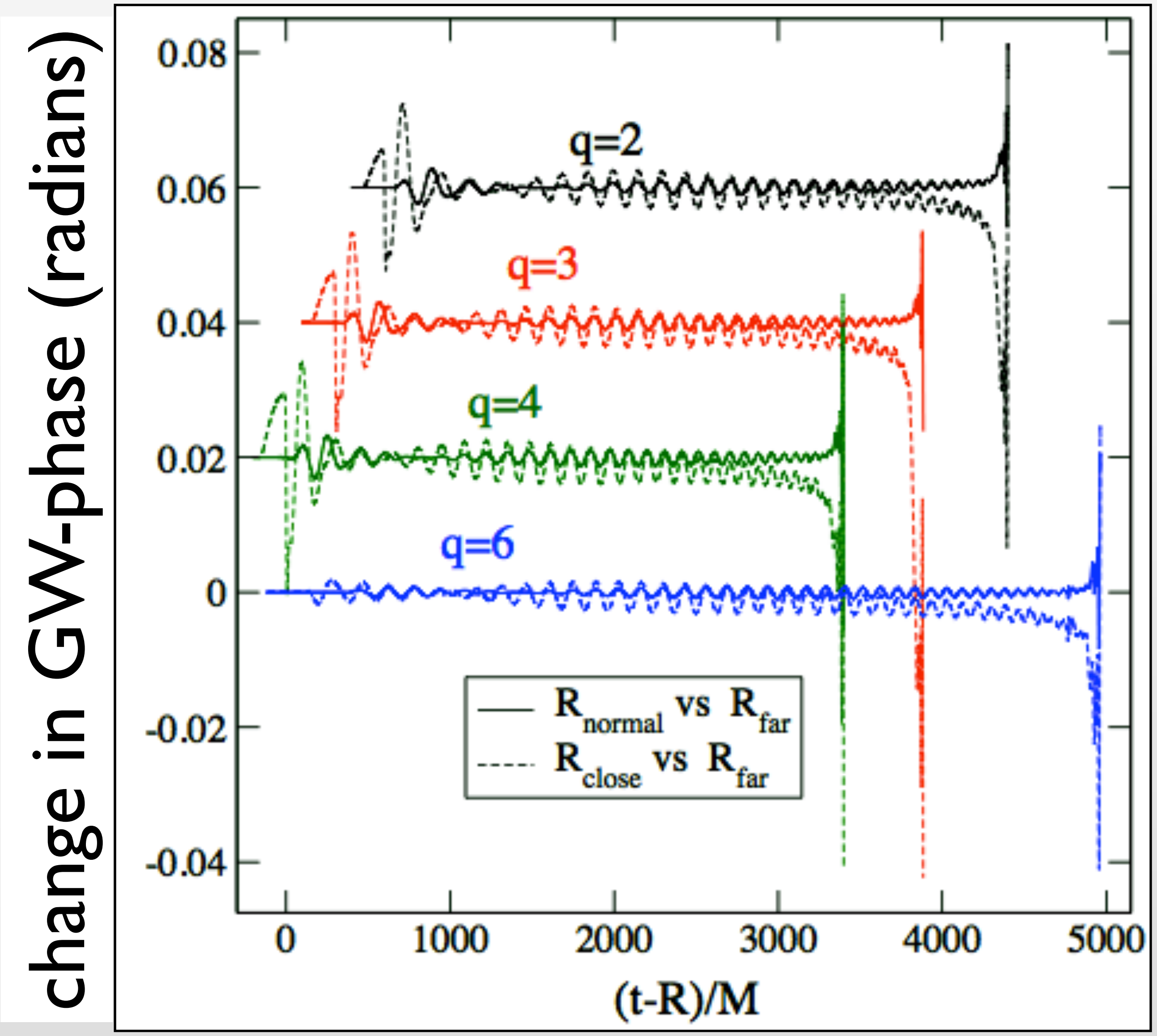


Buchman, HP, Scheel, Szilagyi, 2012

Accuracy

- Sum of irreducible masses ~1 part in 10^6
- (non-)effect of artificial outer boundary

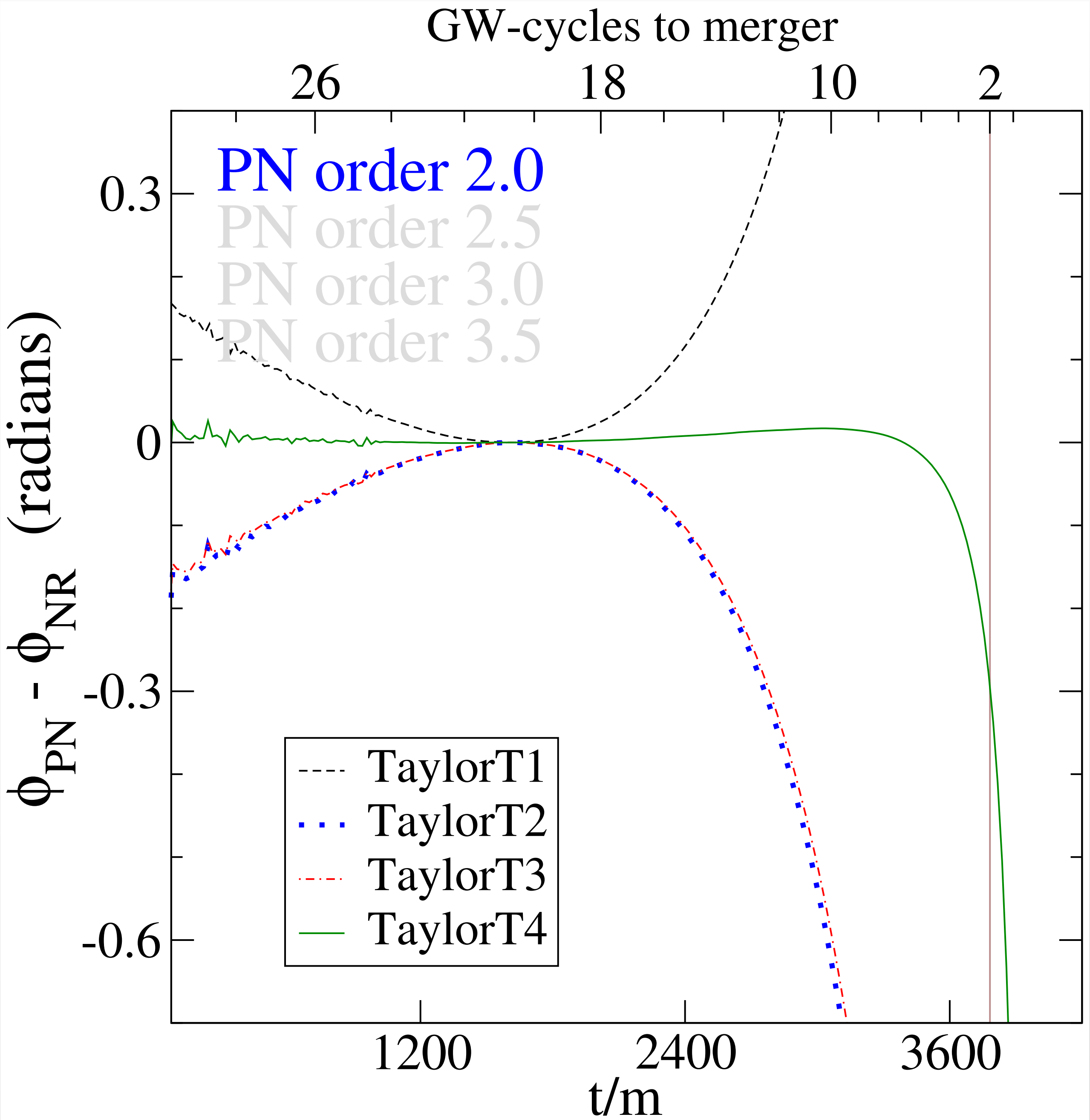




Buchman, HP, Scheel, Szilagyi, 2012

Validate post-Newtonian

- NR & PN agree!



Boyle, ... HP ea, 2007

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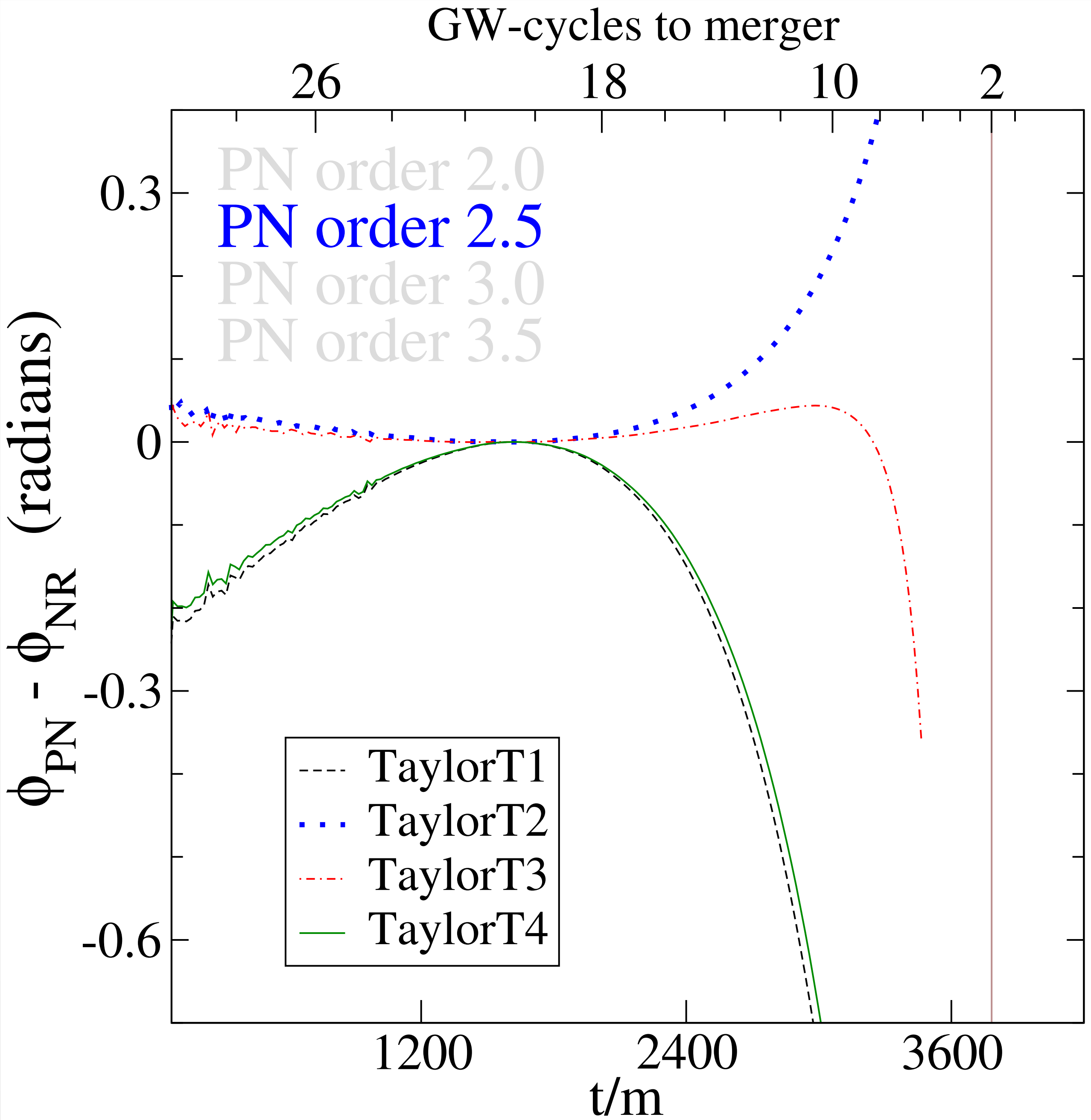

Boyle, ... HP ea, 2007

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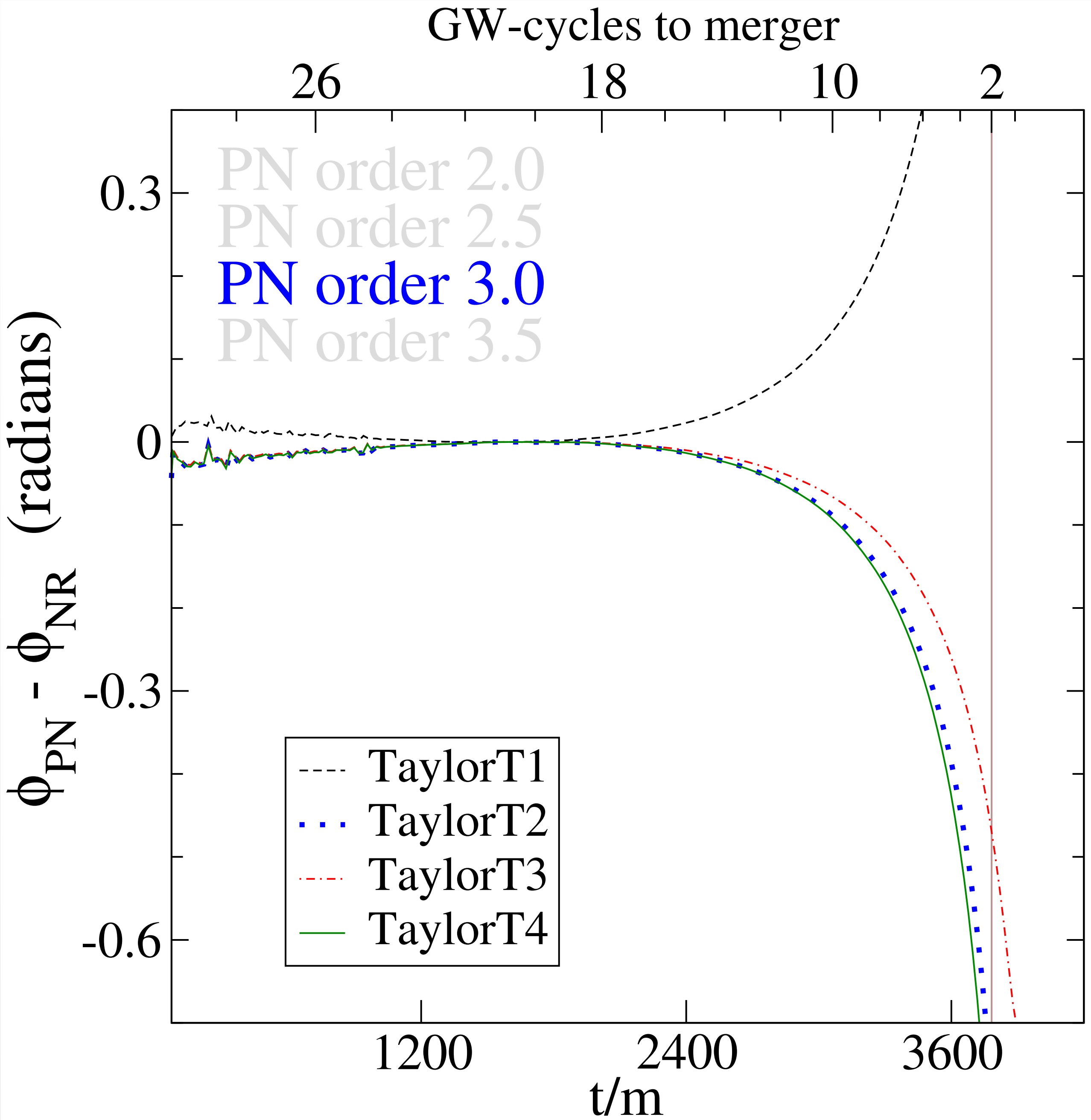

Boyle, ... HP ea, 2007

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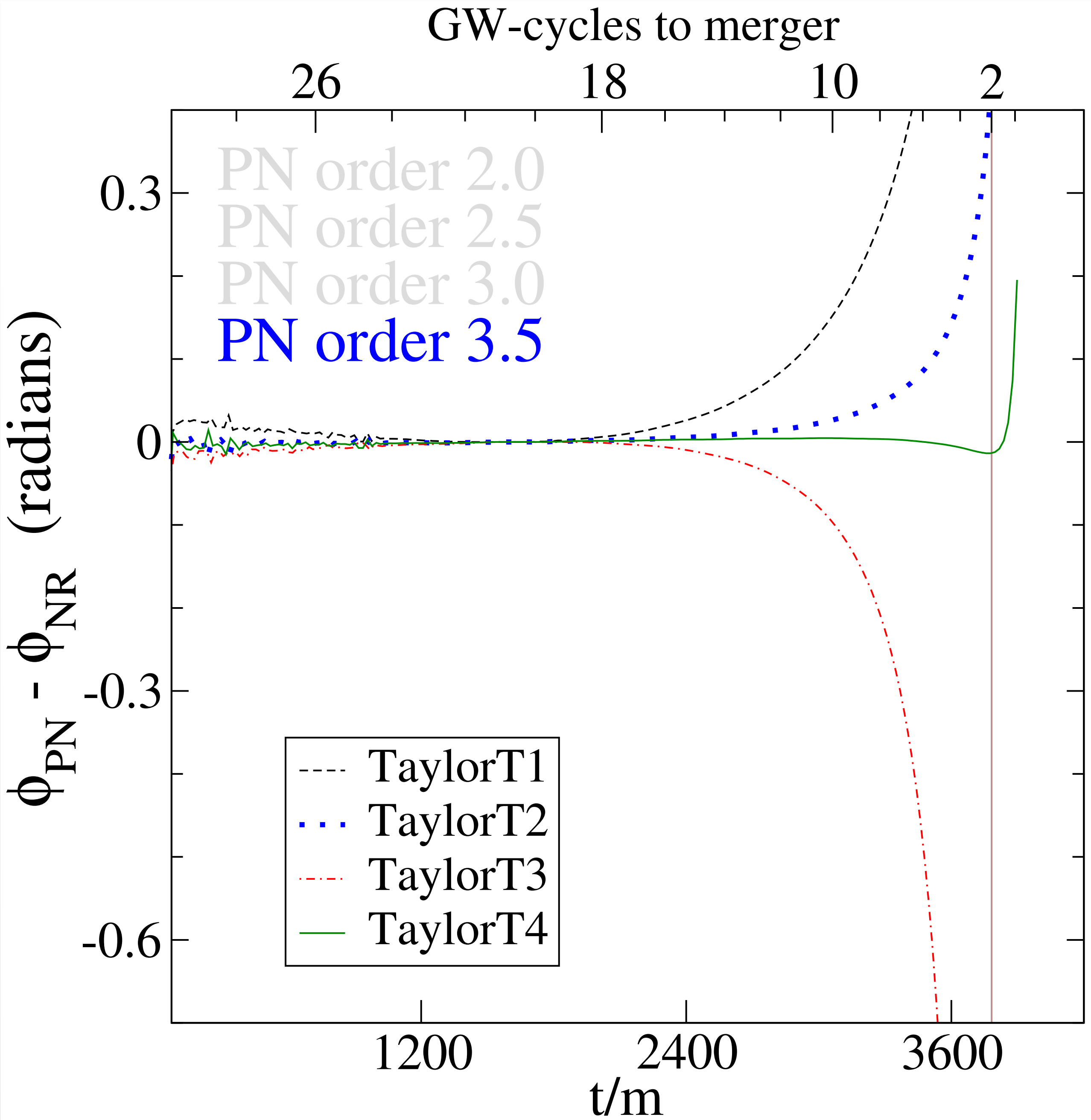
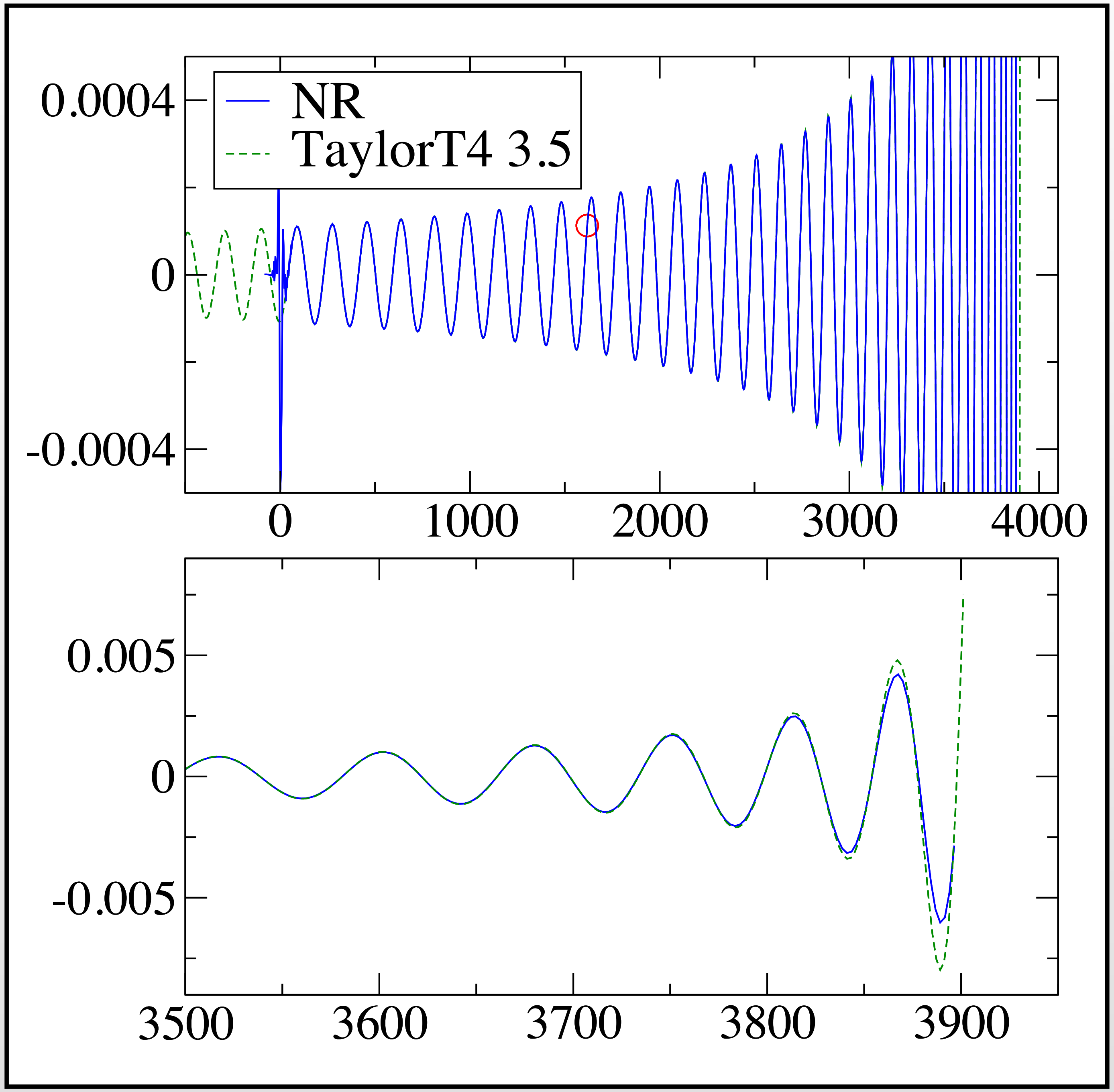


Boyle, ... HP ea, 2007

Validate post-Newtonian

- ❖ NR & PN agree!

- ❖ Or do they?
  - SOME versions of PN match very well
  - NO a priori knowledge which ones work (if any)



Boyle, ... HP ea, 2007

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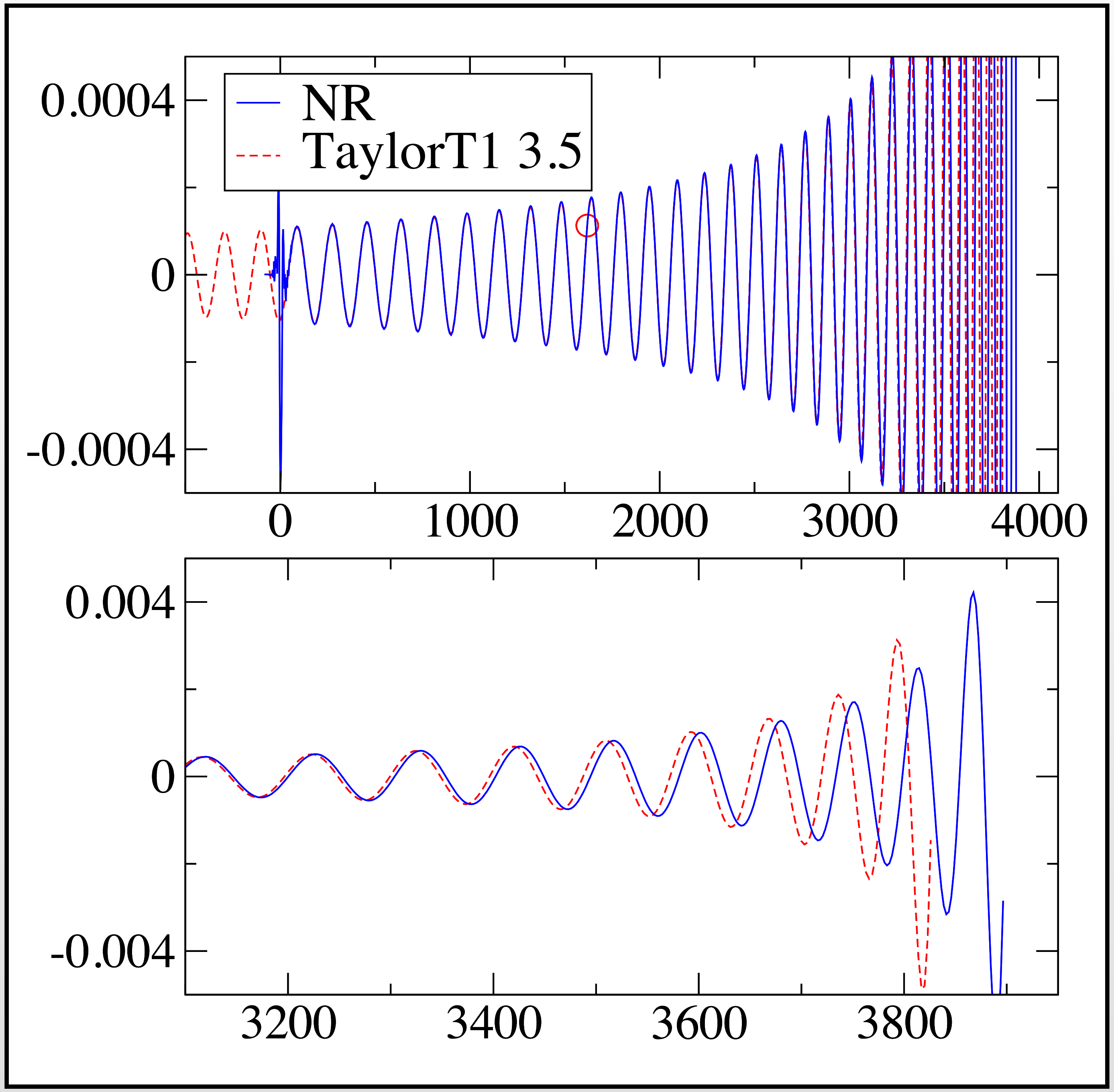

Boyle, ... HP ea, 2007

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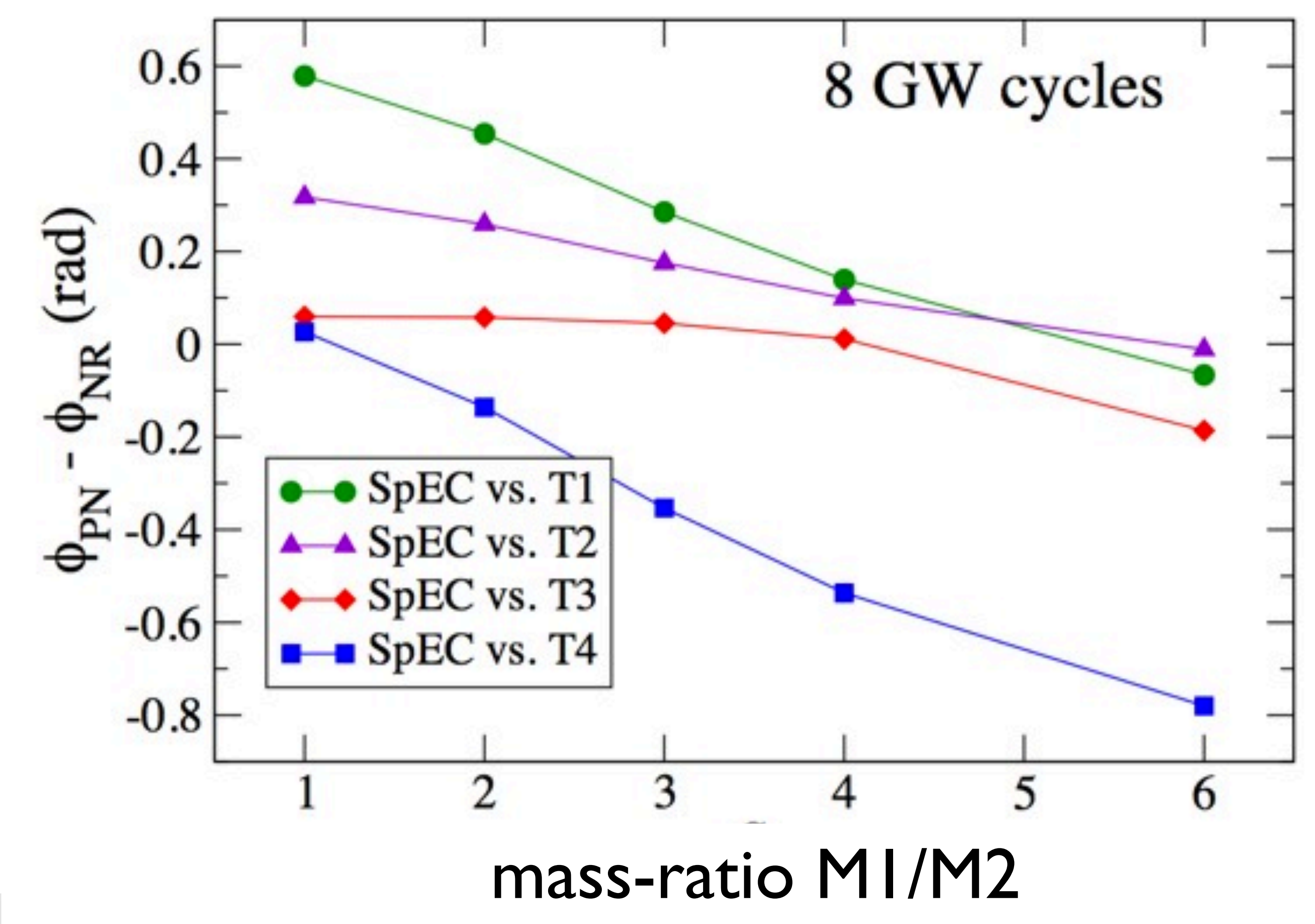

MacDonald, Mroue, HP ea, 2012

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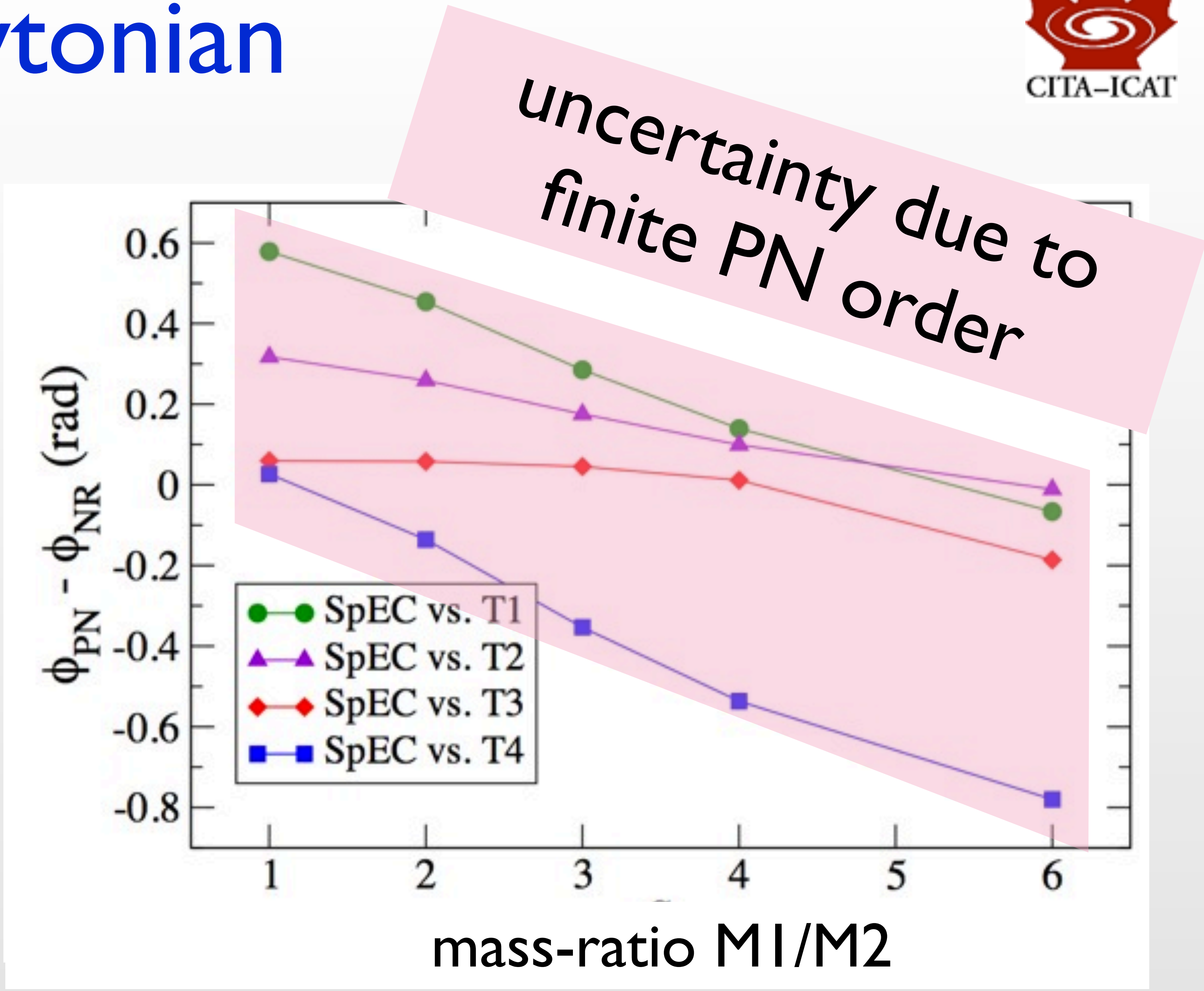
  - SOME versions of PN match very well
  - NO a priori knowledge which ones work (if any)

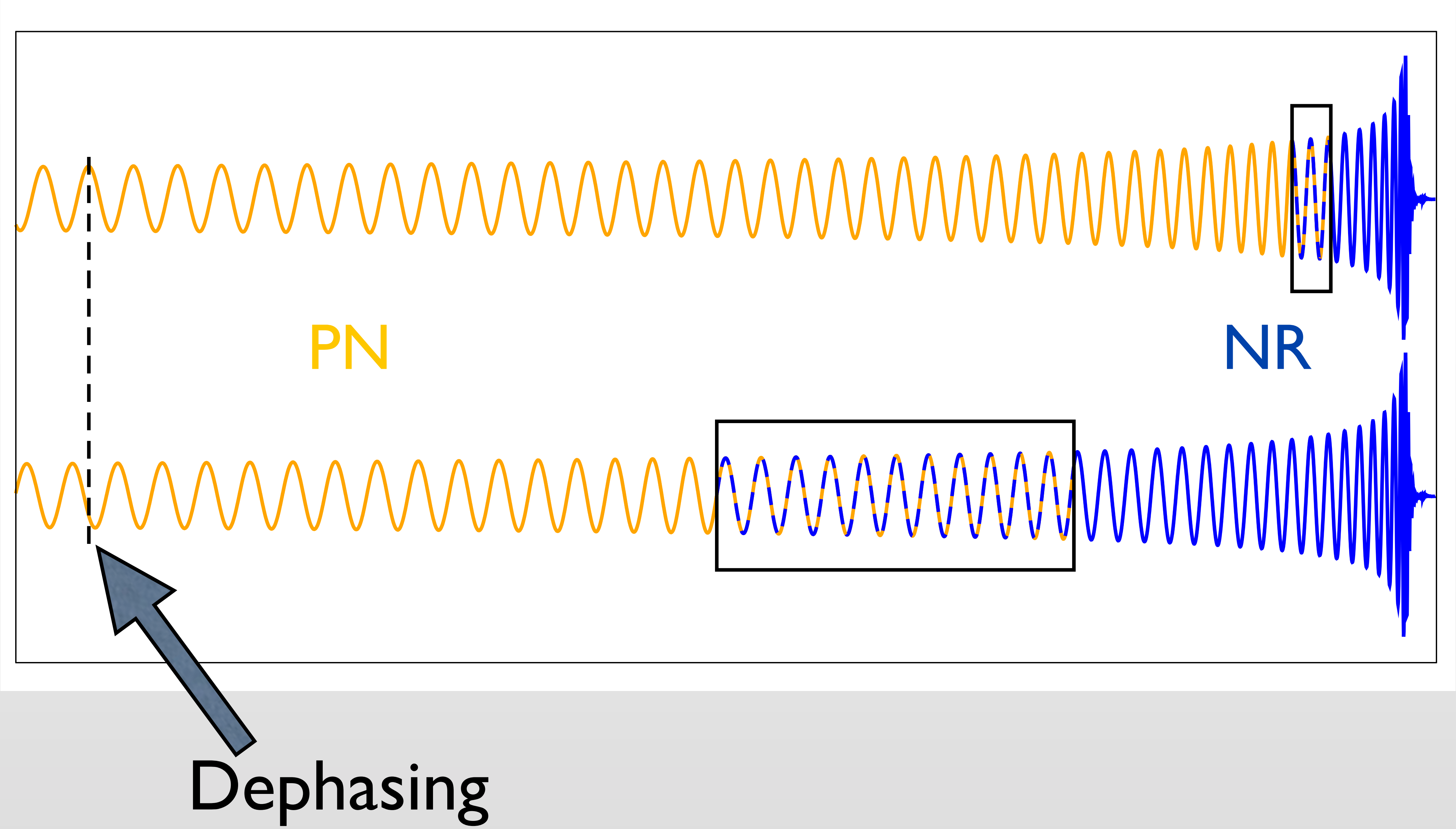


uncertainty due to finite PN order

MacDonald, Mroue, HP ea, 2012

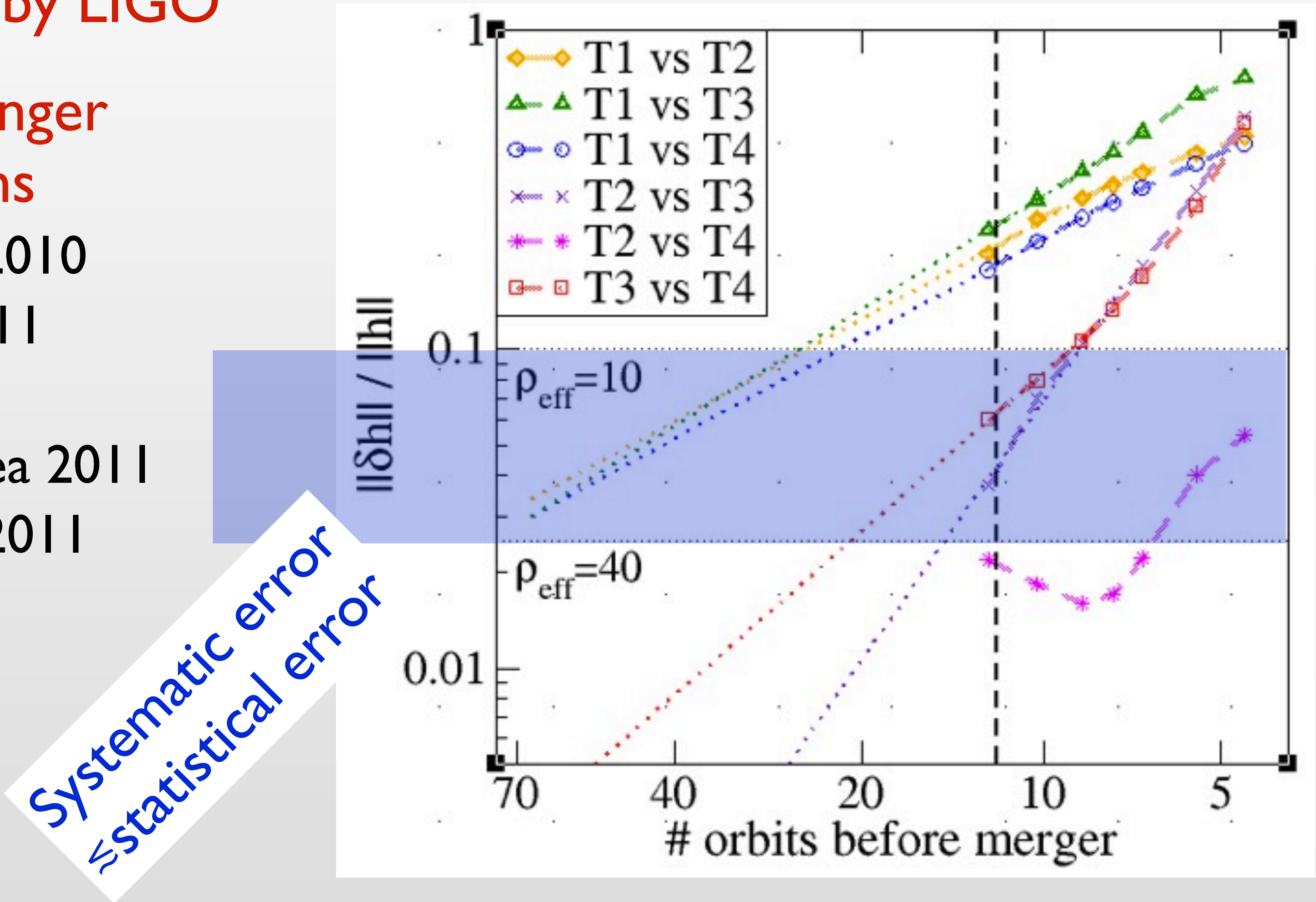
Length of NR simulations

- Must switch to NR early enough to avoid large PN errors



Length of NR simulations

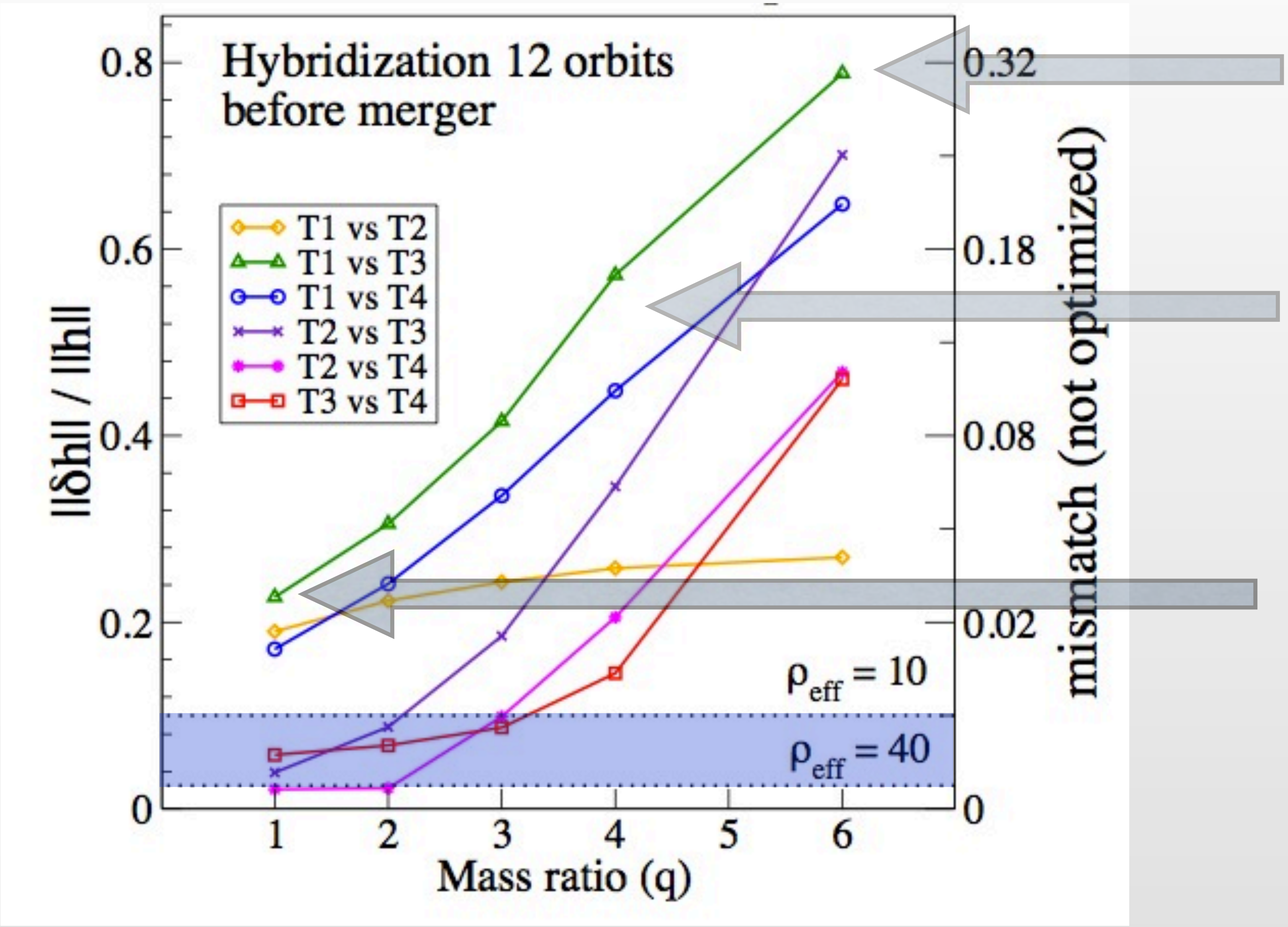
- Desire: Start NR so early that different PN versions cannot be distinguished by LIGO
- need _much_ longer NR waveforms
  - Hannam ea 2010
  - Ohme ea 2011
  - Boyle 2011
  - MacDonald ea 2011
  - Damour ea 2011



Systematic error ≲statistical error

MacDonald, Nissanke, HP 2011

Non-spinning, unequal masses



q=6, MM=0.3

q=4, MM=0.2

q=1, MM=0.03

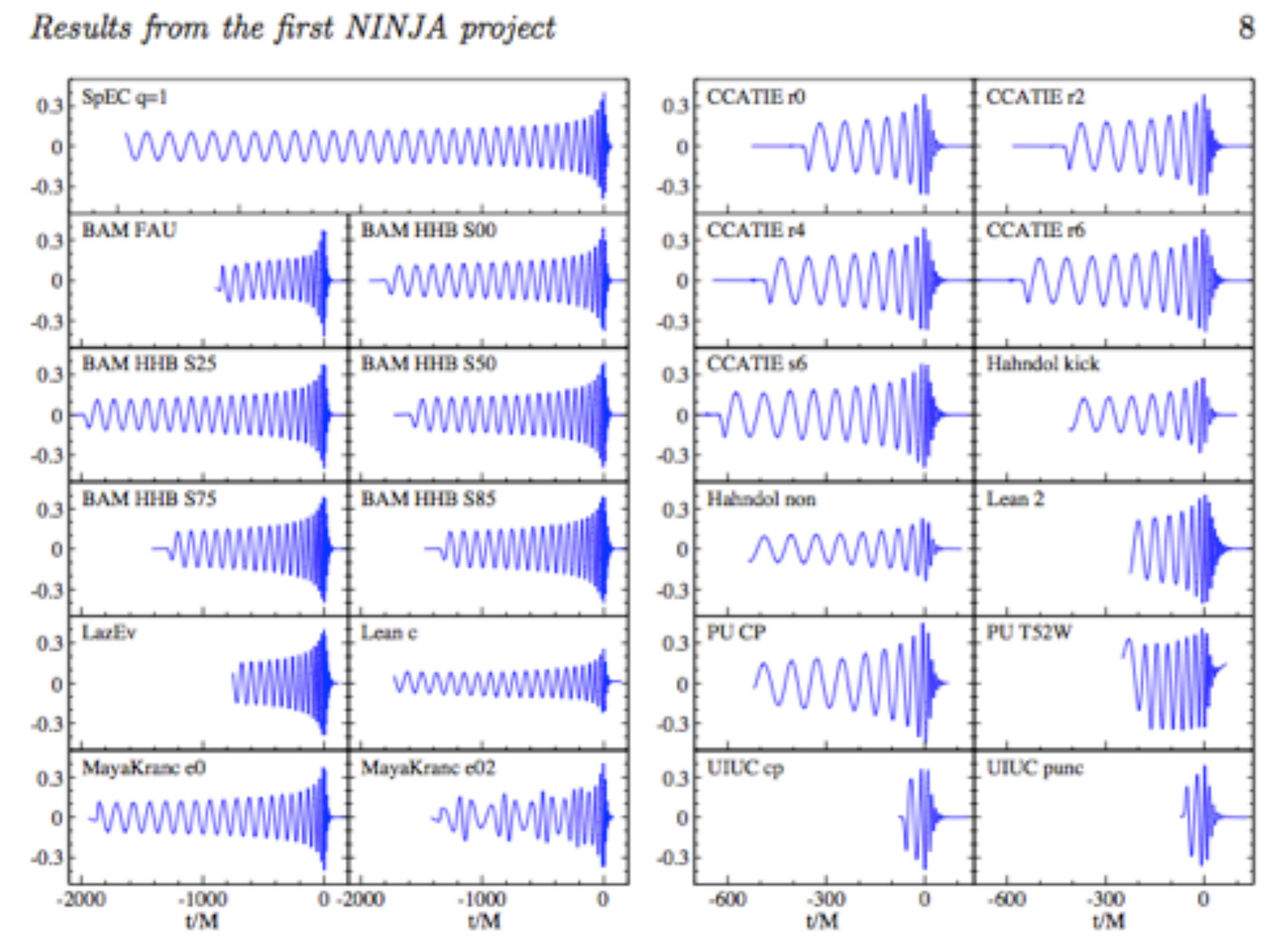
MacDonald, Mroue, HP ea 2012
(similar results in Ohme ea, 2011)

Exploring Parameter Space & Precession

Completed Waveform Catalog Efforts

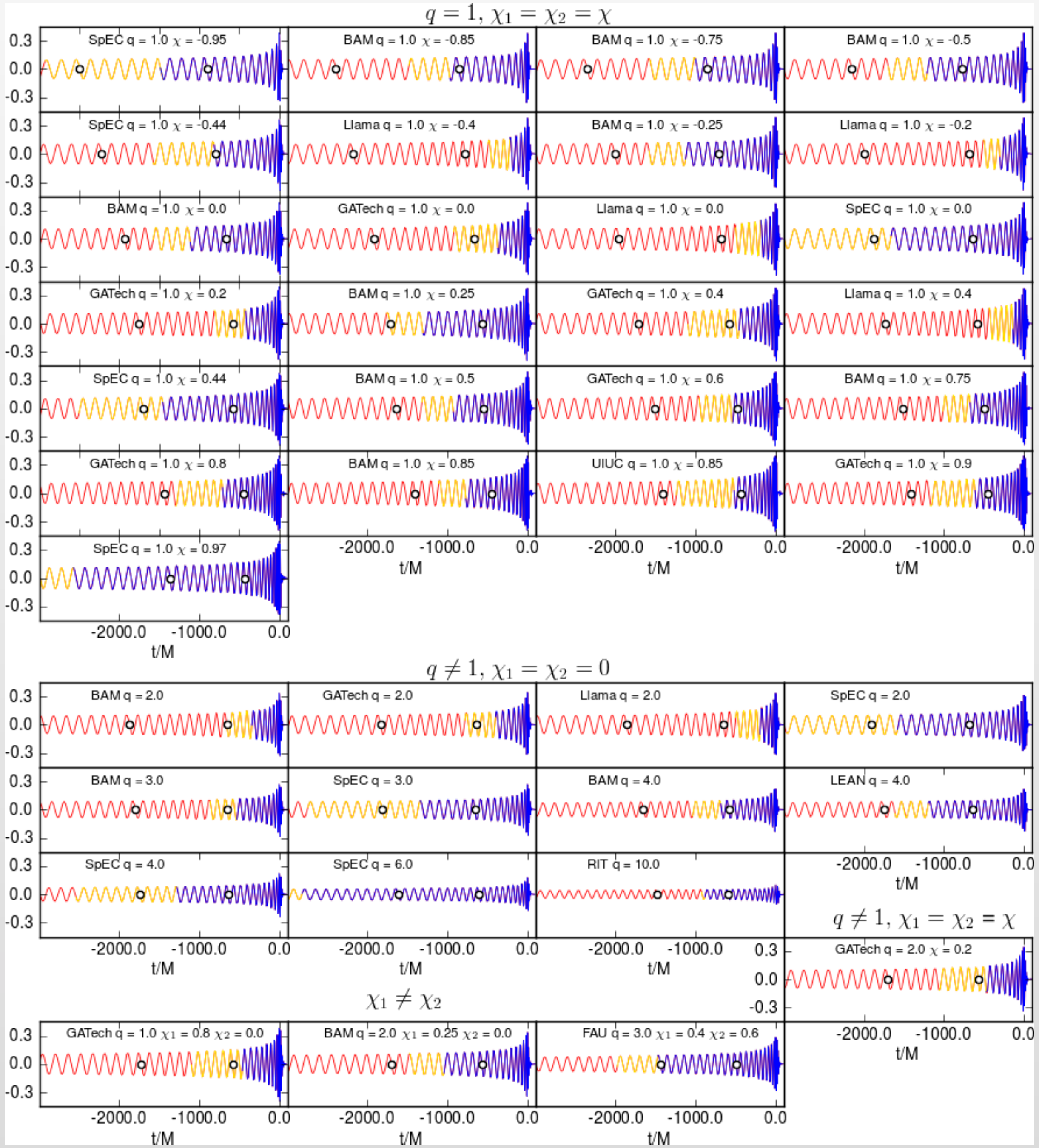
Ninja1 (2008)

Ailott + 77 co-authors



Ninja2 (2012)

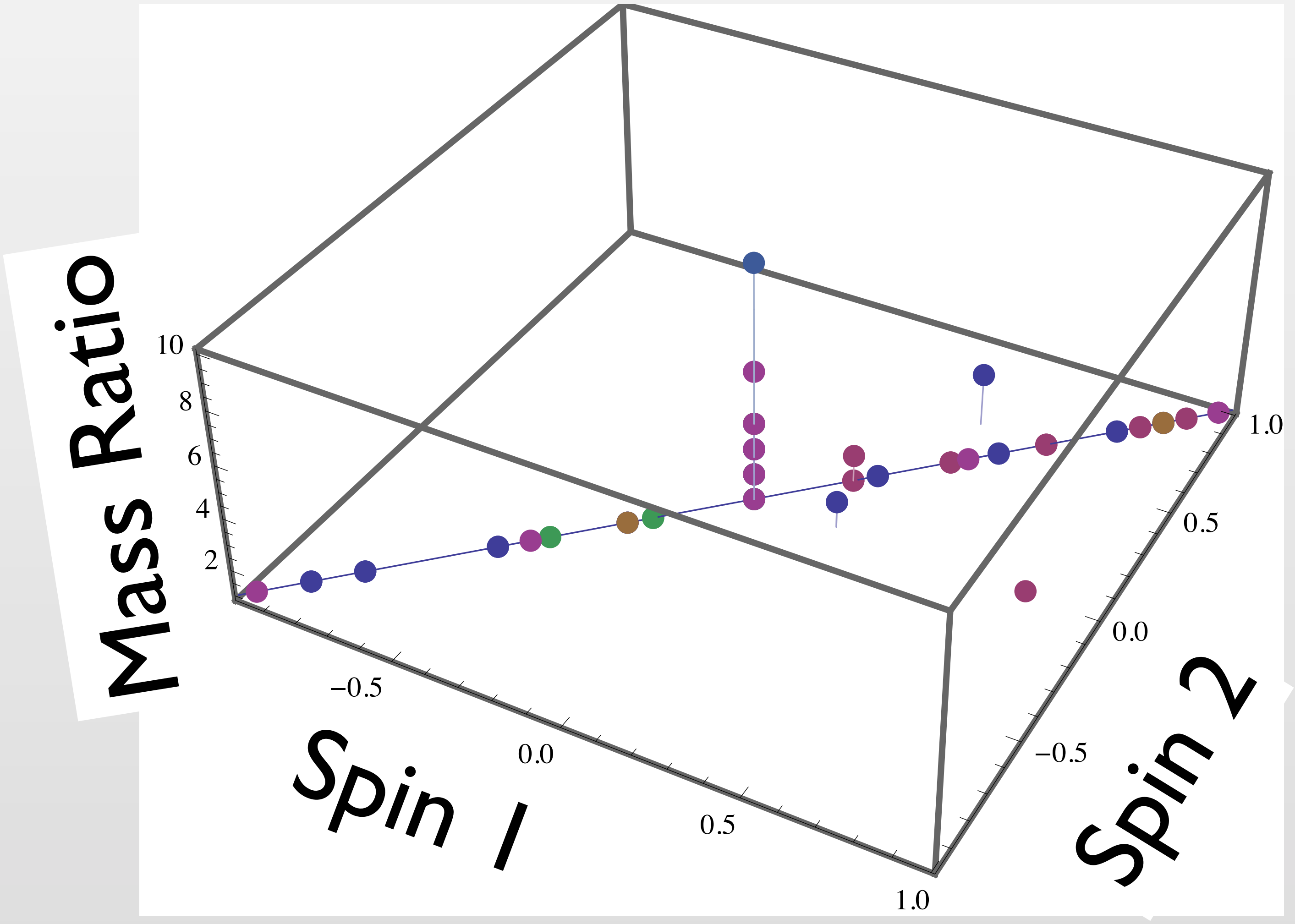
Ajith + 47 co-authors

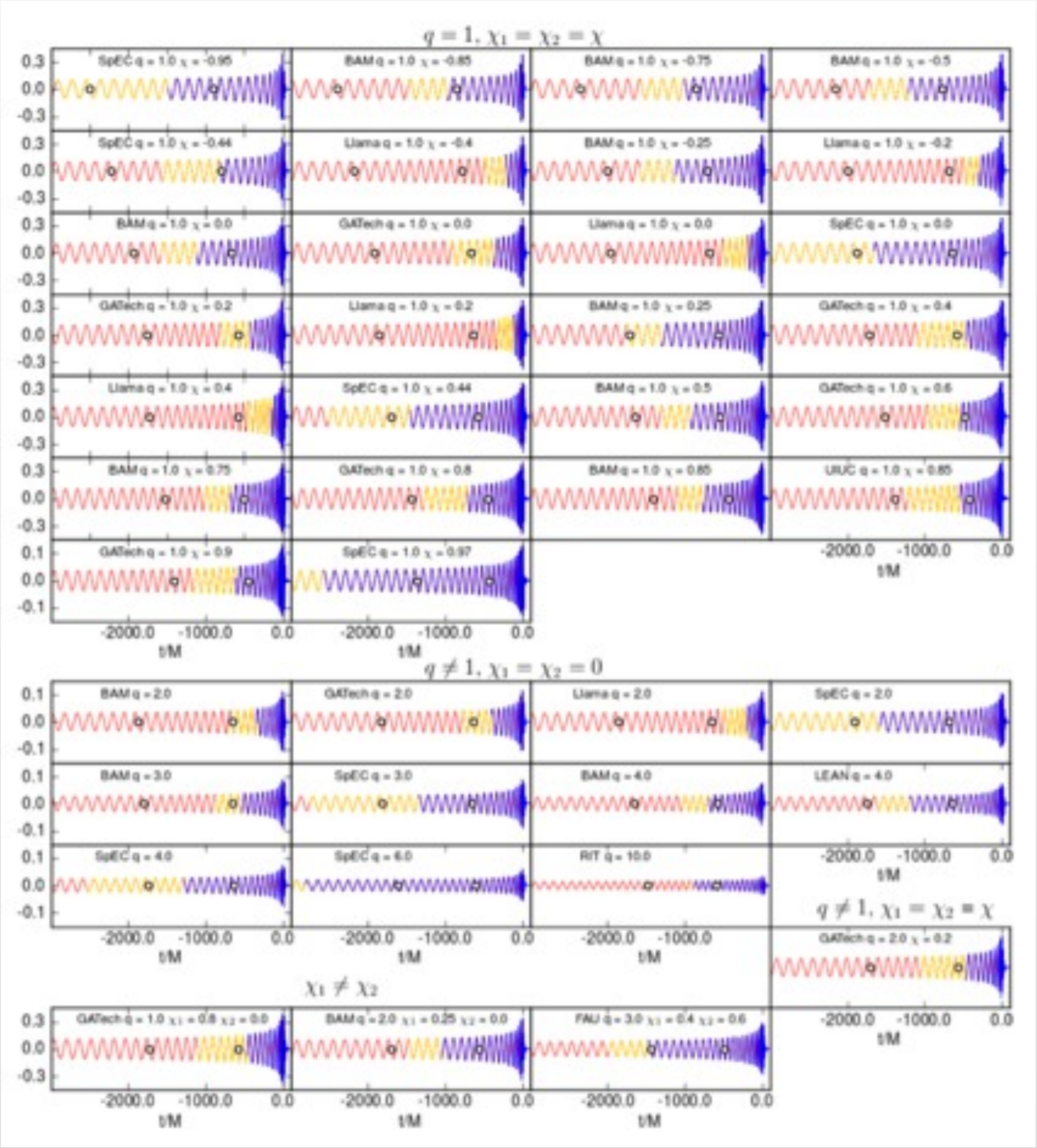


Lack of parameter space coverage

❖ BH-BH simulations are hard

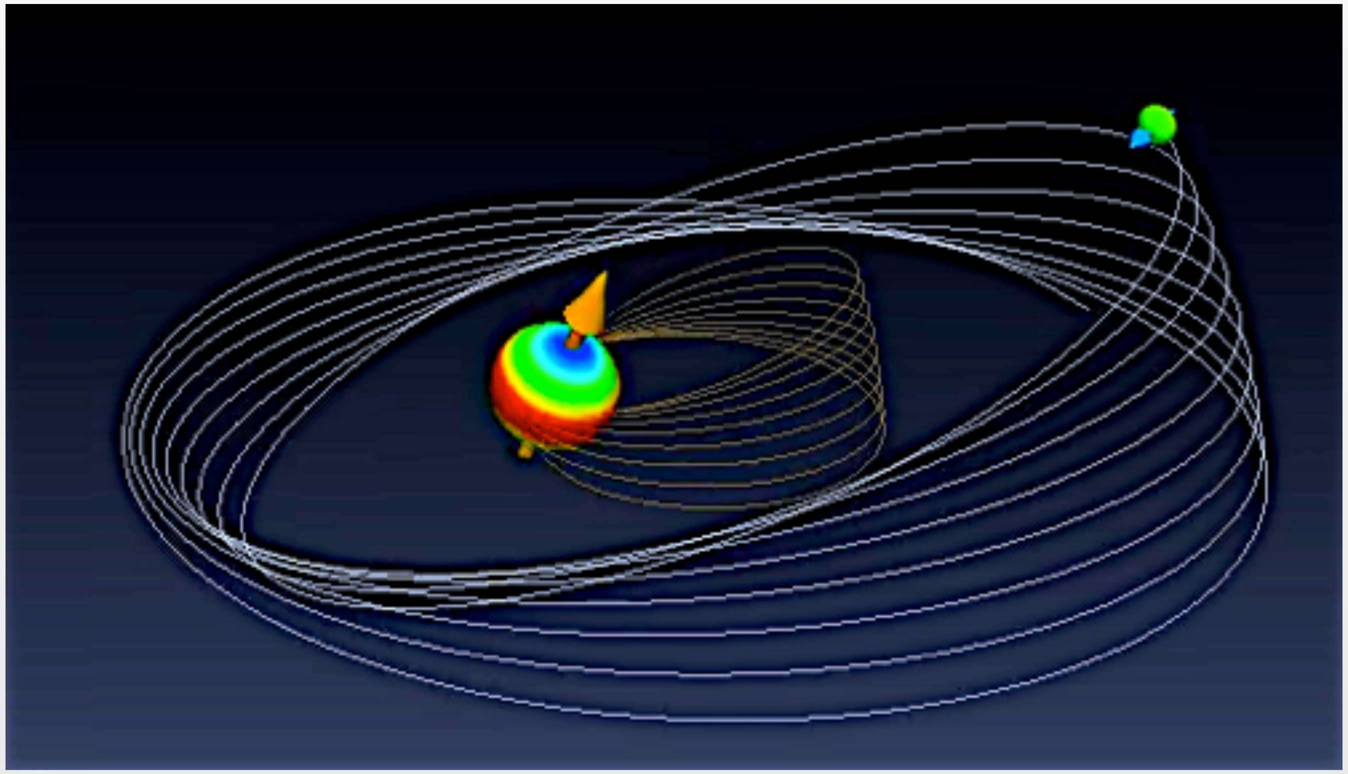
- World-wide NINJA-2 collaboration computed 40 spin-alinged systems (no precession at all)





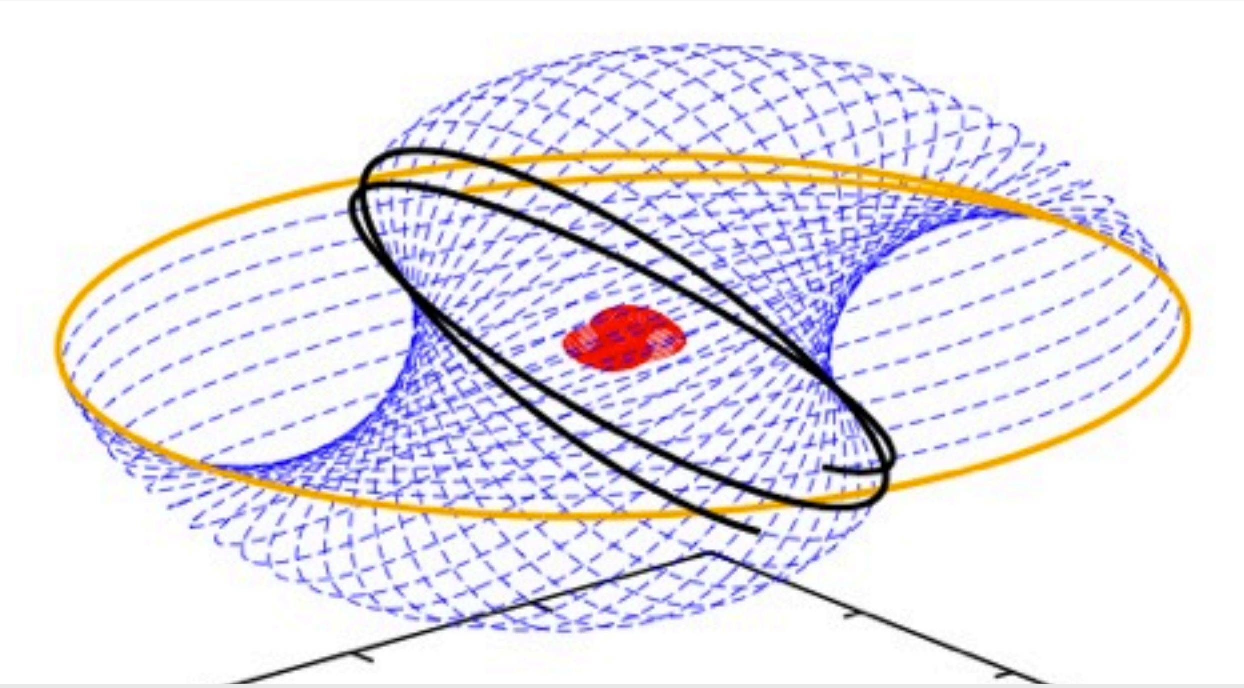
Ajith ea, 1211.5319

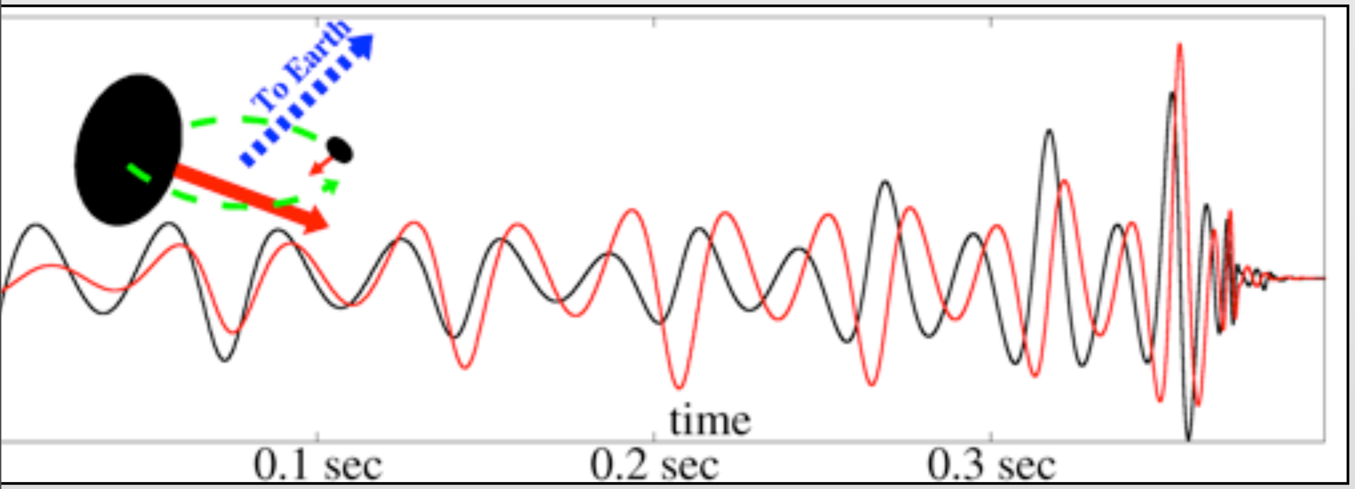
Precessing waveforms



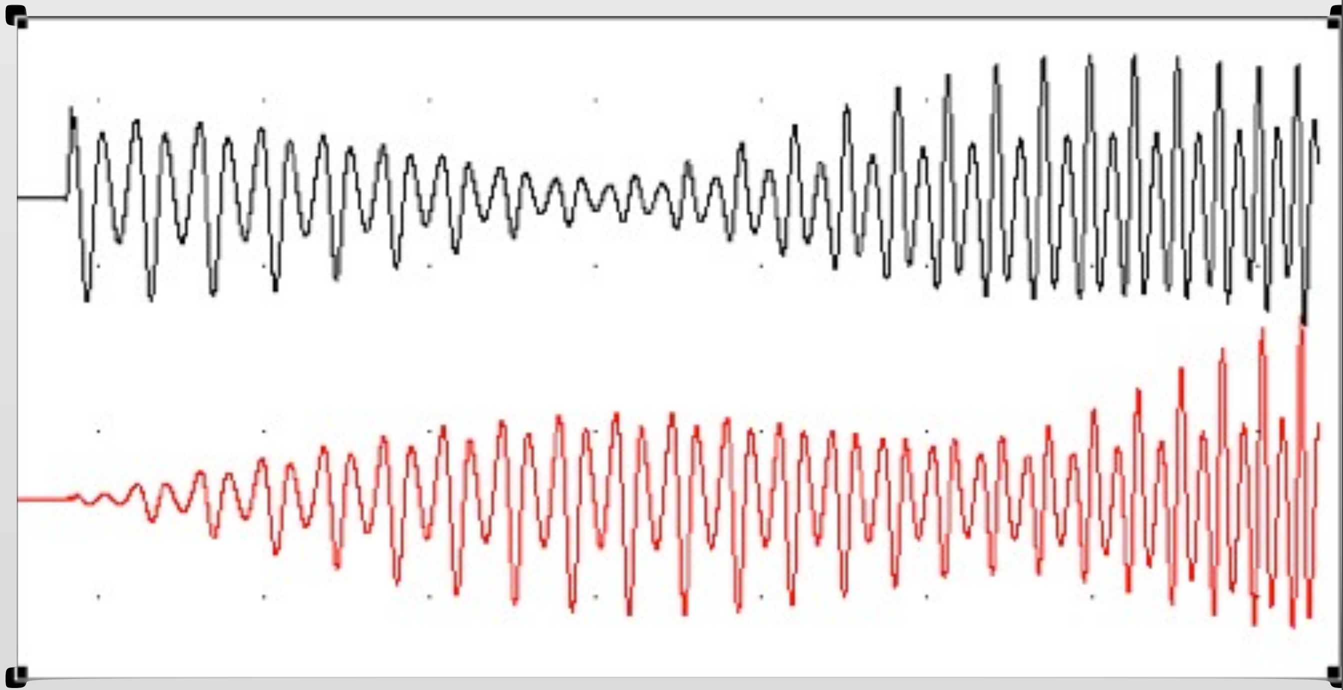
Code improvements
(Quaternions)

Ossokine, Kidder, HP 1304.3067



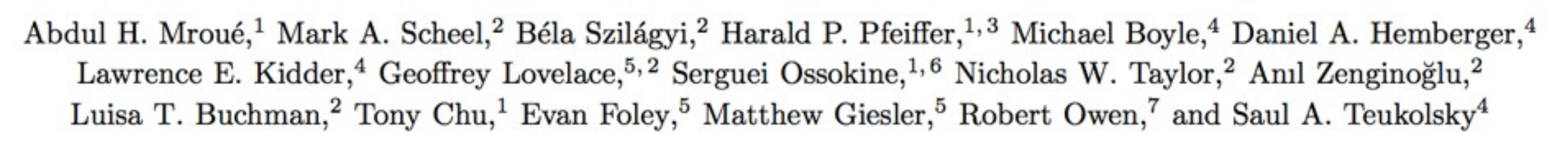


Kidder & SXS

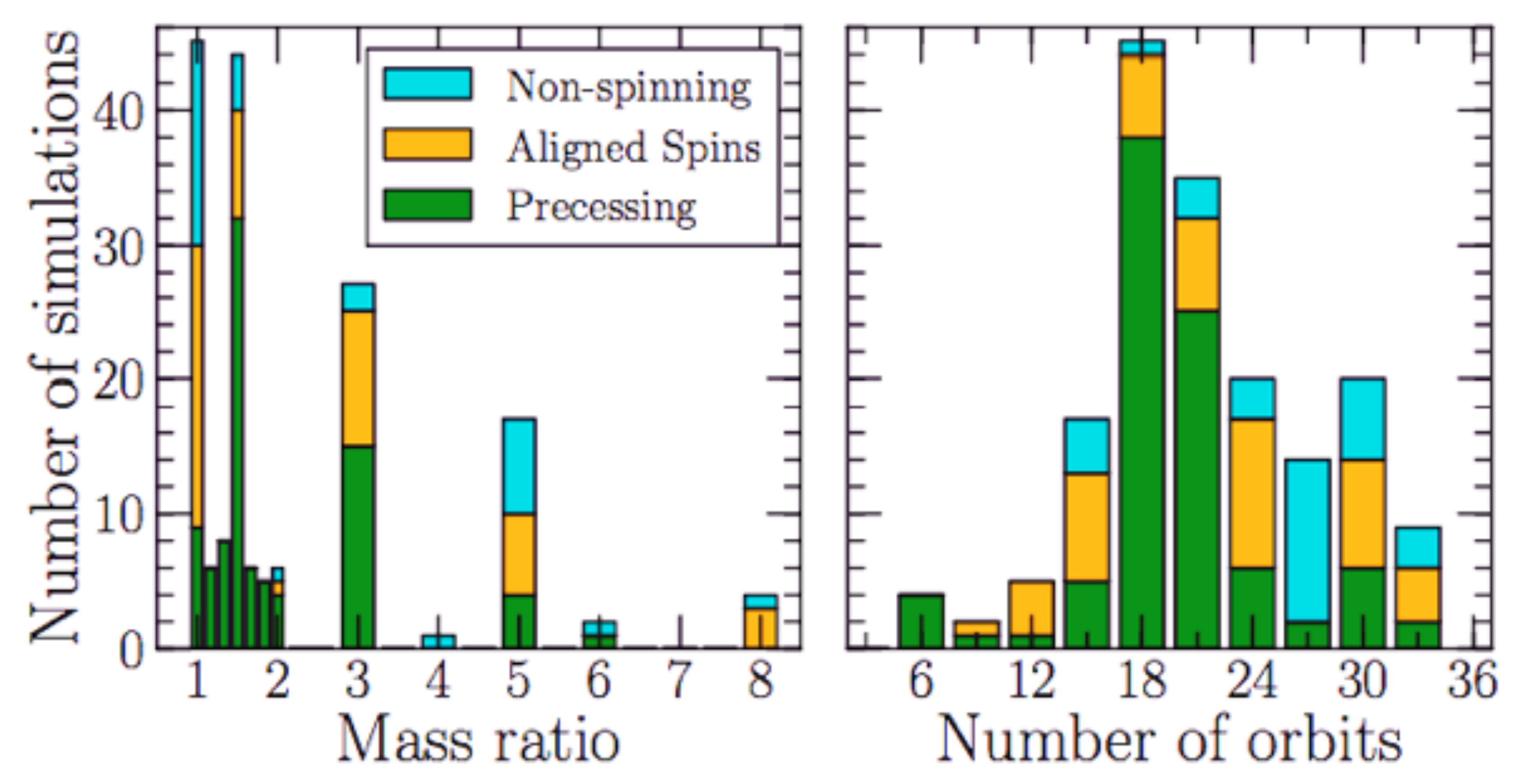


Exploring parameter space

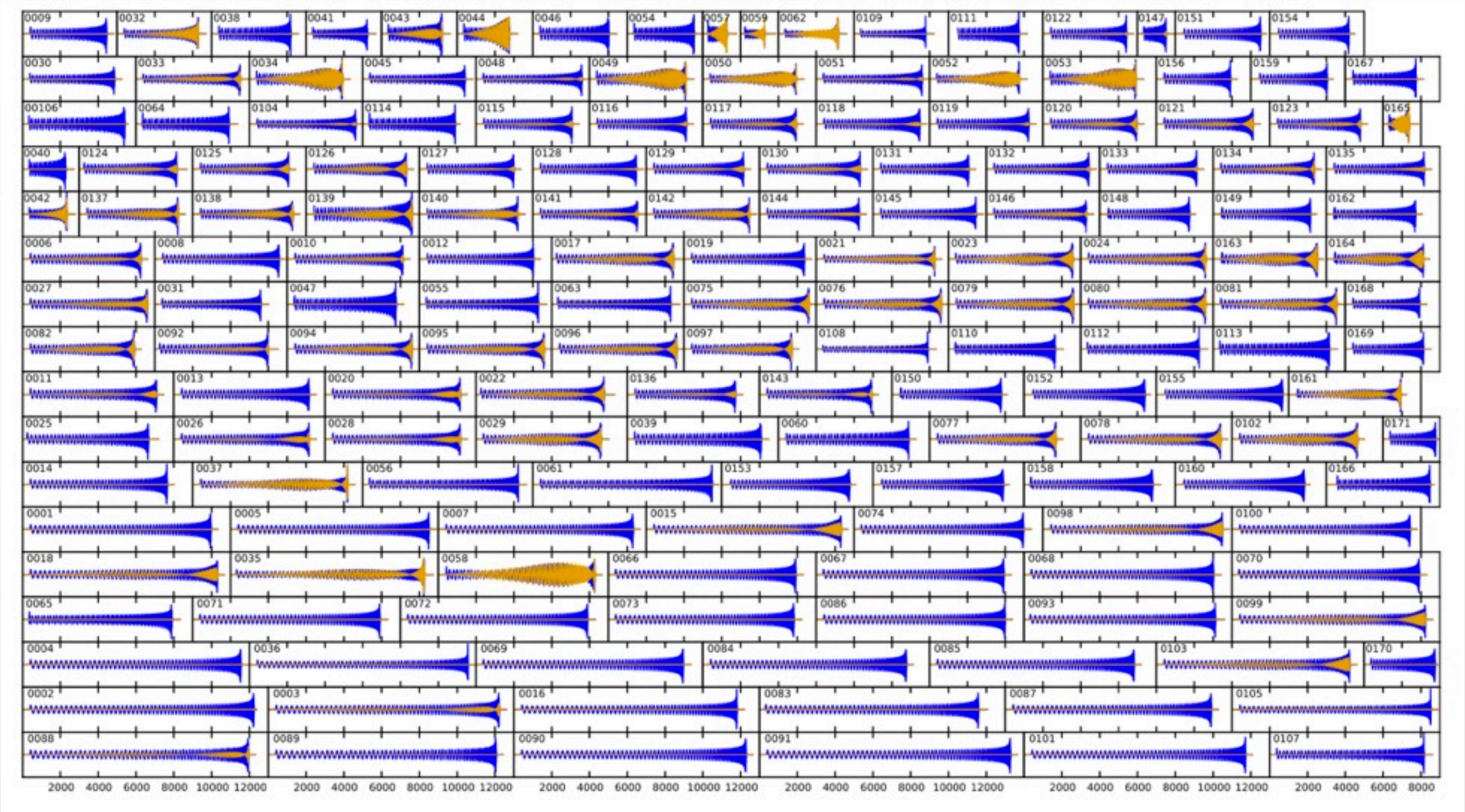
- ❖ 700 configurations quasi-circularized (Mroue, HP 1210.2958)
- ❖ 171 simulations completed
  - Mroue ea, arXiv:1304.6077



Abdul H. Mroué,[1] Mark A. Scheel,[2] Béla Szilágyi,[2] Harald P. Pfeiffer,[1,3] Michael Boyle,[4] Daniel A. Hemberger,[4] Lawrence E. Kidder,[4] Geoffrey Lovelace,[5,2] Serguei Ossokine,[1,6] Nicholas W. Taylor,[2] Anıl Zenginoğlu,[2] Luisa T. Buchman,[2] Tony Chu,[1] Evan Foley,[5] Matthew Giesler,[5] Robert Owen,[7] and Saul A. Teukolsky[4]



171 waveform catalog

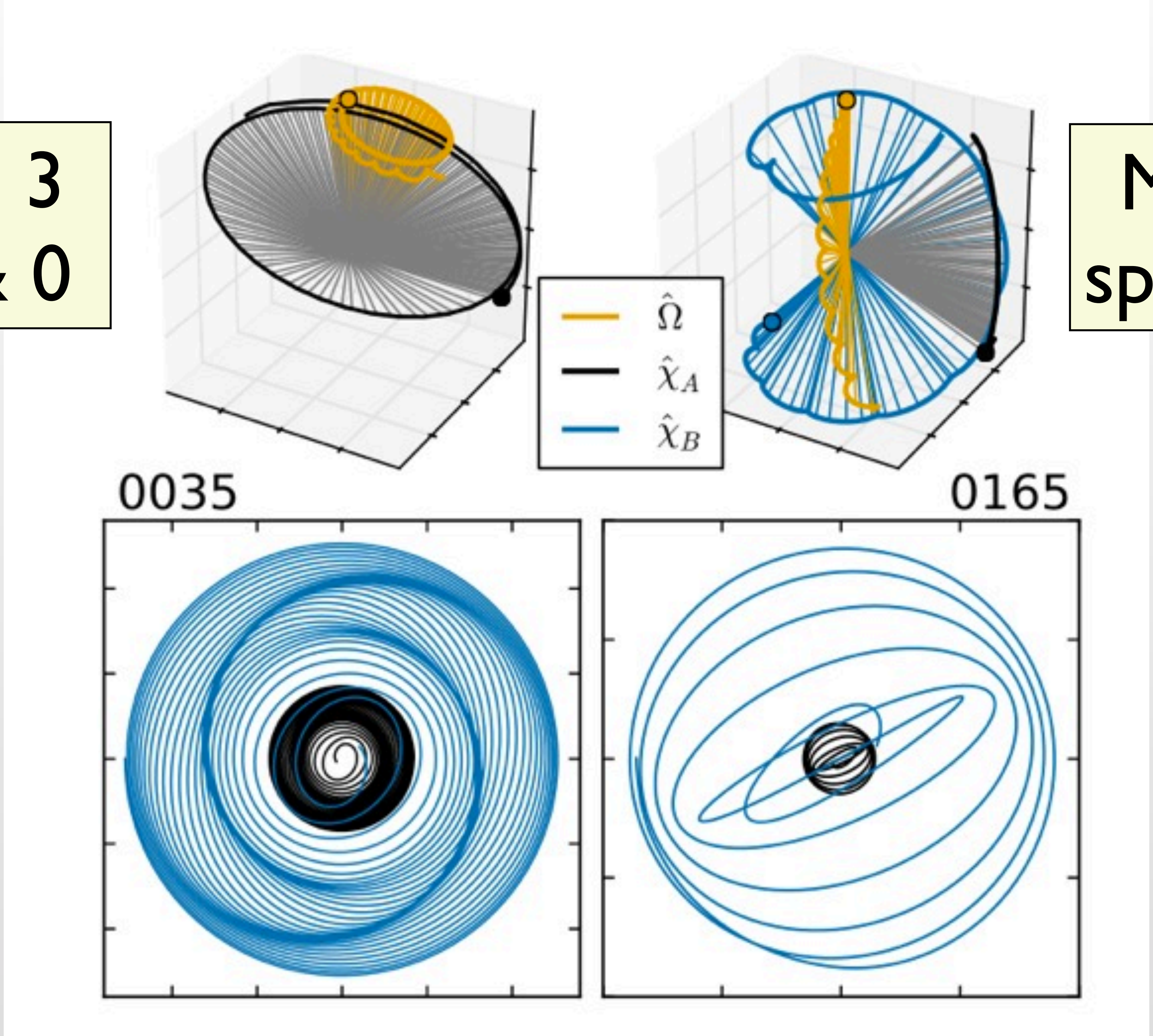


3 years, 50 Mio CPU-hours

Mroue ea, arXiv:1304.6077

Examples of precessing binaries

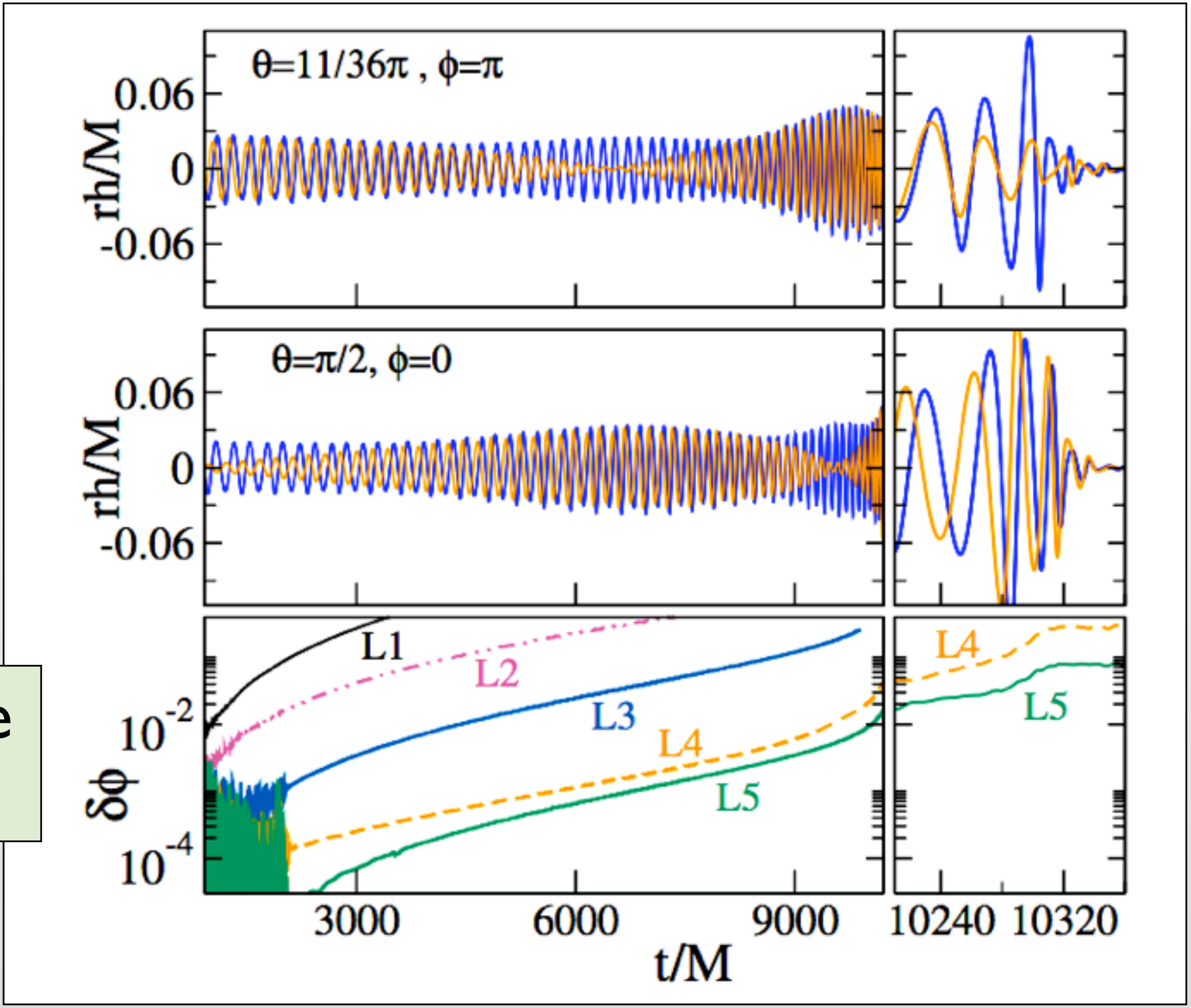
Mass-ratio 3
spins 0.5 & 0



Mass-ratio 6
spins 0.9 & 0.3

Mroue ea, arXiv:1304.6077

Orientation-dependence of waveform



Good phase accuracy

Mroue ea, arXiv:1304.6077

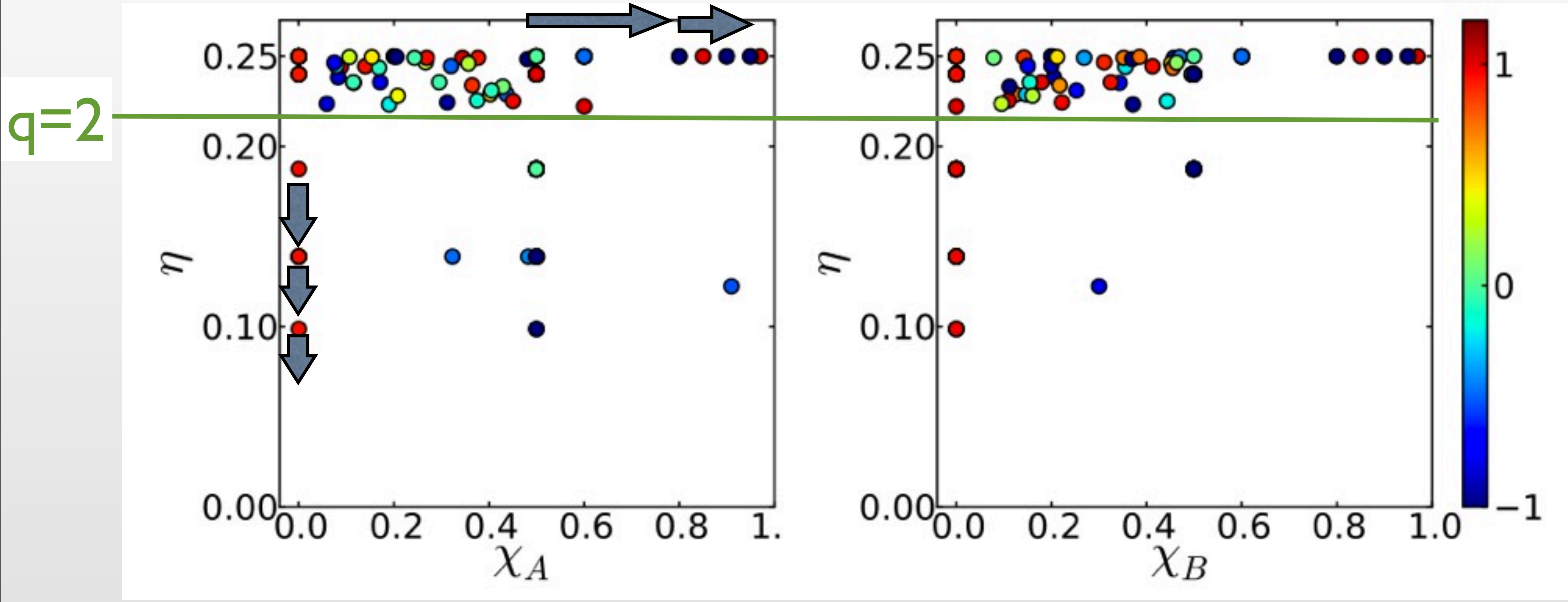
Alternative study: Georgia Tech

❖ Pekowsky ea 1304.3176, continuing sequence of papers

- 191 generic BBH waveforms
- precessing waveform = [non-precessing waveform] x [Rotation]
- IMRPhenomB fits to better than 95% for 200Msun<M<2500Msun
- At low masses, GW's can measure BH-BH properties
- At high masses, GW's can measure remnant properties

Expanding parameter space coverage

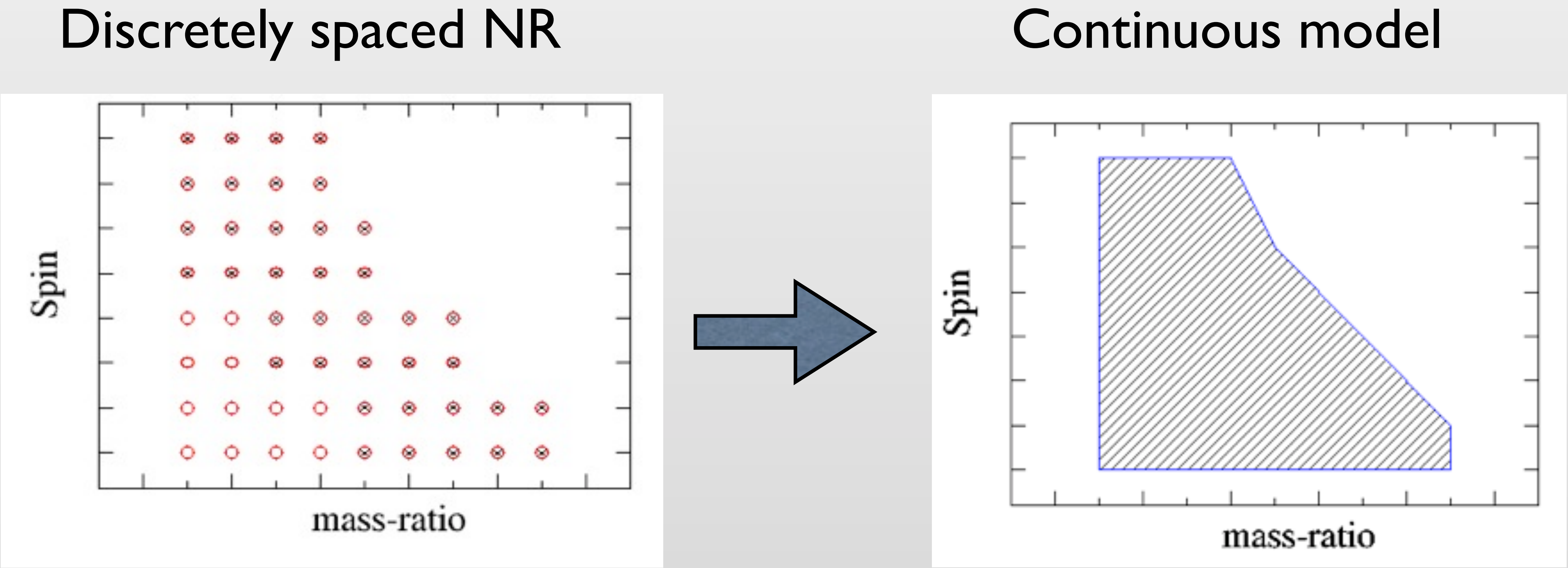
❖ Most spinning runs at q<2



❖ So far, pushing parameters was *always* difficult

- Each arrow 1-2years hard work

Waveform modeling

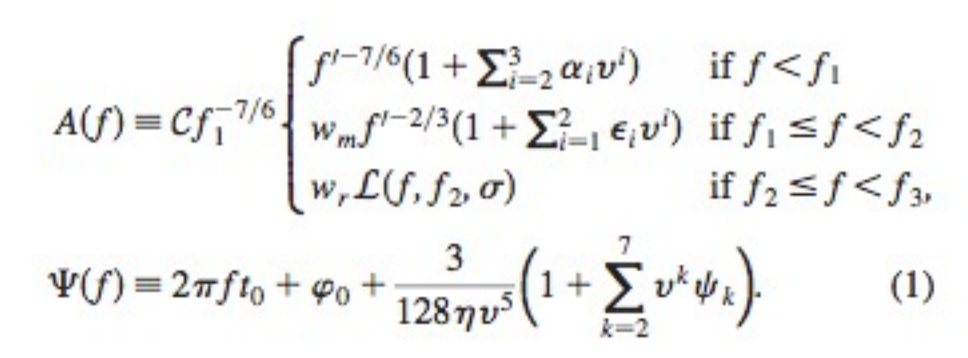


Phenomenological, aligned spins

CITA–ICAT

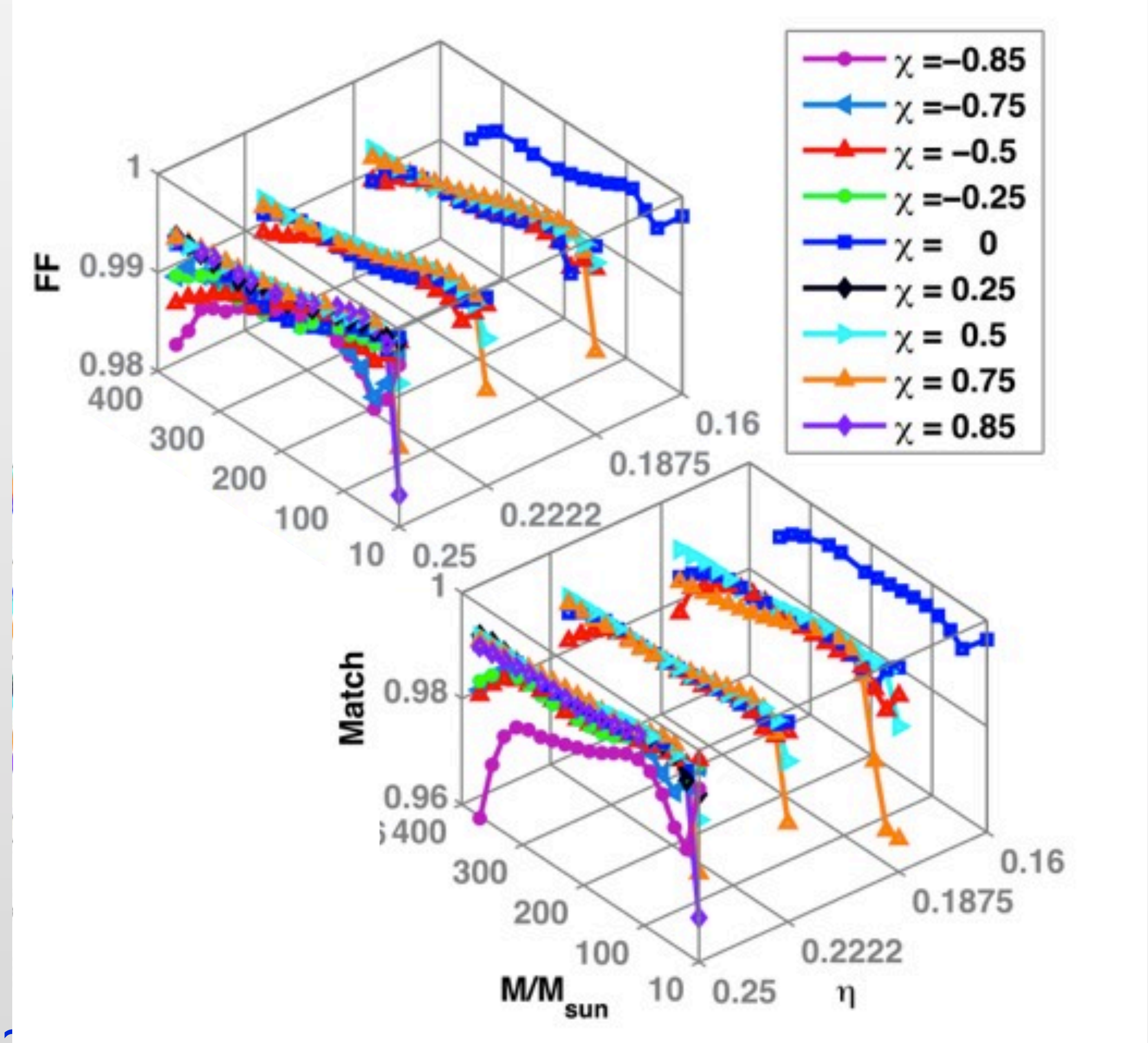
- **Unequal-mass, aligned spins (Ajith ea 2011) "IMRPhenomC"**
  - 2-dim waveform family (mass-ratio, effective spin)
  - (2,2) mode calibrated against 24 sims (BAM, Ccatie, Llama)

- **Two stages:**
  1. construct TaylorT1+NR hybrids
  2. fit model to hybrids



$$A(f) \equiv C f_1^{-7/6} \begin{cases} f'^{-7/6}(1+\sum_{i=2}^{3}\alpha_i v^i) & \text{if } f < f_1 \\ w_m f'^{-2/3}(1+\sum_{i=1}^{2}\epsilon_i v^i) & \text{if } f_1 \le f < f_2 \\ w_r \mathcal{L}(f, f_2, \sigma) & \text{if } f_2 \le f < f_3, \end{cases}$$

$$\Psi(f) \equiv 2\pi f t_0 + \varphi_0 + \frac{3}{128\eta v^5}\left(1+\sum_{k=2}^{7} v^k \psi_k\right). \qquad (1)$$



EOB + NR

- ❖ Effective one body
  - Buonanno, Damour 1999; many papers since
- ❖ Inspiral-Merger-Ringdown waveform model based on
  - Effective Hamiltonian to capture conservative dynamics

$$H = \mu\sqrt{p_r^2 + A(r)\left[1 + \frac{p_r^2}{r^2} + 2(4-3\nu)\nu\frac{p_r^4}{r^2}\right]}, \qquad A(r) = \sum_{k=0}^{4}\frac{a_k(\nu)}{r^k} + \frac{a_5(\nu)}{r^5}$$

  - Radiation reaction terms

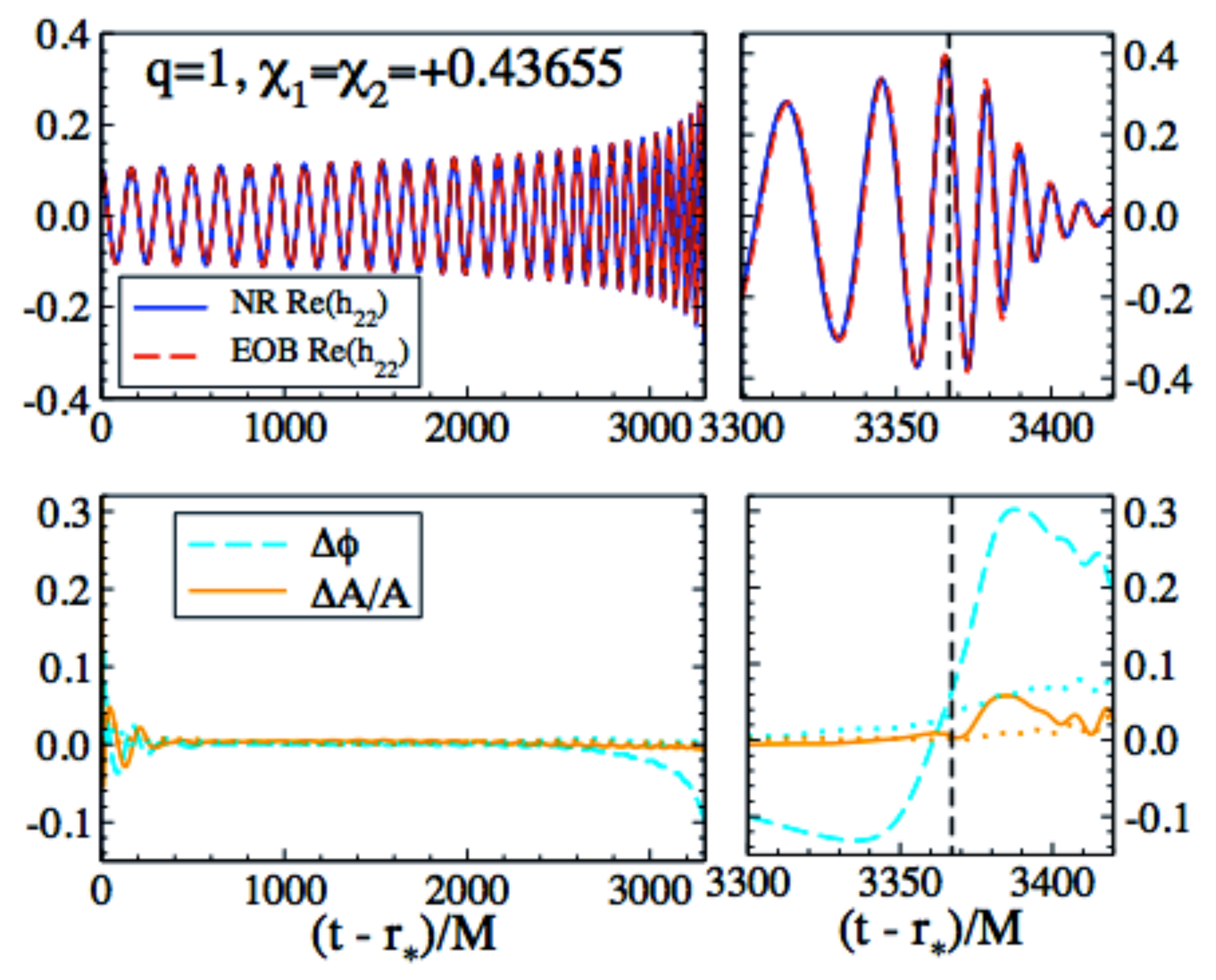
$$\frac{dp_r}{dt} = -\frac{\partial H}{\partial p_r} + a_{\rm RR}^r \frac{\dot r}{r^2\Omega}\widehat{\mathcal{F}}_\phi$$

$$\frac{dp_\varphi}{dt} = = 0 - \frac{v_\Omega^3}{\nu V_\phi^6} F_4^4(V_\phi;\nu, v_{\rm pole}), \quad \text{using 4-PN term } \mathcal{F}_{8,\nu=0} + \nu A_8$$

  - Attach ringdown modes
- ★ Fit ***parameters*** to NR simulations

EOB for aligned spins

- ❖ EOB w/ aligned spins “SEOBNRv1”
  - Taracchini ea 2012
  - (2,2) mode calibrated against
    - 7x SpEC
    - Teukolsky code
- ❖ Prototype-model:
  - Recalibration with more NR sims



Taracchini ea 2012

Precessing EOB model

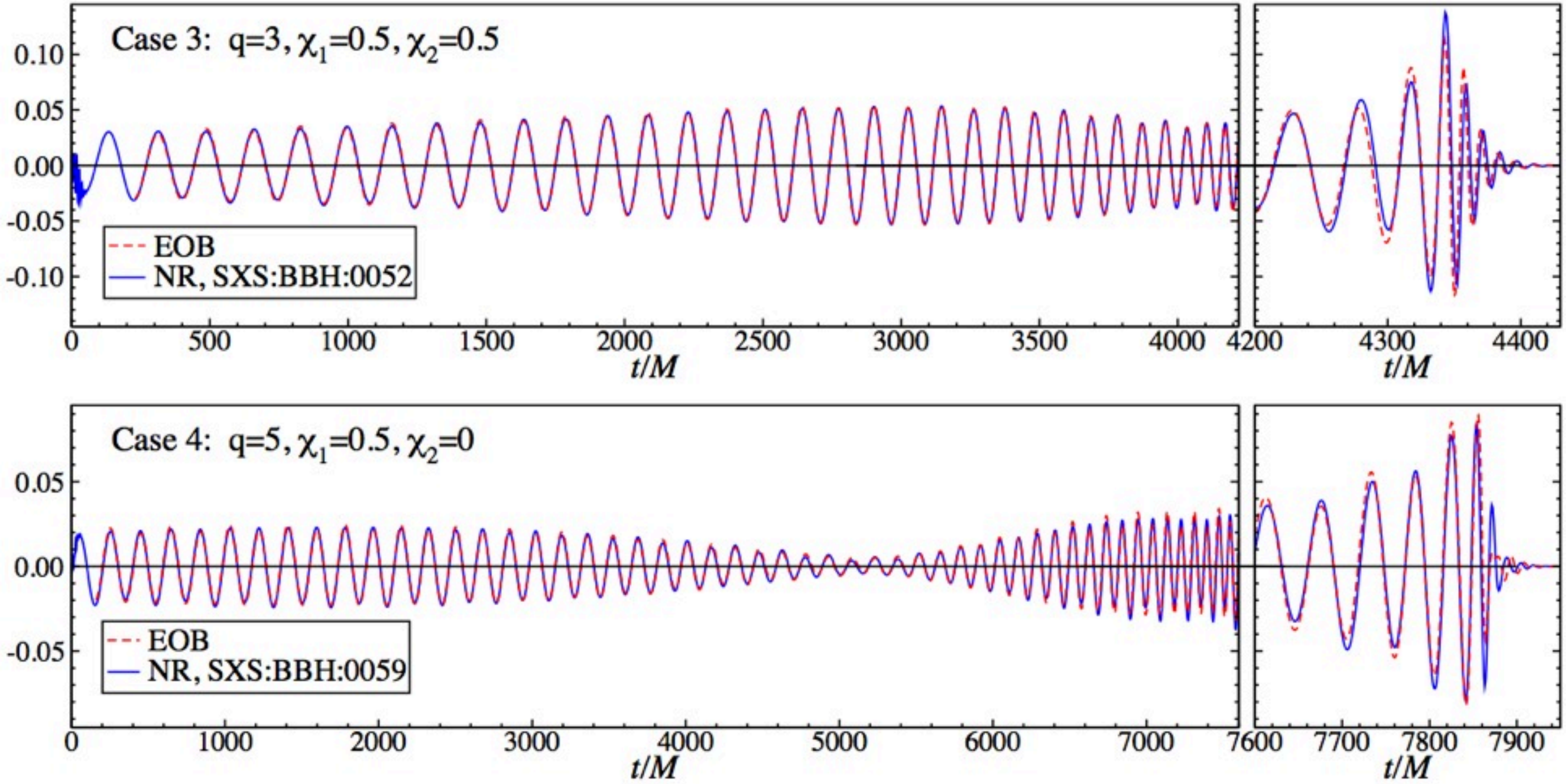
- Pan, Buonanno, Tarracchini, & SpEC (in prep)
- Evolve EOB orbital dynamics w/ precession

$$\Phi(t), \vec{L}(t), \vec{S}_1(t), \vec{S}_2(t)$$

- Compute aligned-spin SEOBNRv1 waveforms
- Rotate into precessing frame

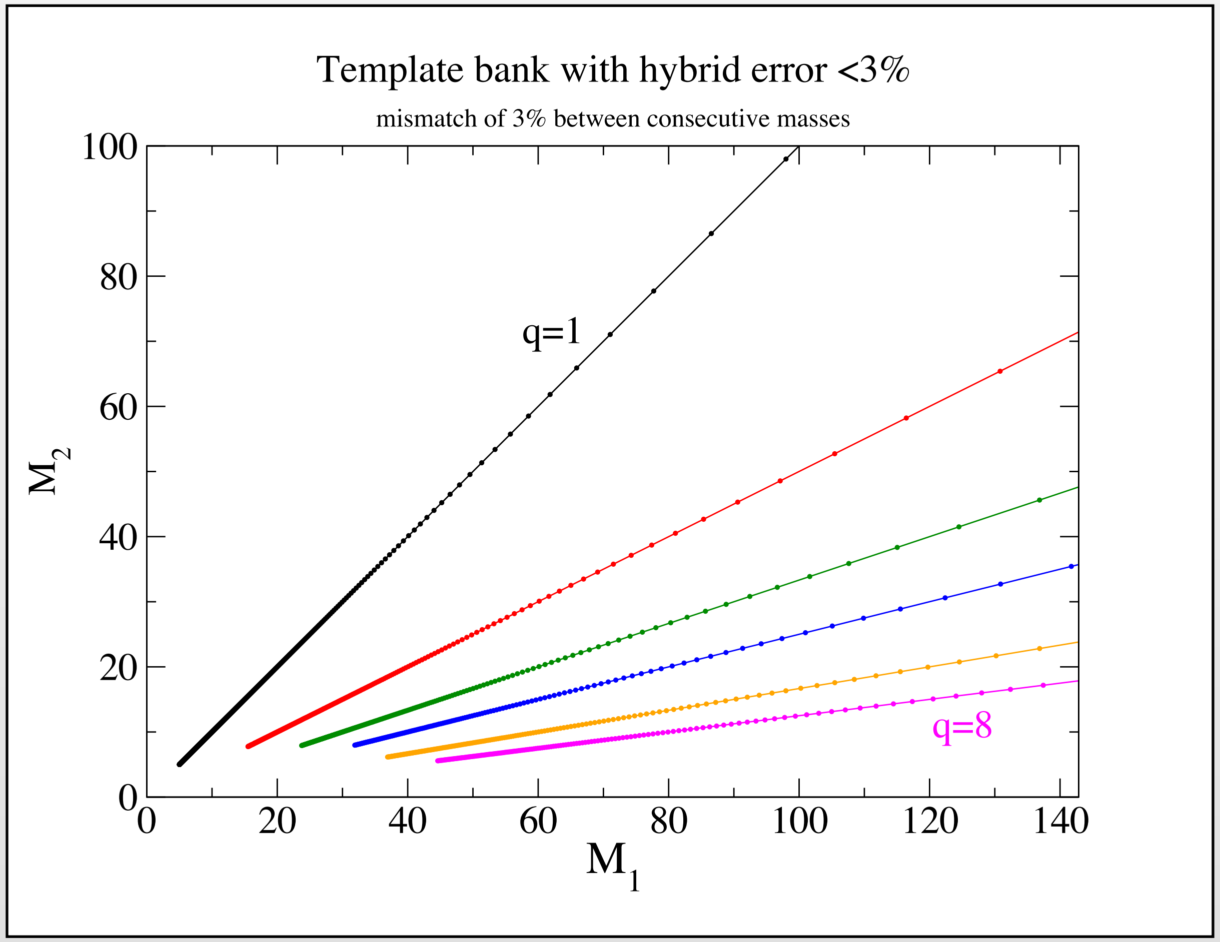
- Note: EOB tuned only with
  - Non-spinning q=1,2,3,4,6 and Teukolsky code
  - Equal-mass, aligned spins: $(\chi_1, \chi_2)=(0.44,0.44), (-0.44, 0.44)$

Precessing EOB model: NR comparison

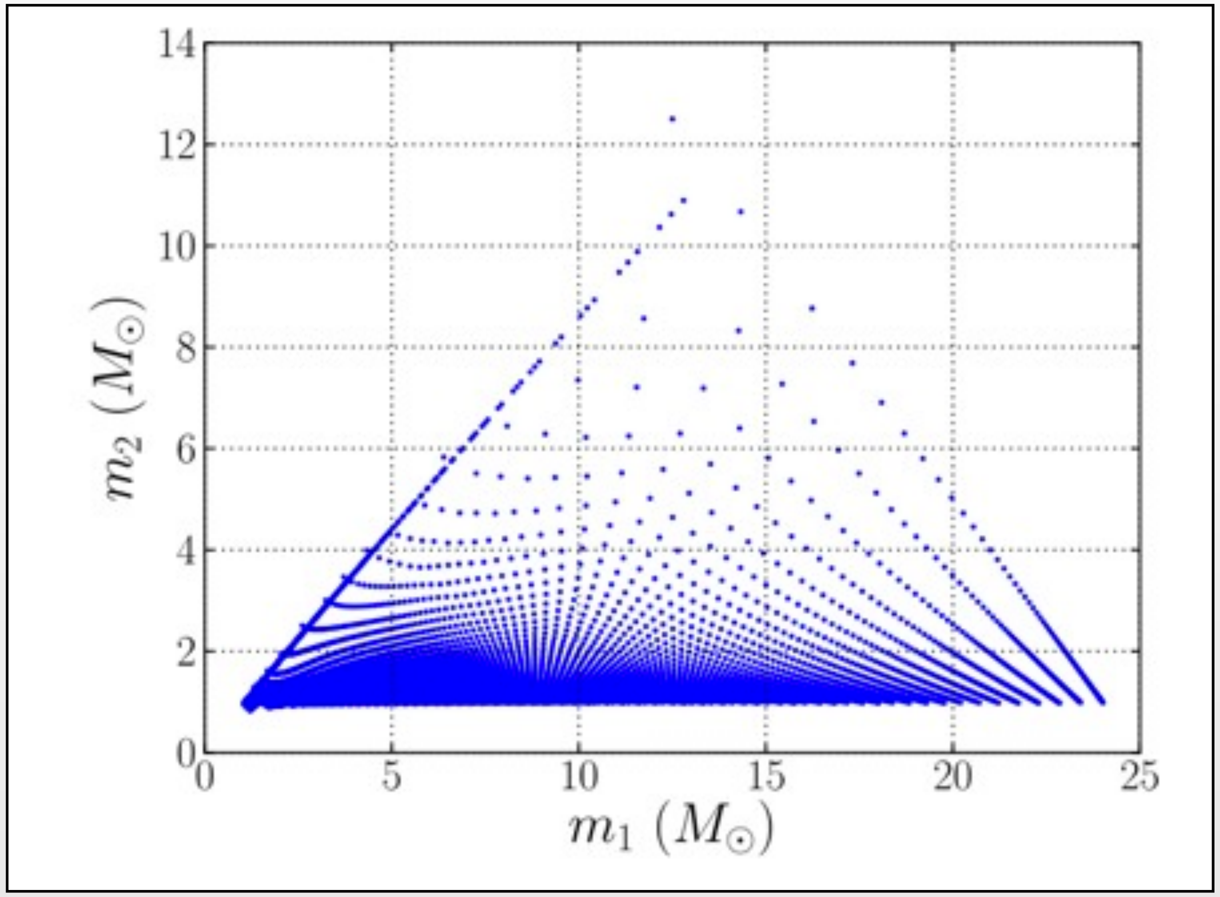


Purely NR template bank

- BH-BH waveforms
  scale invariant



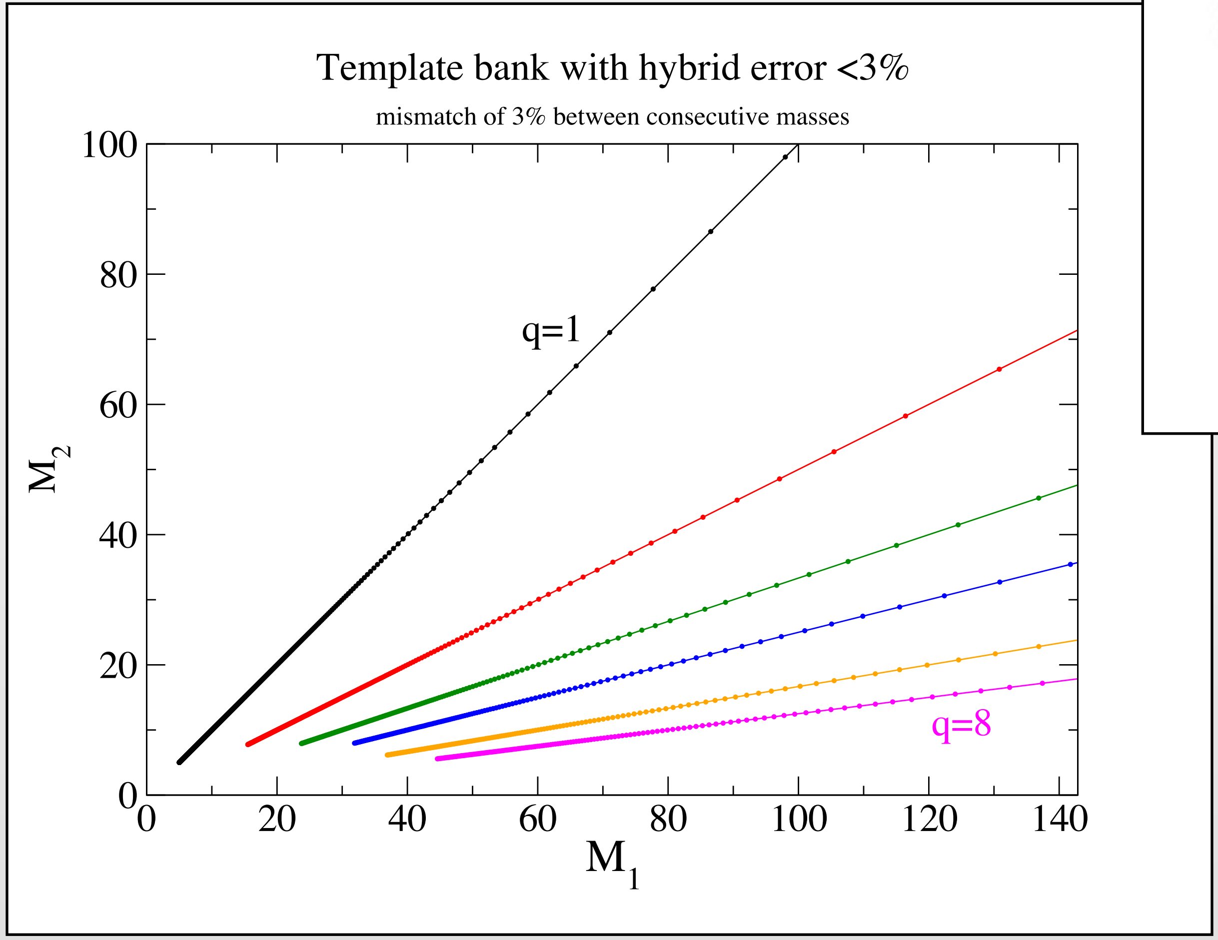
NR-only template bank
MacDonald



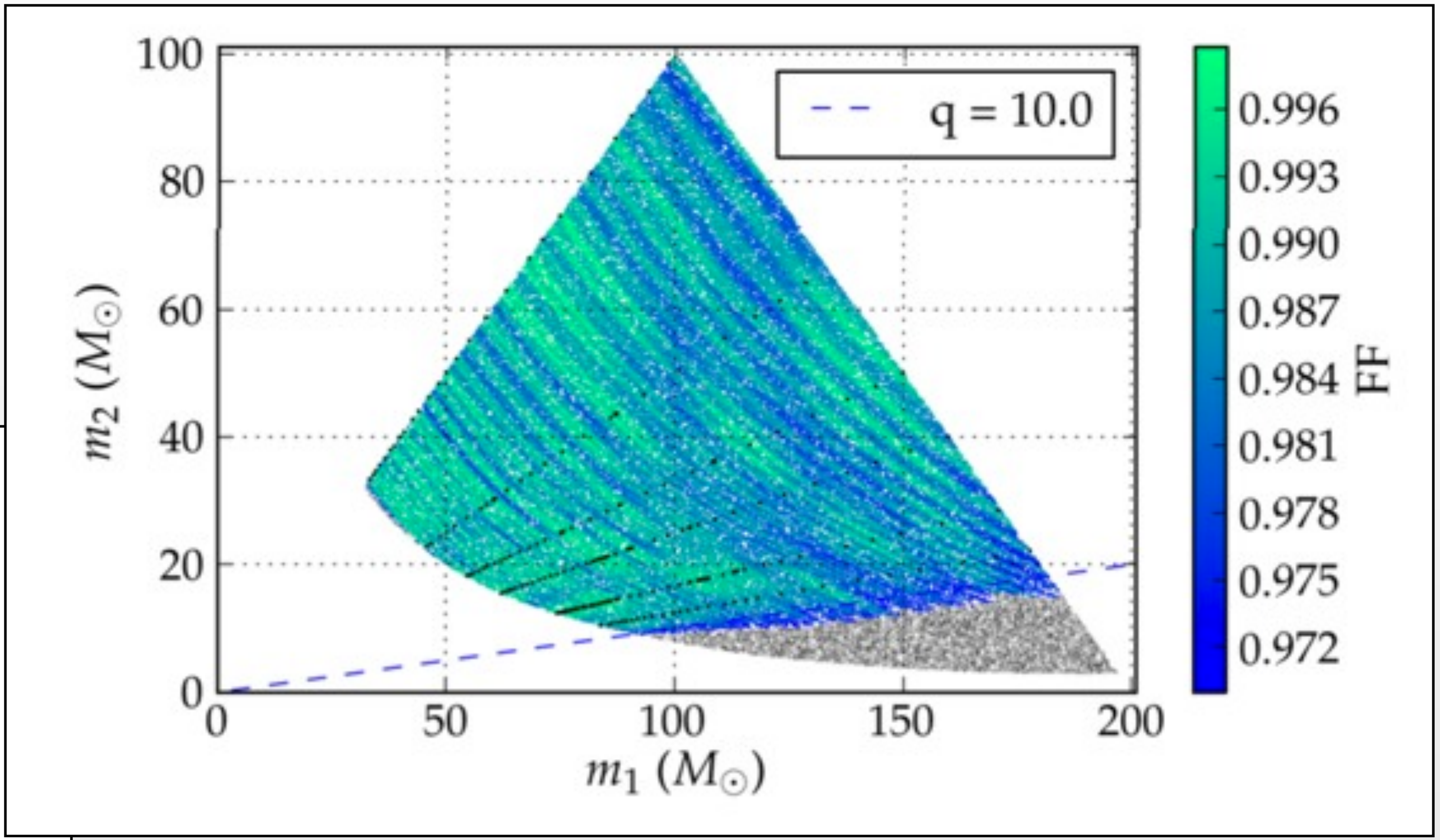
Standard template bank

Purely NR template bank

- BH-BH waveforms
  scale invariant



NR-only template bank
MacDonald



With 26 NR runs, each 40 orbits:
Cover M>12Msun
Kumar, MacDonald, ea (in prep)

The end. Thank you!