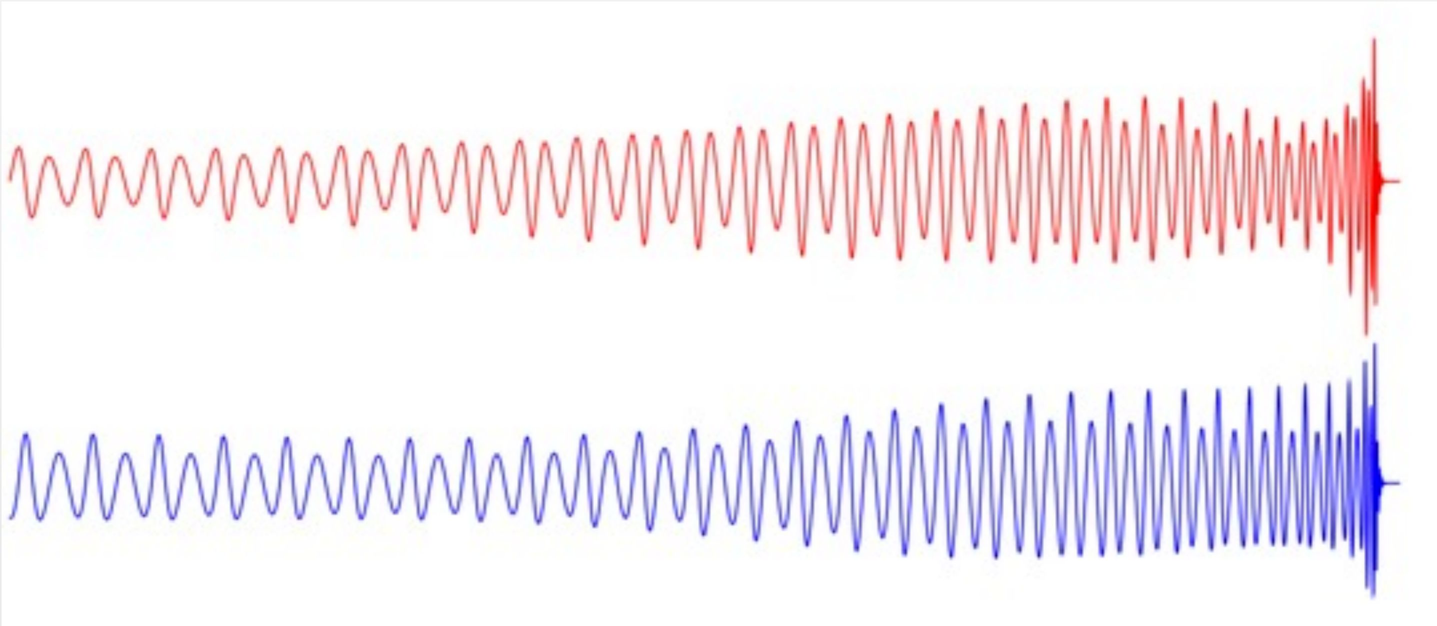
Precessing NR simulations

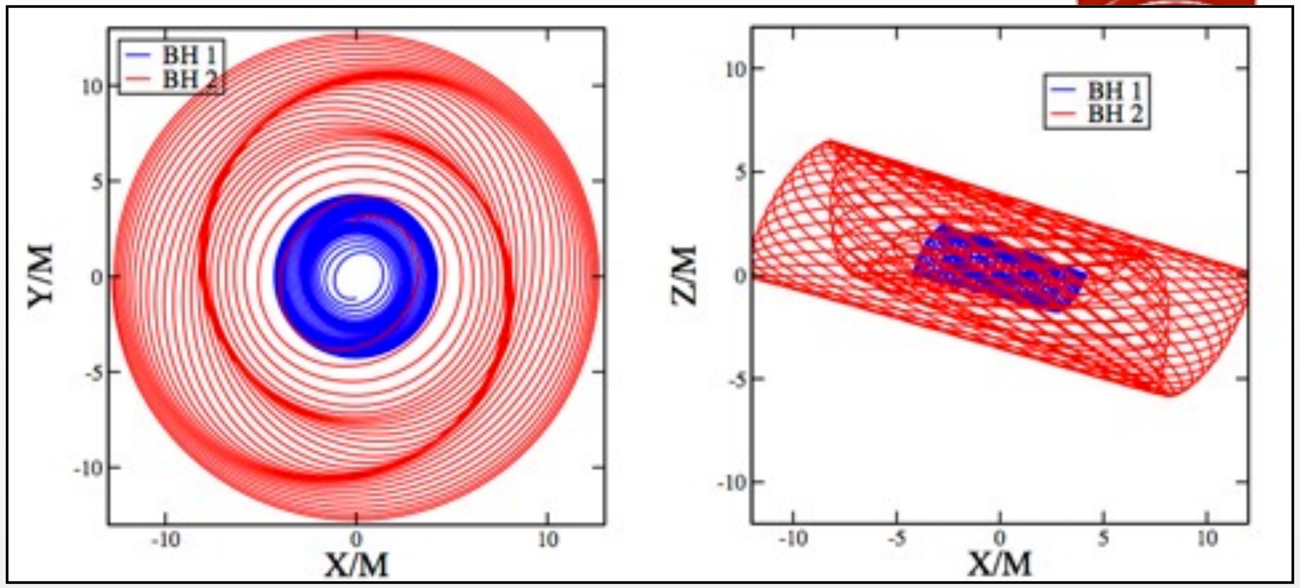
Harald Pfeiffer, CITA

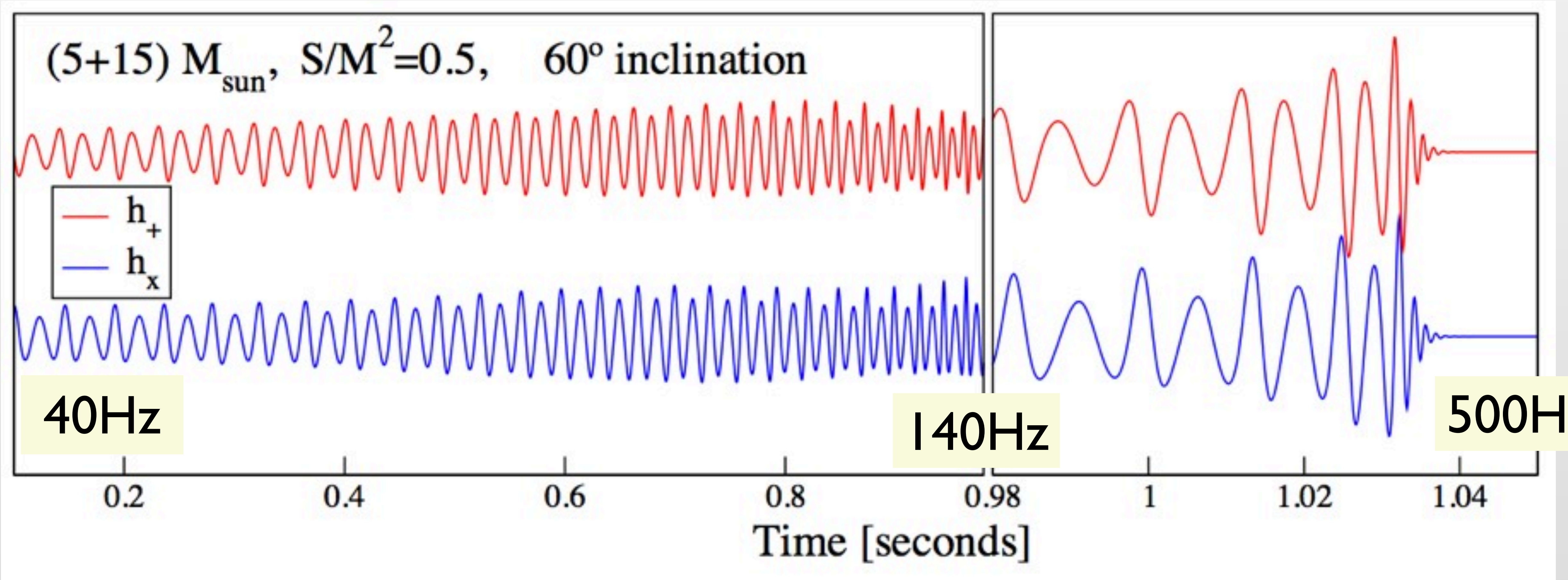
ICTS Program on Numerical Relativity
ICTS/TIFR Bengaluru
June 26, 2013



Today's goal:

- ❖ How does one compute these?
- ❖ Where are we in terms of parameter space exploration?





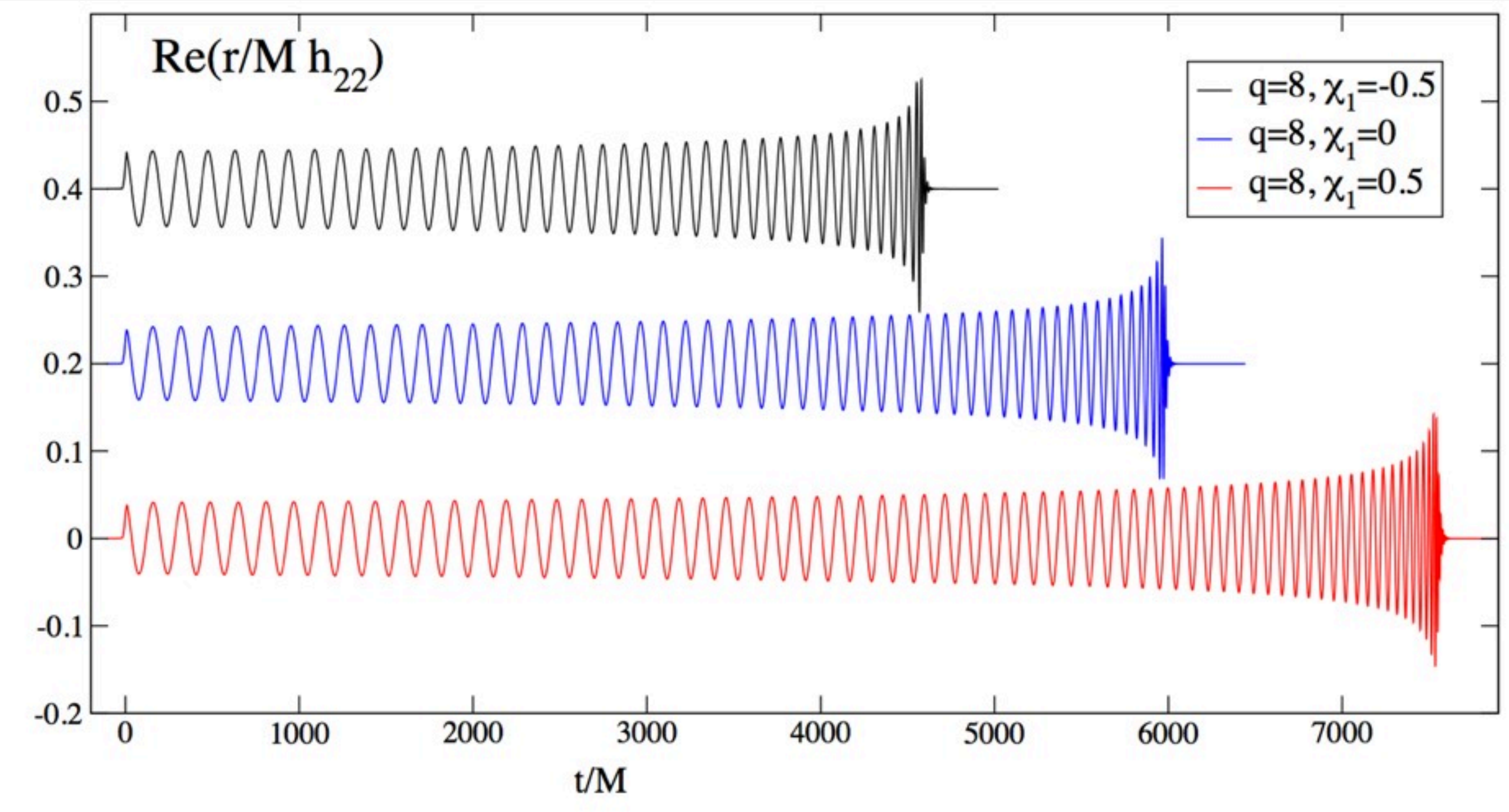
Abdul Mroue (CITA)

Special feature

per Sathya's request

Spinning NR-AR comparison

- SpinTaylorT4 w/ 2.5 PN spin and WRONG χ_2=+-0.5
  - Thanks, Ajith!
- Numerical Relativity: q=8, χ_1=-0.5 / 0 / +0.5. χ_2=0



q=8 spinning PN-NR comparison

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❖ PRELIMINARY: COMPARISON PERFORMED THIS MORNING

❖ Procedure

- Extract φ(t) from time-domain waveform

$$h_{22}(t) = A(t) \exp(-i\phi(t))$$

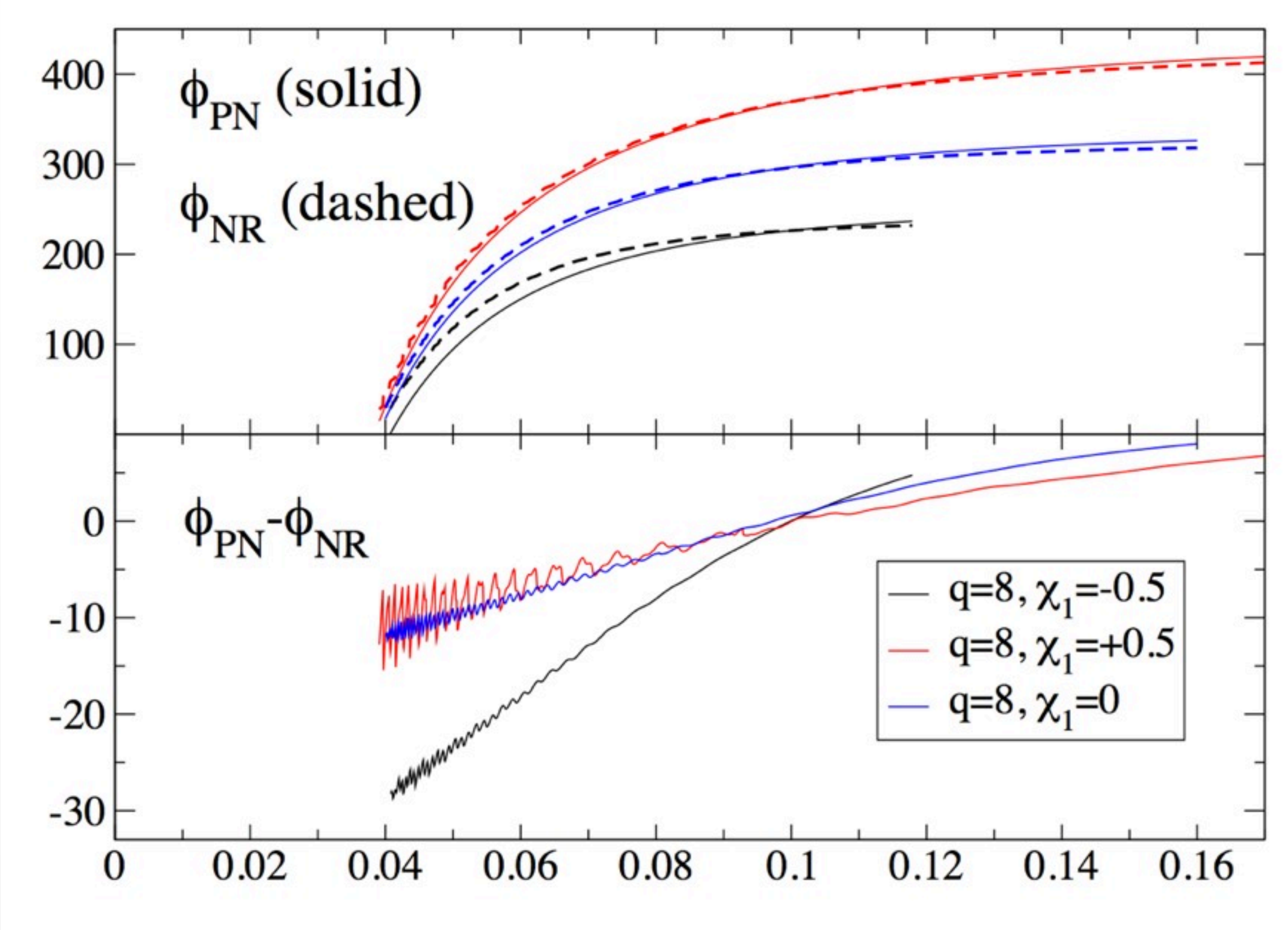
- Compute frequency

$$\omega(t) = \frac{\phi(t)}{dt}$$

- Eliminate time t, to yield

$$\phi(\omega)$$

PRELIMINARY RESULTS



Background NR

SXS collaboration

***S**imulation of e**X**treme **S**pacetimes*

- Goal: Simulate compact object binaries to satisfy LIGO's data-analysis needs
- Cornell, Caltech, CITA, WashU, Fullerton, Oberlin
- Work presented here involves:

<u>Numerics:</u> L. Buchman[1], T. Chu[2], L. Kidder[3], S. Lau[4], G. Lovelace[5], A. Mroue[2], S. Ossokine[2], R. Owen[6], M. Scheel[1], B. Szilagyi[1], N. Taylor[1], S. Teukolsky[2]
<u>Analysis:</u> M. Boyle[3], D. Brown[7], A. Buonanno[8], I. MacDonald[1], S. Nissanke[1], Y. Pan[8], A. Taracchini[8]

1 Caltech, 2 CITA, 3 Cornell, 4 Albuquerque, 5 Fullerton, 6 Oberlin, 7 Syracuse, 8 Maryland

Techniques I: Generalized Harmonic

- Einstein's equations

$$0 = R_{ab}[g_{ab}] = -\frac{1}{2}\Box g_{ab} + \nabla_{(a}\Gamma_{b)} + \text{lower order terms}, \qquad \Gamma_a = -g_{ab}\Box x^b.$$

- Generalized harmonic coordinates $g_{ab}\Box x^b \equiv H_a(x^a, g_{ab})$
  (Friedrich 1985, Pretorius 2005; $H = 0$ used since 1920's)

$$\Box g_{ab} = \text{lower order terms.}$$

$\Rightarrow$ Constraint $C_a \equiv H_a - g_{ab}\Box x^b = 0$

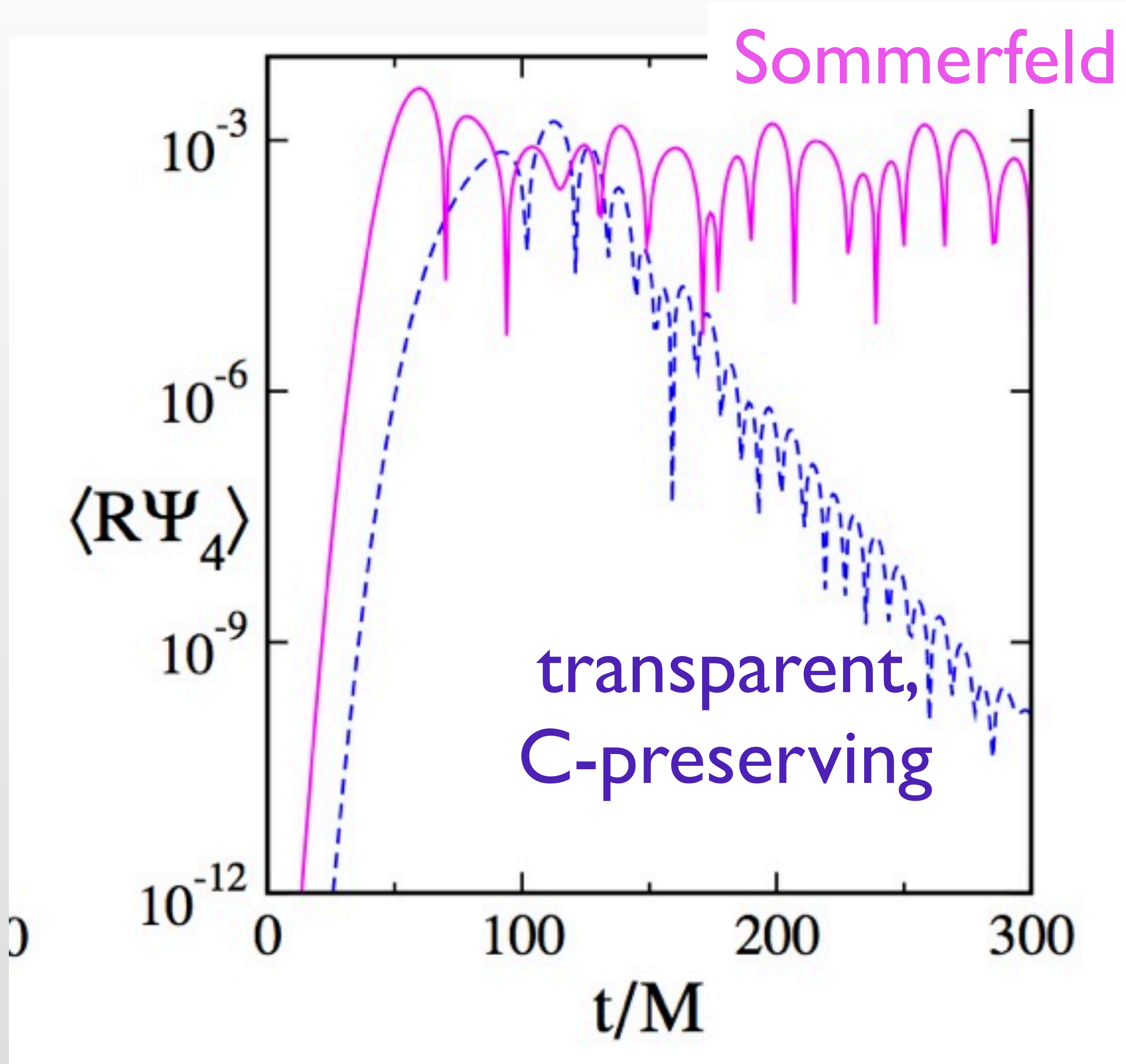
- Constraint damping (Gundlach, et *al.*, Pretorius, 2005)

$$\Box g_{ab} = \gamma \left[t_{(a}C_{b)} - \frac{1}{2} g_{ab} t^c C_c \right] + \text{lower order terms}$$

$$\partial_t C_a \sim -\gamma C_a.$$

II. Boundary conditions

- ❖ Constraint preserving
- ❖ Nearly transparent to outgoing radiation



Lindblom, Scheel, Kidder, Owen, Rinne 2006

III: Spectral methods

- Expand in basis-functions, solve for coefficients

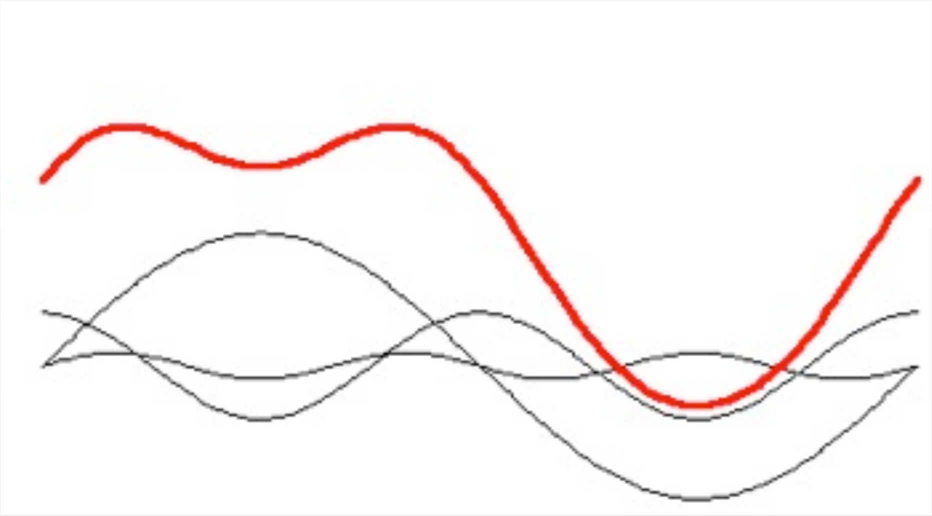
$$u(x,t) = \sum_{k=1}^{N} \tilde{u}(t)_k \Phi_k(x)$$

- Compute derivatives analytically

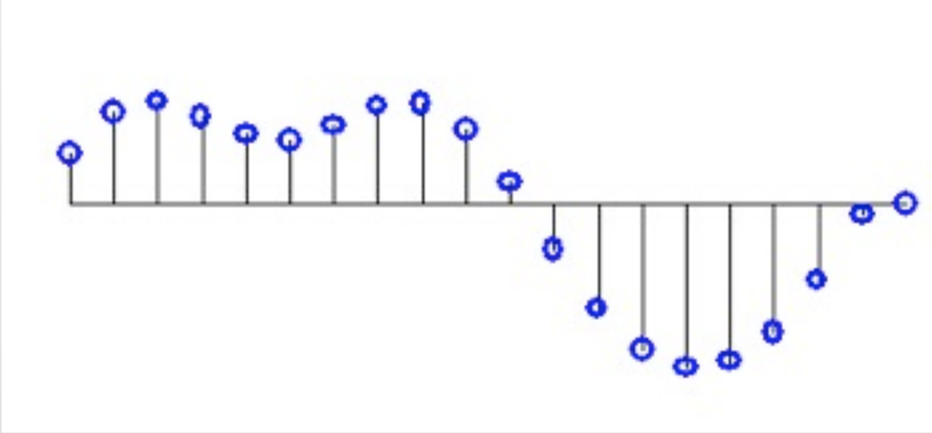
$$u'(x,t) = \sum_{k=1}^{N} \tilde{u}(t)_k \Phi'_k(x)$$

- Compute nonlinearities in physical space

Spectral

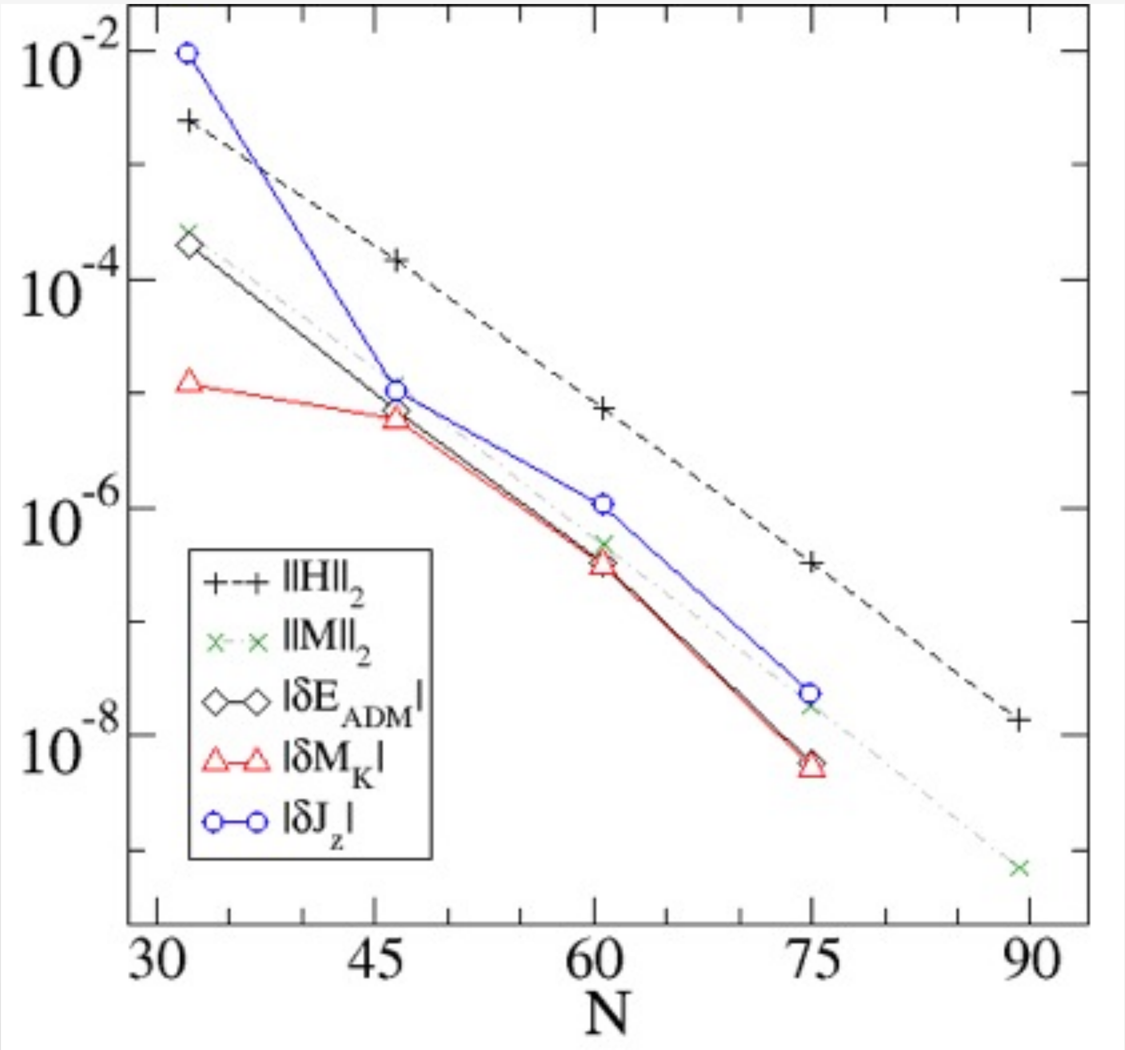


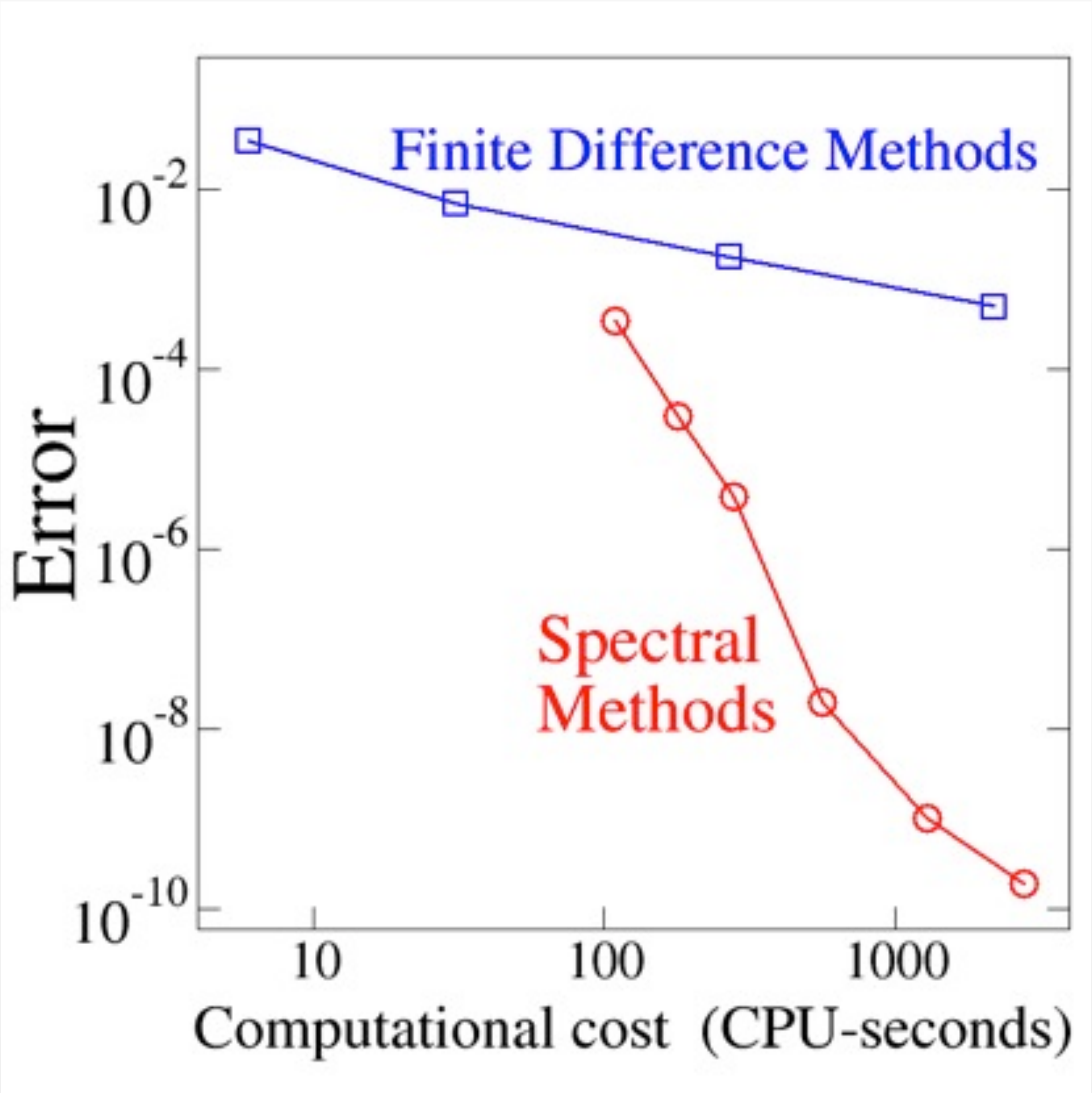
Finite differences



Why spectral methods?

❖ Smooth solutions ⇒ exponential convergence





HP et al, 2002

... but more difficult than finite differences

IV: Domain-decomposition

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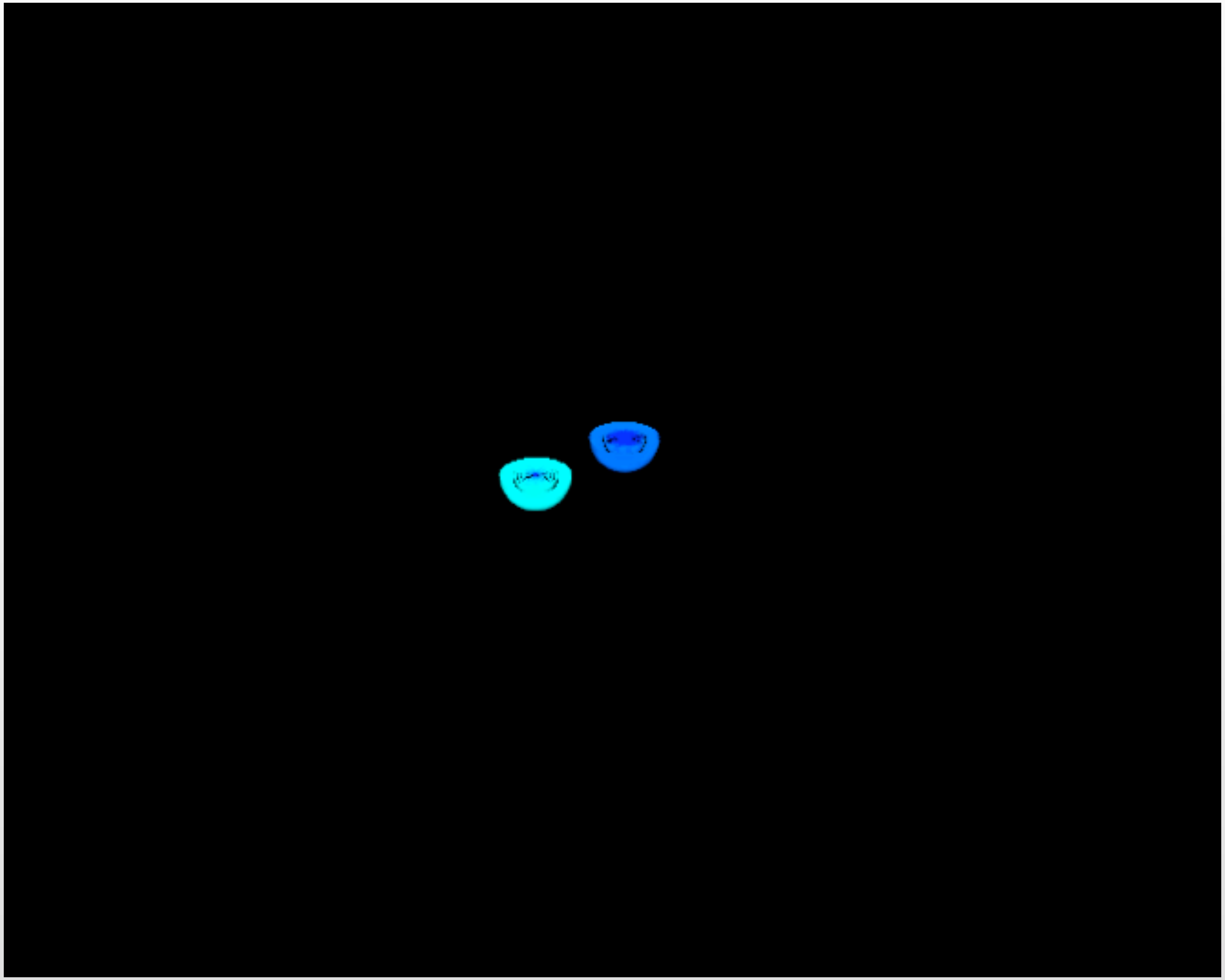
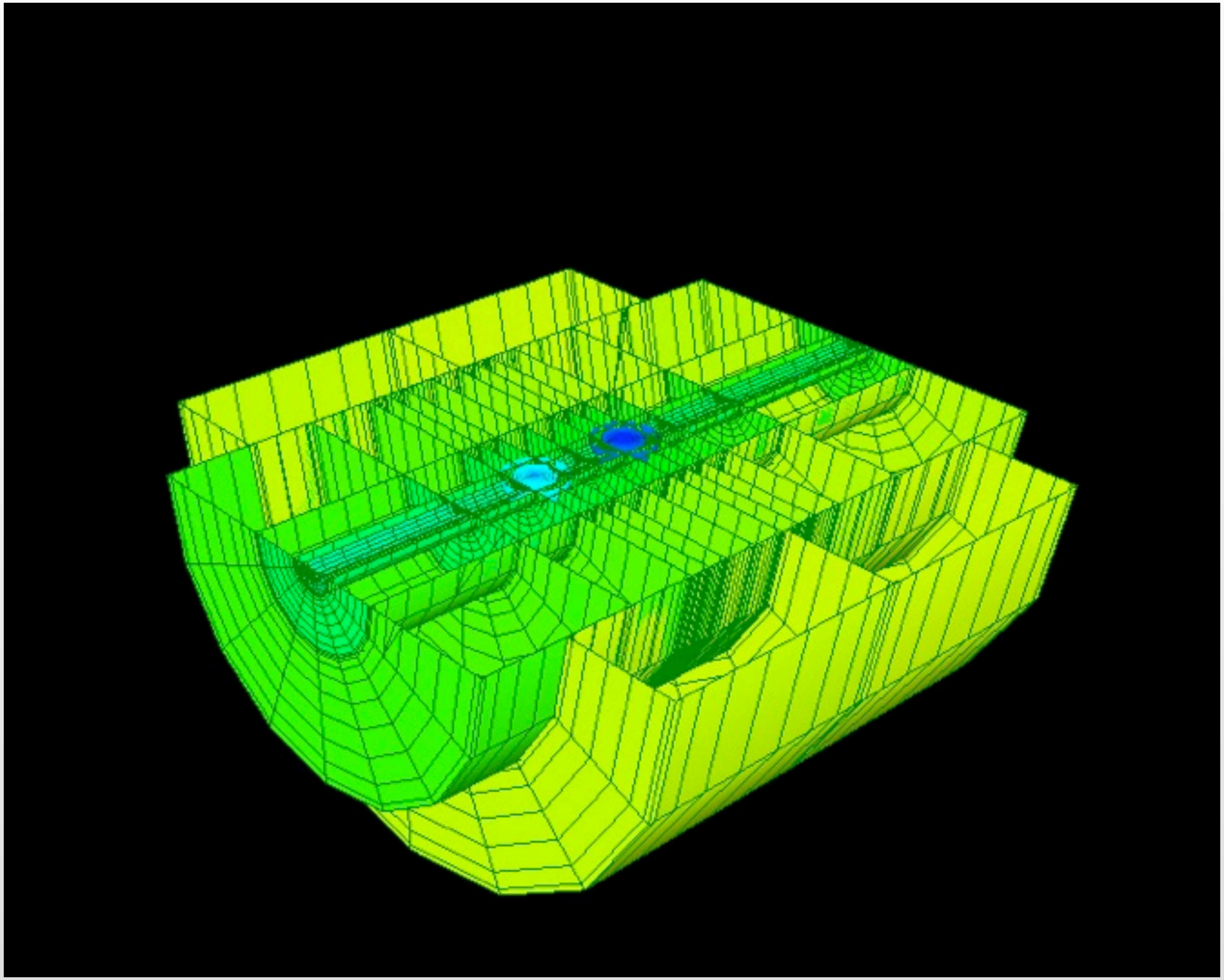


Spectral Einstein Code *SpEC* (Caltech-Cornell-CITA)
http://www.black-holes.org/SpEC.html

IV: Domain-decomposition



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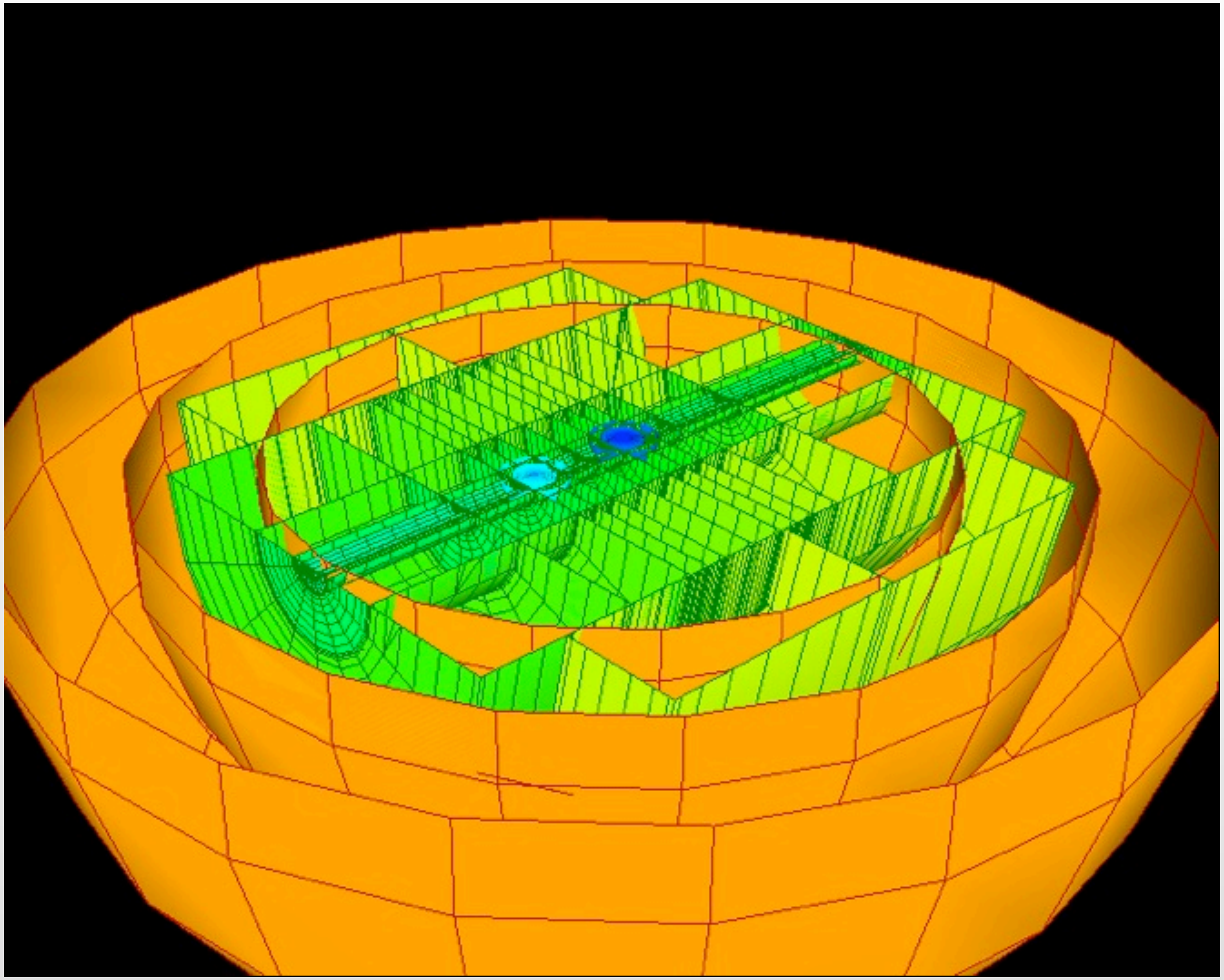

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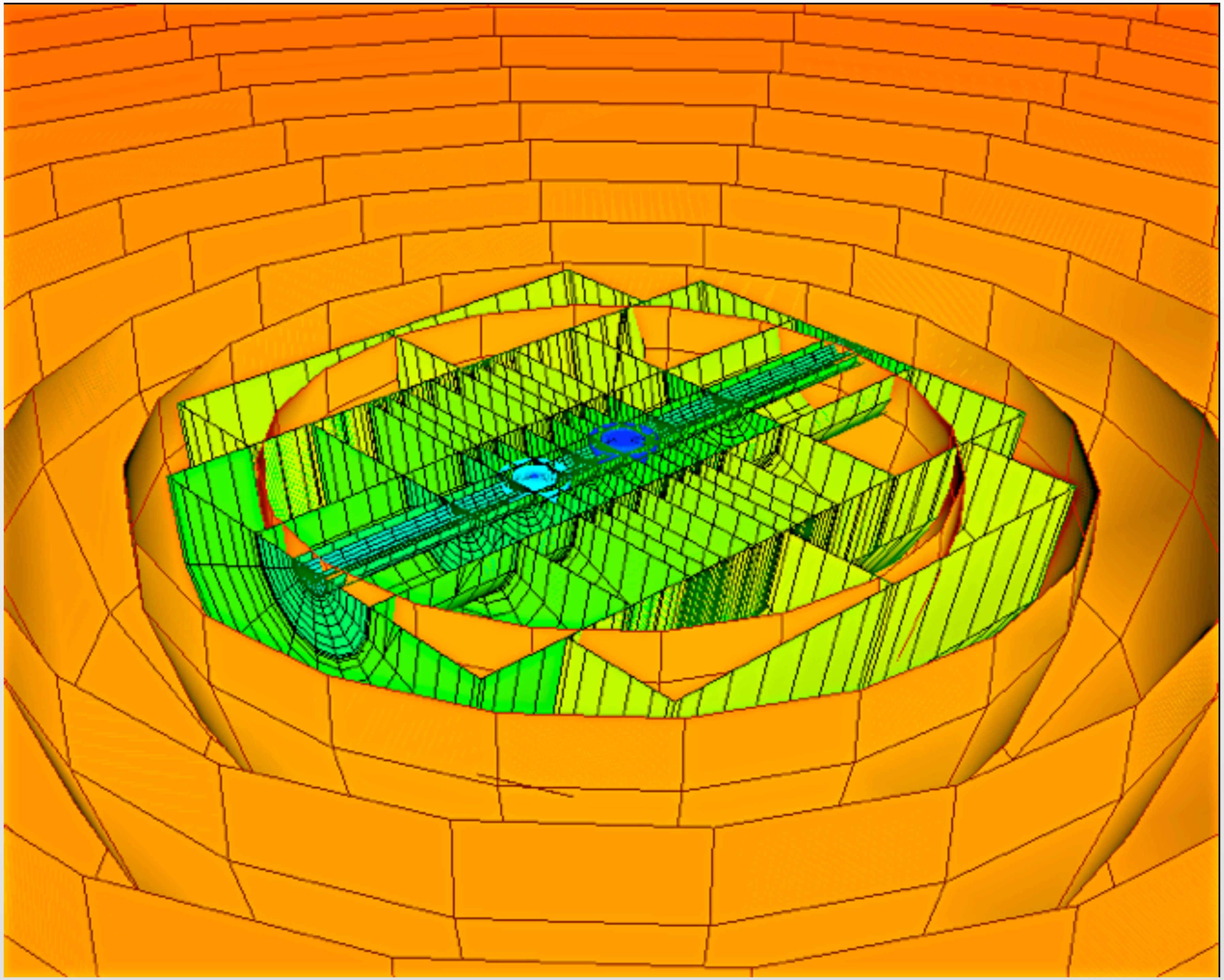

Spectral Einstein Code *SpEC* (Caltech-Cornell-CITA)
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V. Solve constraints

❖ Spins above ~0.8 ... 0.9 require special techniques

$$0 = \tilde{\nabla}^2\psi - \frac{1}{8}\tilde{R}\psi - \frac{1}{12}K^2\psi^5 + \frac{1}{8}\psi^{-7}\tilde{A}^{ij}\tilde{A}_{ij},$$

$$0 = \tilde{\nabla}_j\left(\frac{\psi^7}{2(\alpha\psi)}(\tilde{\mathbb{L}}\beta)^{ij}\right) - \frac{2}{3}\psi^6\tilde{\nabla}^i K - \tilde{\nabla}_j\left(\frac{\psi^7}{2(\alpha\psi)}\tilde{u}^{ij}\right), \quad (37b)$$

$$0 = \tilde{\nabla}^2(\alpha\psi) - (\alpha\psi)\left[\frac{\tilde{R}}{8} + \frac{5}{12}K^4\psi^4 + \frac{7}{8}\psi^{-8}\tilde{A}^{ij}\tilde{A}_{ij}\right] + \psi^5(\partial_t K - \beta^k\partial_k K), \quad (37c)$$

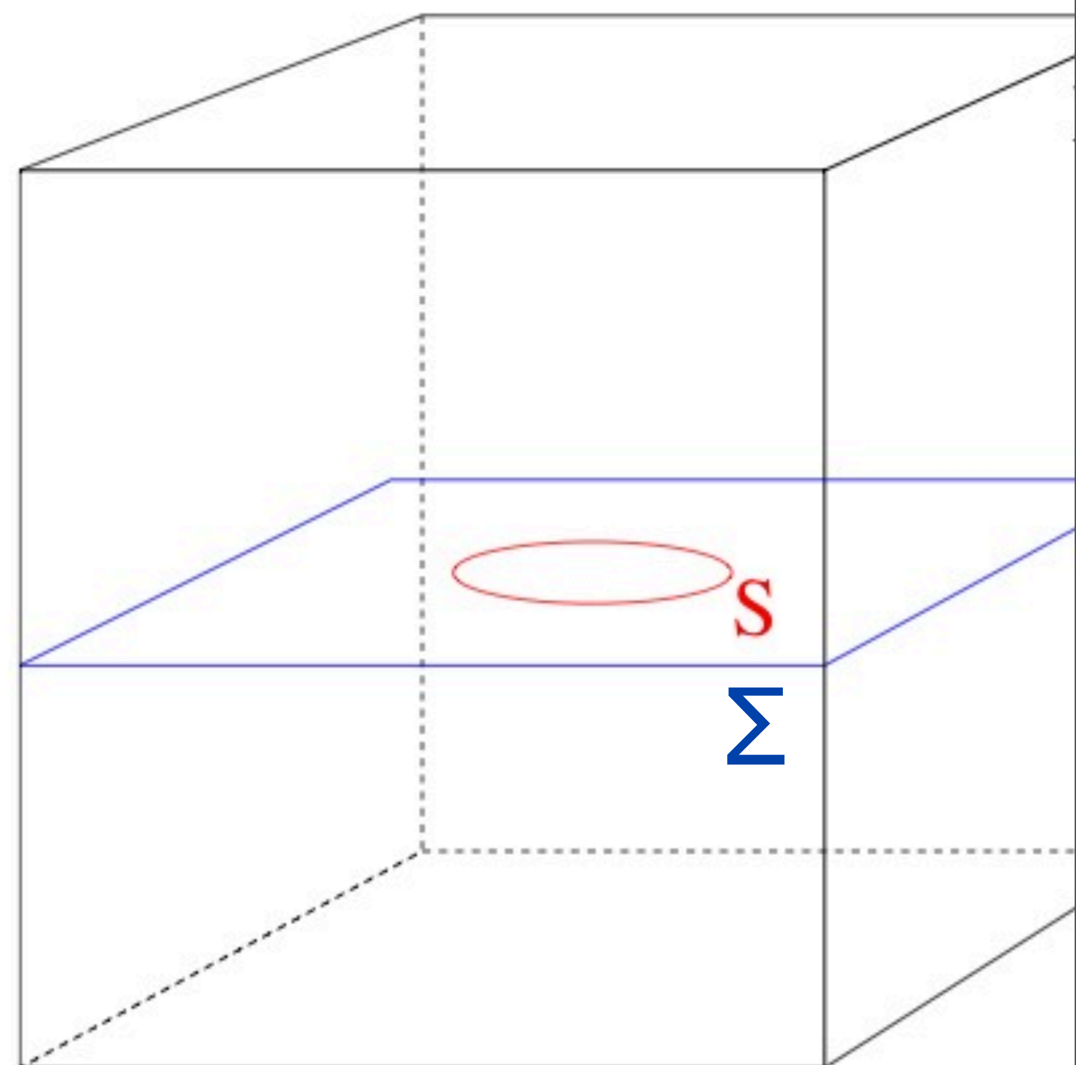
HP ea 02,03
Cook, HP 03, 04
Lovelace ea 08

VI. Define Spin

- In axisymmetry anglar momentum rigourously defined e.g. via the Hamiltonian that generates the rotation (Brown & York, 1993; isolated/dynamical horizon framework)

$$J = \frac{1}{8\pi} \oint_S (K_{ij} - g_{ij} K)\, \phi^i s^j \, dA$$

- ϕ^i rotational Killing vector
  s^i unit-normal to $\mathcal{S}$ in Σ
  g_{ij} metric in Σ
  K_{ij} extrinsic curvature of Σ in M
- $\mathcal{S}$ sphere at $\infty \Rightarrow$ ADM angular momentum
- $\mathcal{S}$ 2-sphere at finite distance $\Rightarrow$ quasi-local spin



Spin in non-axisymmetric spacetimes

- Would like to define "spin" in absence of axisymmetry.
- Choose "approximate Killing vector" ϕ^i; evaluate

$$J_\phi = \frac{1}{8\pi}\oint_{\mathcal{S}} (K_{ij} - g_{ij}K)\,\phi^i s^j\, dA$$

- Q: How to choose ϕ^i?
  - ϕ^i coordinate rotation, $\vec{\phi} = x\hat{e}_y - y\hat{e}_x$ (depends on coordinate system)
  - Integrate Killing transport equation (Dreyer et al, 2003) (depends on integration path; ϕ^i not smooth)
  - A variational approach

Variational approx. Killing vectors

Cook & Whiting 07
Owen 07
Lovelace, Chu, HP, Owen 08

- Require $D_A\phi^A = 0 \Rightarrow \phi^A = \varepsilon^{AB}\partial_B z$ for some potential z.
  (A, B: coordinates within $\mathcal{S}$, D_A derivative within $\mathcal{S}$)
- Minimize functional

$$\mathcal{I} = \oint_{\mathcal{S}} (D_{(A}\phi_{B)})(D^{(A}\phi^{B)})\, dA + \lambda\left(\oint_{\mathcal{S}} \phi_A\phi^A\, dA - N\right)$$

- Results in generalized Eigenvalue problem

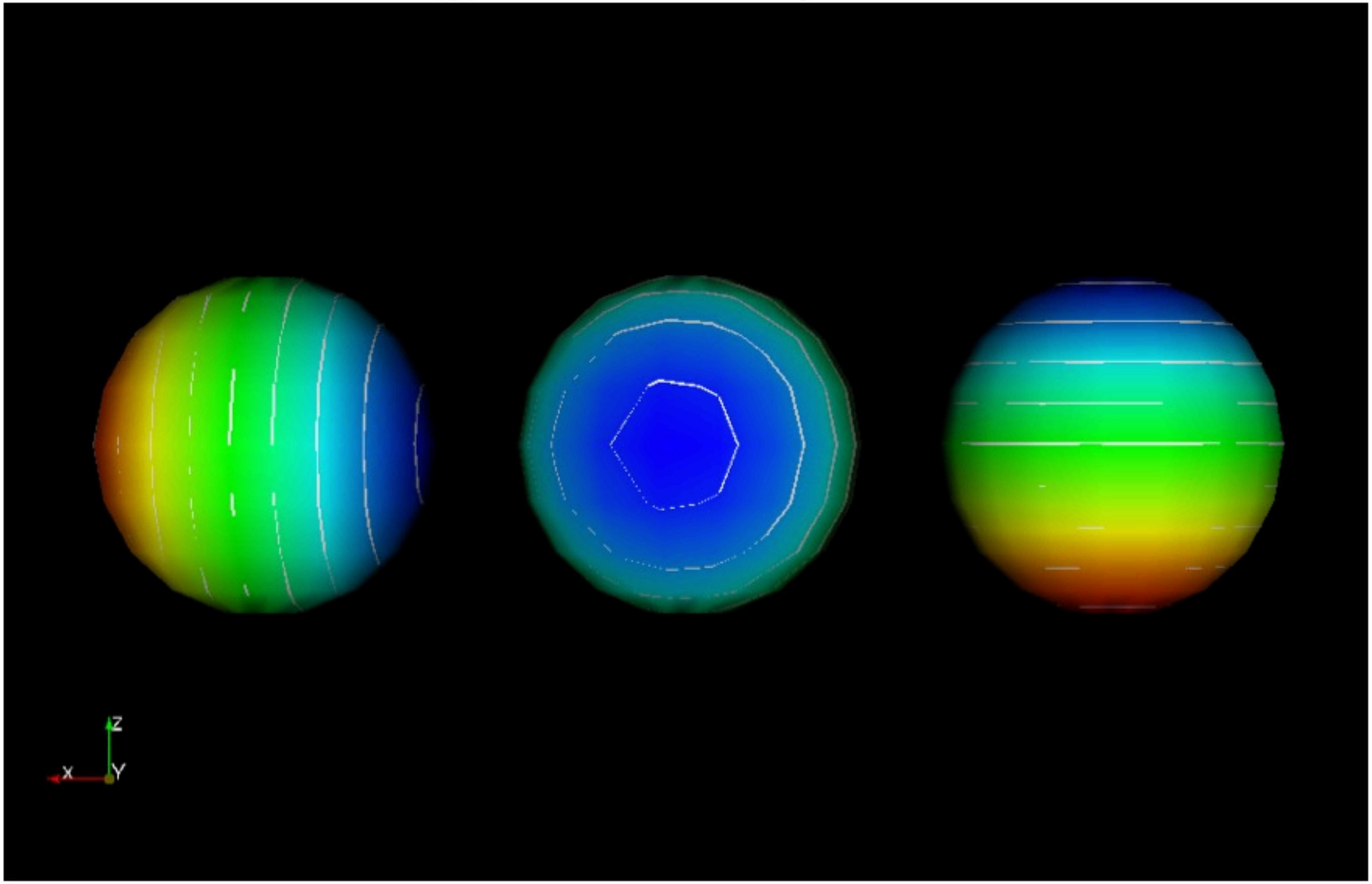
$$\left(D^2 + {}^2R\right) D^2 z + (D_A{}^2R)D^A z = \lambda D^2 z$$

- Spectral expansion $z(\theta,\varphi) = \sum_{l=1}^{L}\sum_{|m|\le l} A^{lm} Y_{lm}(\theta,\varphi)$ results in matrix-equation for coefficients A^{lm}:

$$\Rightarrow M^{lm}{}_{l'm'}A^{l'm'} = \lambda N^{lm}{}_{l'm'}A^{l'm'}$$

Example

- The three smallest eigenvalues correspond to rotations

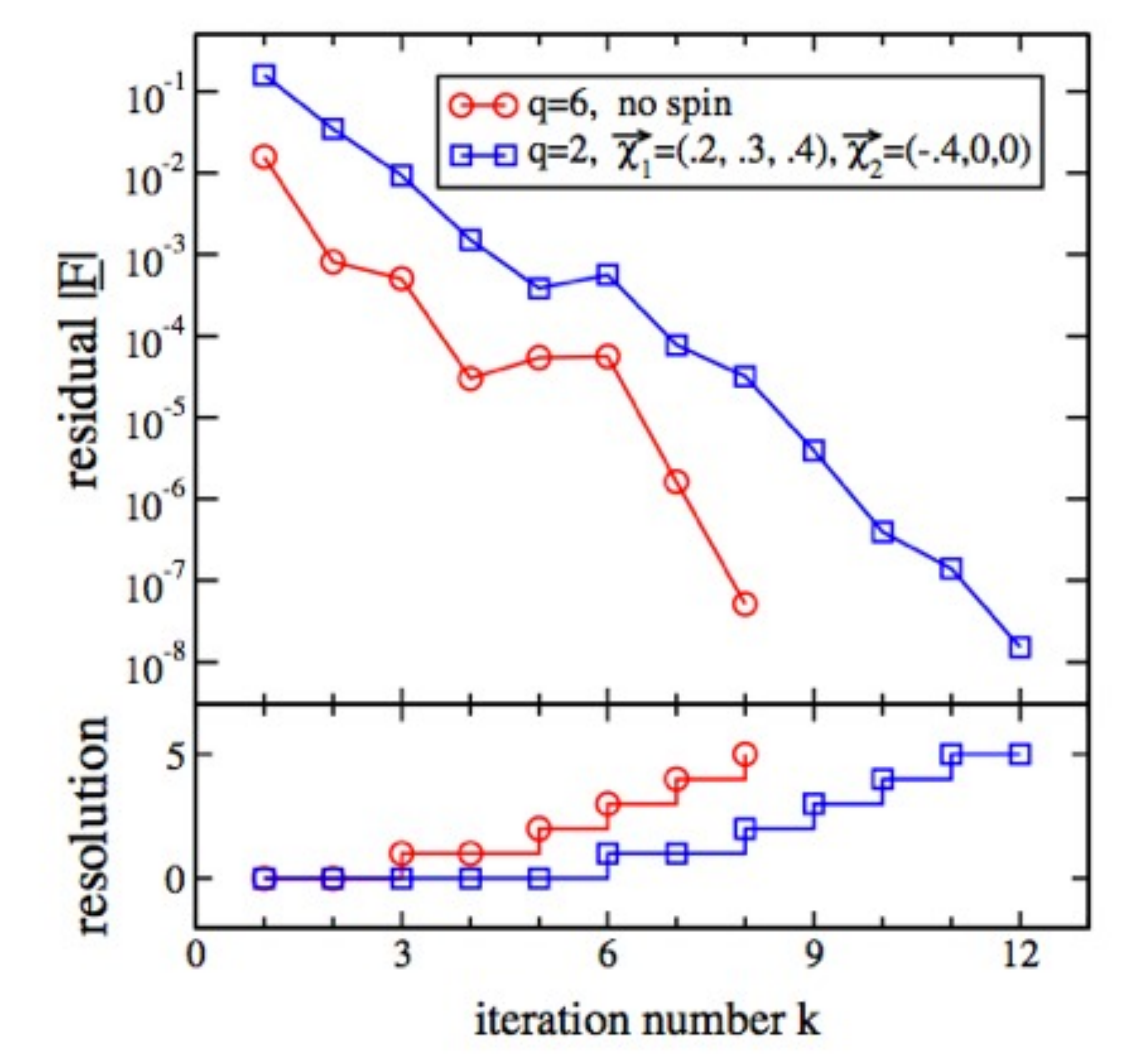


VII Control BH properties

❖ Fix BH-distance, Omega, radial velocity

❖ Iteratively solve the initial-value problem, to adjust

- black hole masses (2 DoF)
- black hole spins (6 DoF)
- center of rotation (2 DoF)

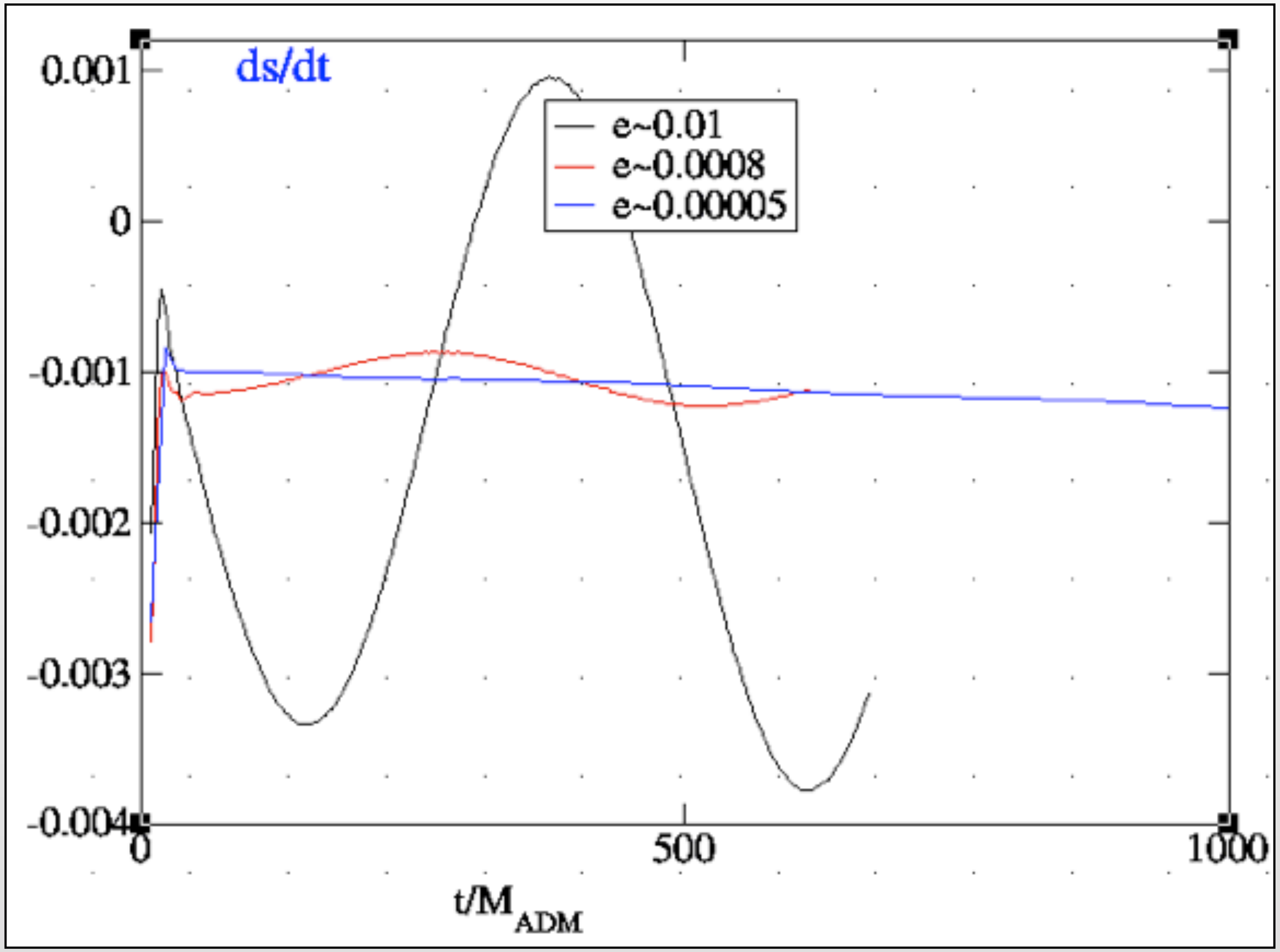


Buchman, HP, Scheel, Szilagyi 1206.3015

VIII Control orbital eccentricity

- Initial data parameters Ω_0, v_r (or p_t, p_r) determine orbital eccentricity and phase at periastron

- Unique values for *zero* eccentricity



HP ea 07, Boyle ea 07, Buchman ea 12

Large-scale eccentricity removal

Mroue, HP
arxiv1210.2958

Large-scale eccentricity removal

❖ Iterative eccentricity removal works, but is tedious.

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- Eccentricity reduce...

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arxiv1210.2958

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❖ Eccentricity reduce...

- 40 non-spinning

Mroue, HP
arxiv1210.2958

Large-scale eccentricity removal

CITA–ICAT

- ❖ Iterative eccentricity removal works, but is tedious.
  - Let's get over with it!
- ❖ Eccentricity reduce...
  - 40 non-spinning
  - 190 single-spin

Mroue, HP
arxiv1210.2958

Large-scale eccentricity removal

CITA–ICAT

❖ Iterative eccentricity removal works, but is tedious.

- Let's get over with it!

❖ Eccentricity reduce...

- 40 non-spinning
- 190 single-spin
- 300 double-spin

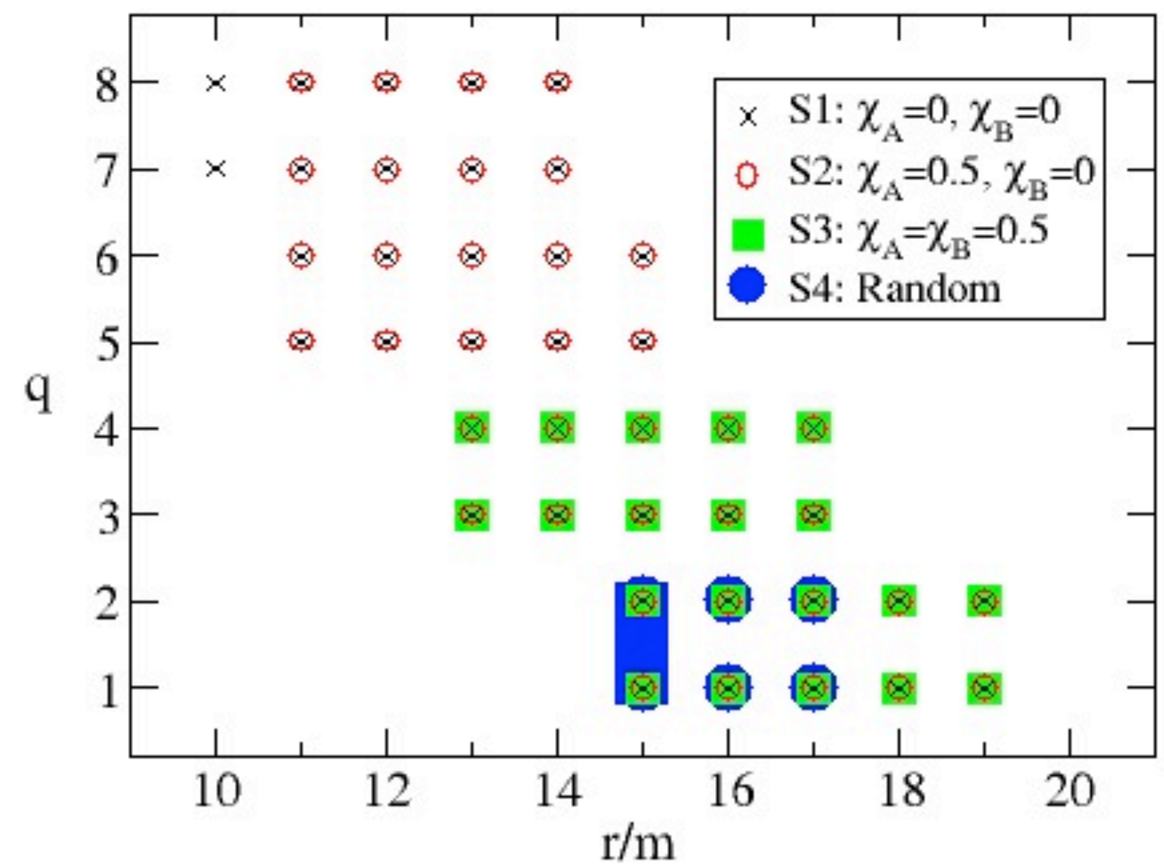
Mroue, HP
arxiv1210.2958

Large-scale eccentricity removal

CITA–ICAT

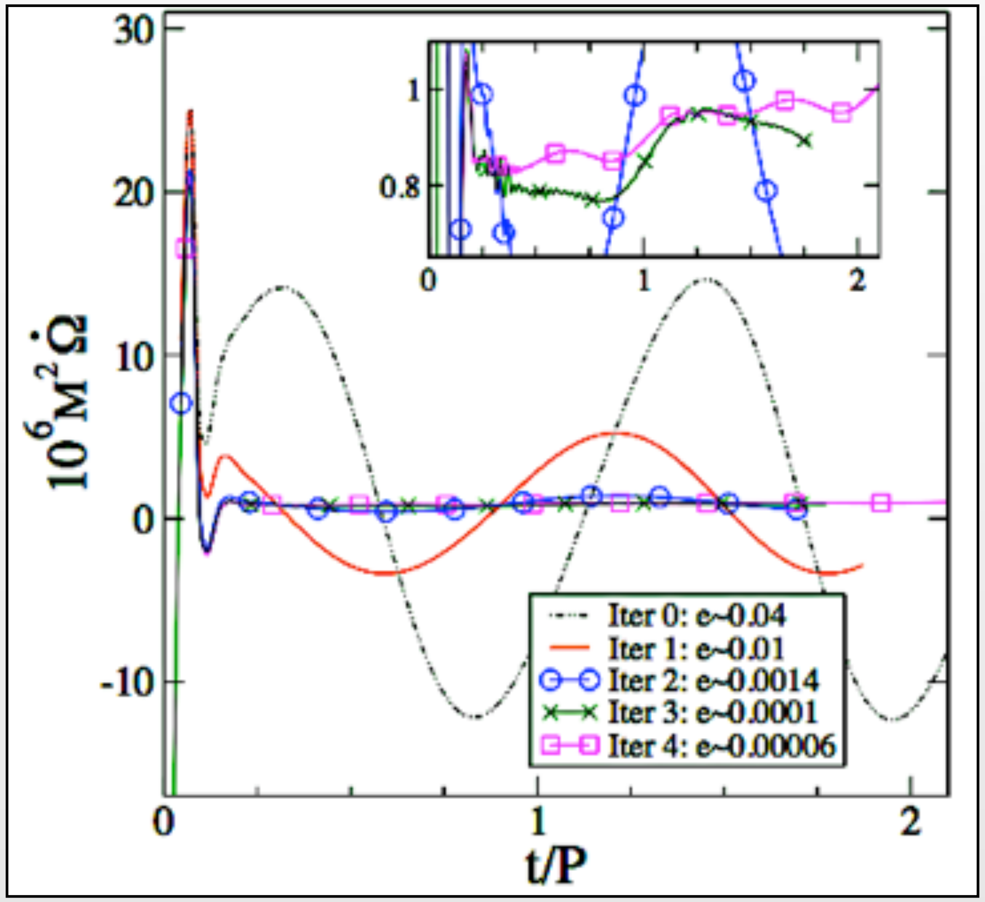
- Iterative eccentricity removal works, but is tedious.
  - Let's get over with it!
- Eccentricity reduce...
  - 40 non-spinning
  - 190 single-spin
  - 300 double-spin
  - 130 random spin directions

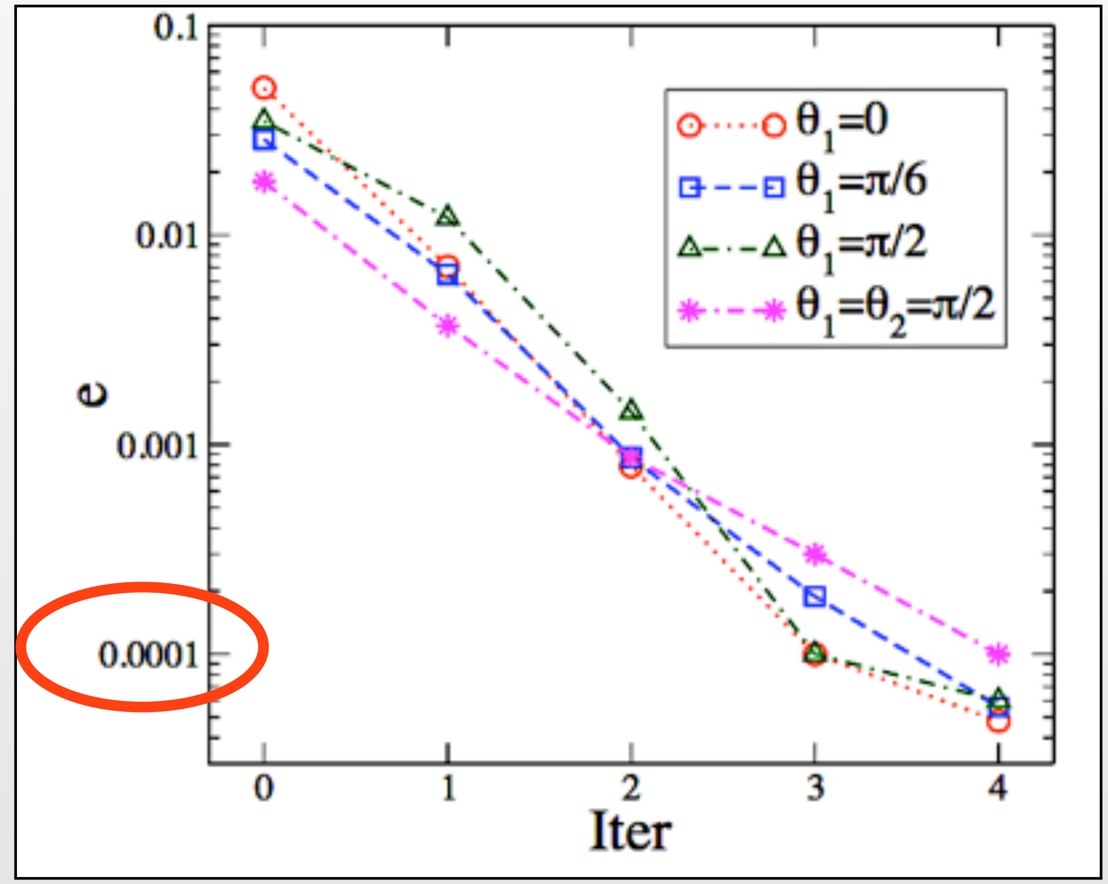
Mroue, HP
arxiv1210.2958



Eccentricity in precessing BH-BH

❖ With enough care, iterative eccentricity removal works!





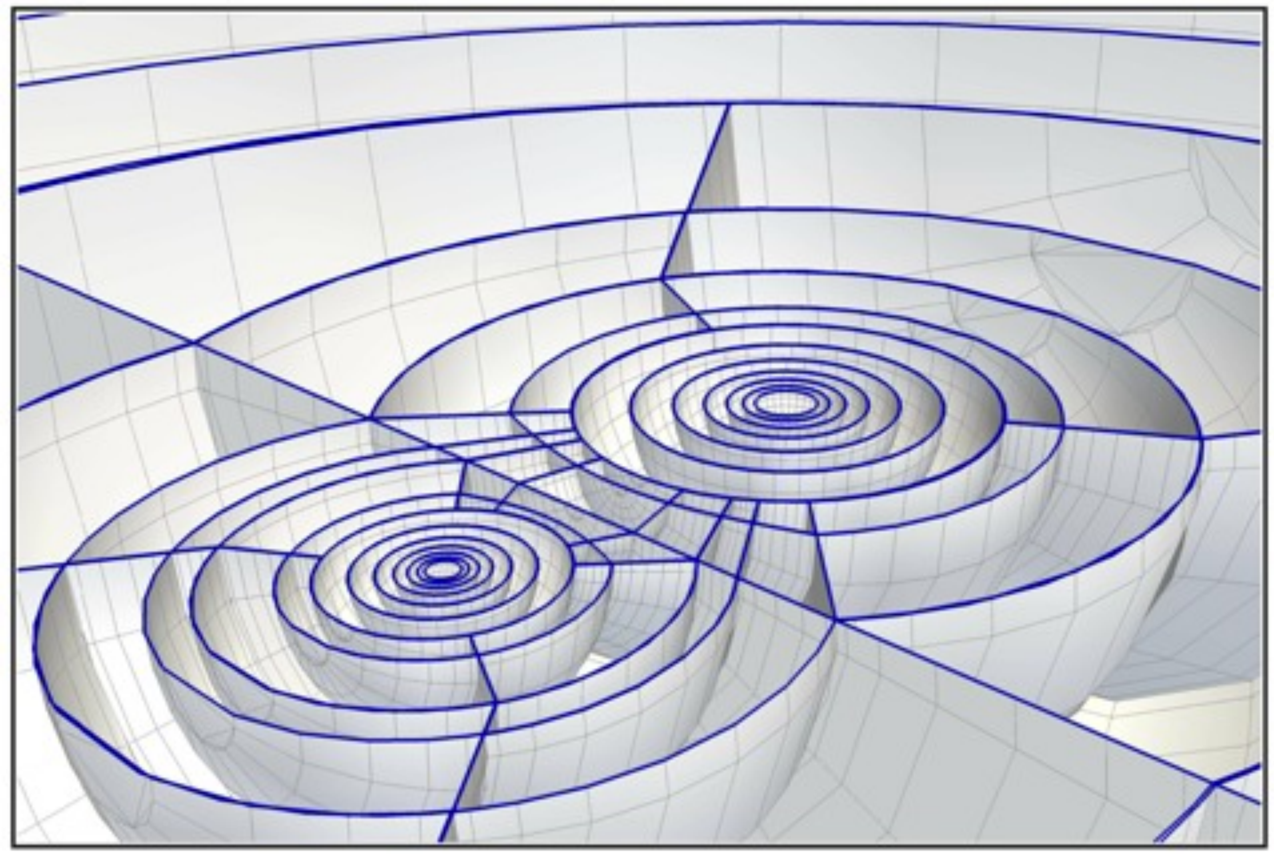
Buonnano, Kidder, Mroue, HP, Tarraccini, 10

IX. Precession & Dual-Frames

- ❖ Map excision boundaries onto BH-location

$$x^{\bar{\imath}} = a(t) R(t)^{\bar{\imath}}_{j} x^{j} + T^{\bar{\imath}}(t)$$

- ❖ Measure location of BHs, and adjust dynamically:
  - Expansion factor a(t)
  - Translation T(t)
  - Rotation R(t)

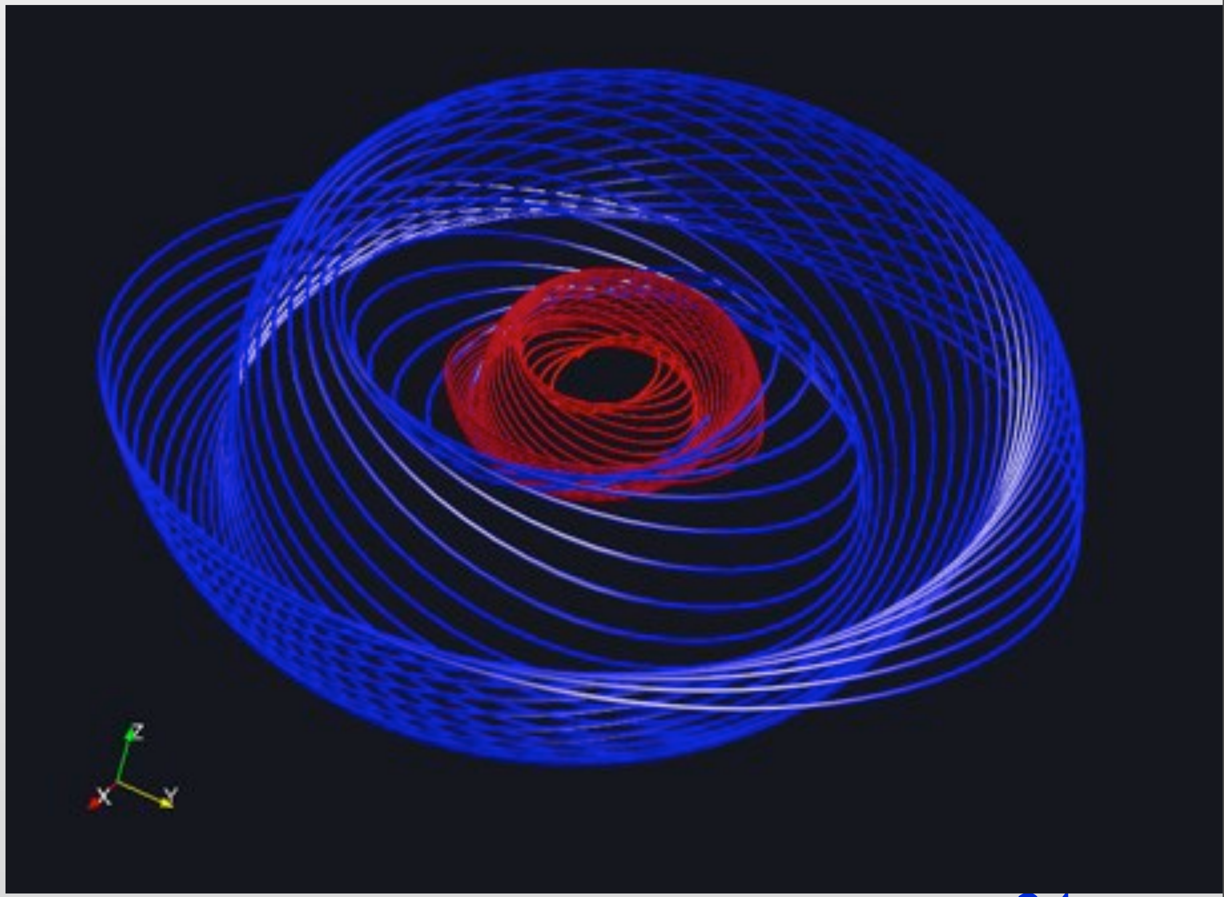


Precession & Dual-Frames

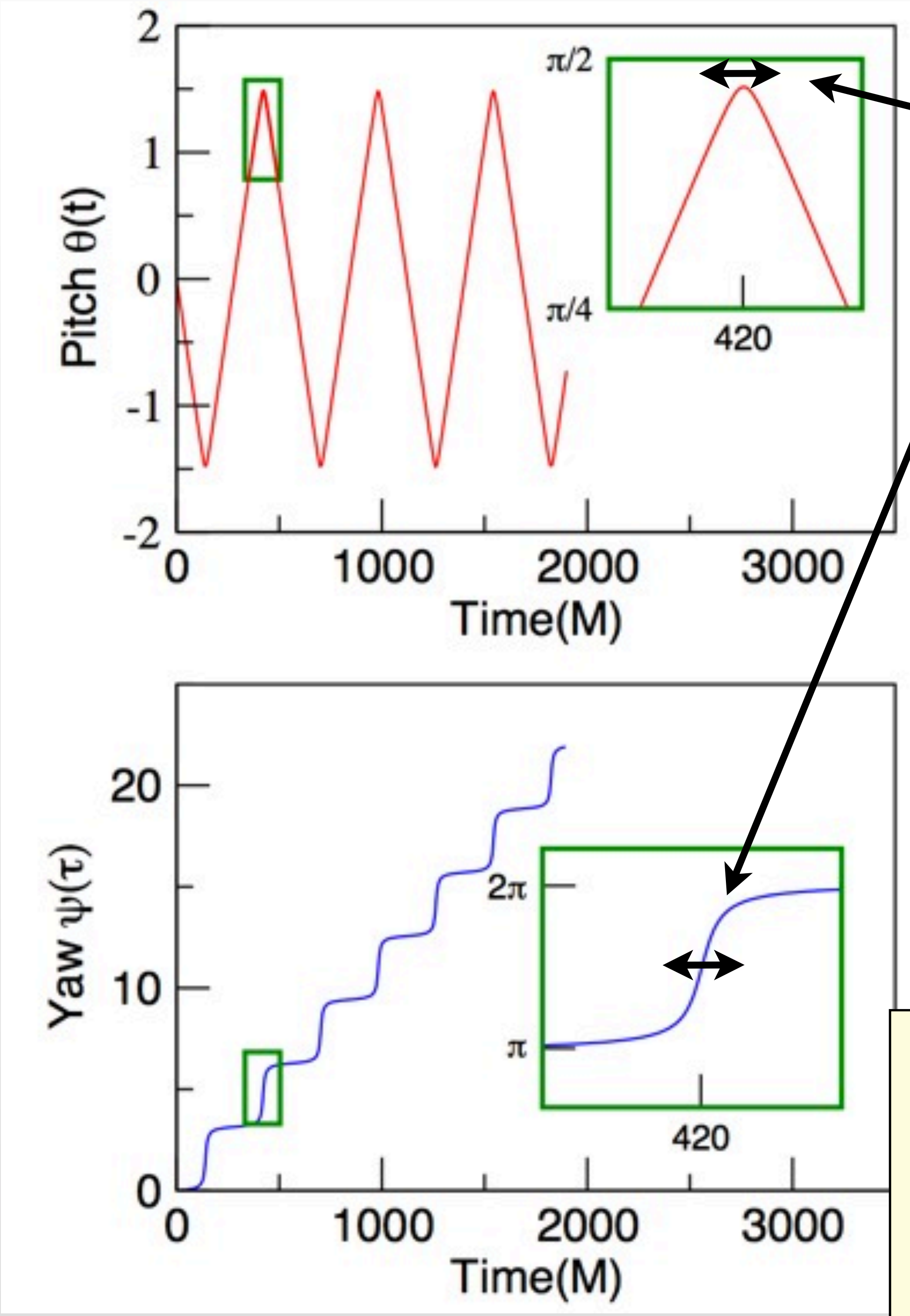
- Early technique: Pitch & Yaw
  - Rotate about y-axis, then about z-axis

$$R = \begin{pmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$$

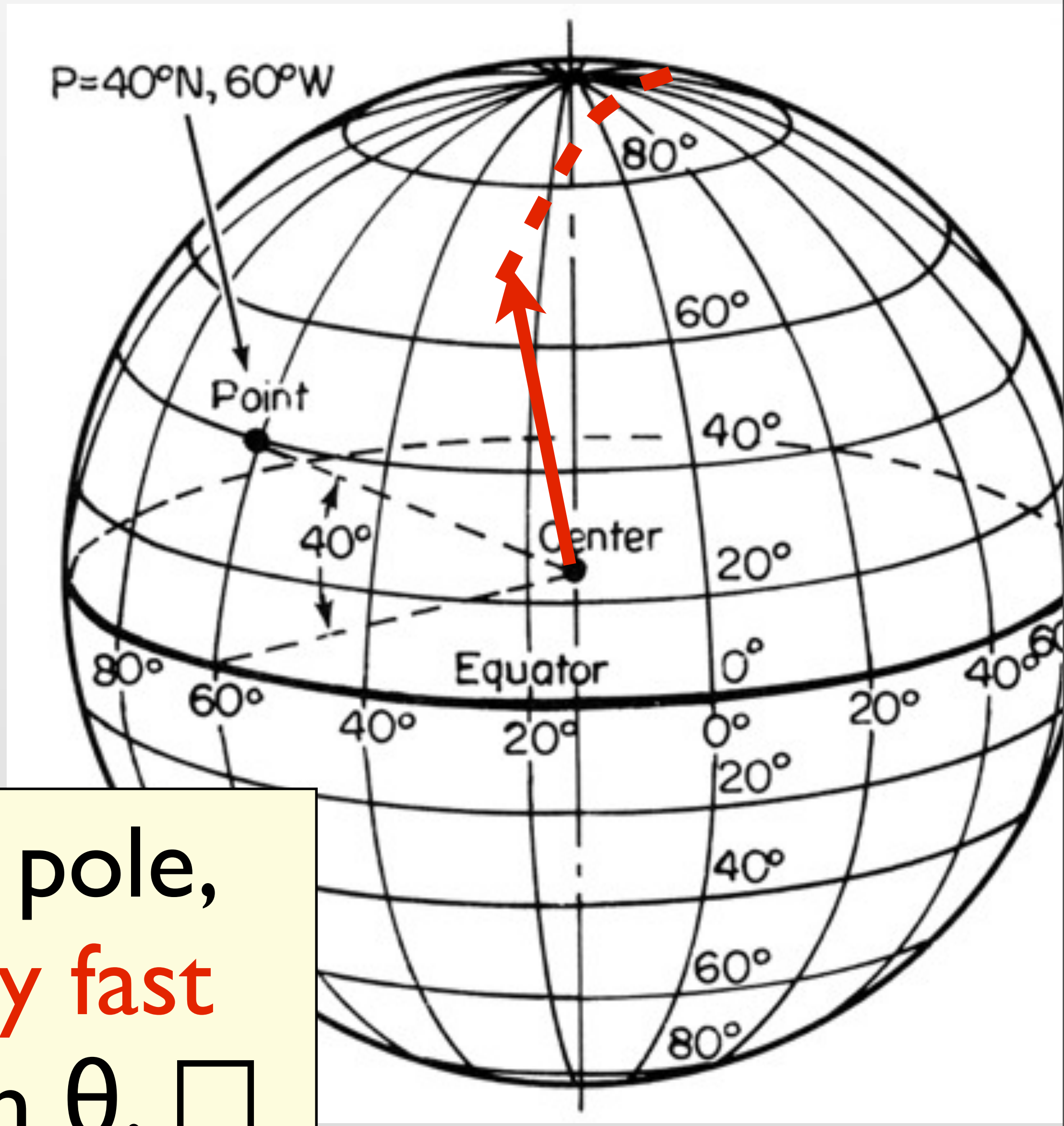
- Works for moderate precession



Polar singularity breaks Pitch&Yaw



$\propto \min(\pi/2 - \theta)$



Close to pole, arbitrarily fast changes in θ, □

Solution: Quaternions

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❖ Represent rotation without preferred axis/singularity

- Unit-Quaternions represent rotations

$$\mathbf{q} = [q_0, \vec{q}\,] = q_0 + iq_1 + jq_2 + kq_3$$

$$R_{\mathbf{q}}\vec{v} : \mathbf{q}[0, \vec{v}]\mathbf{q}^* = [0, R_{\mathbf{q}}\vec{v}]$$

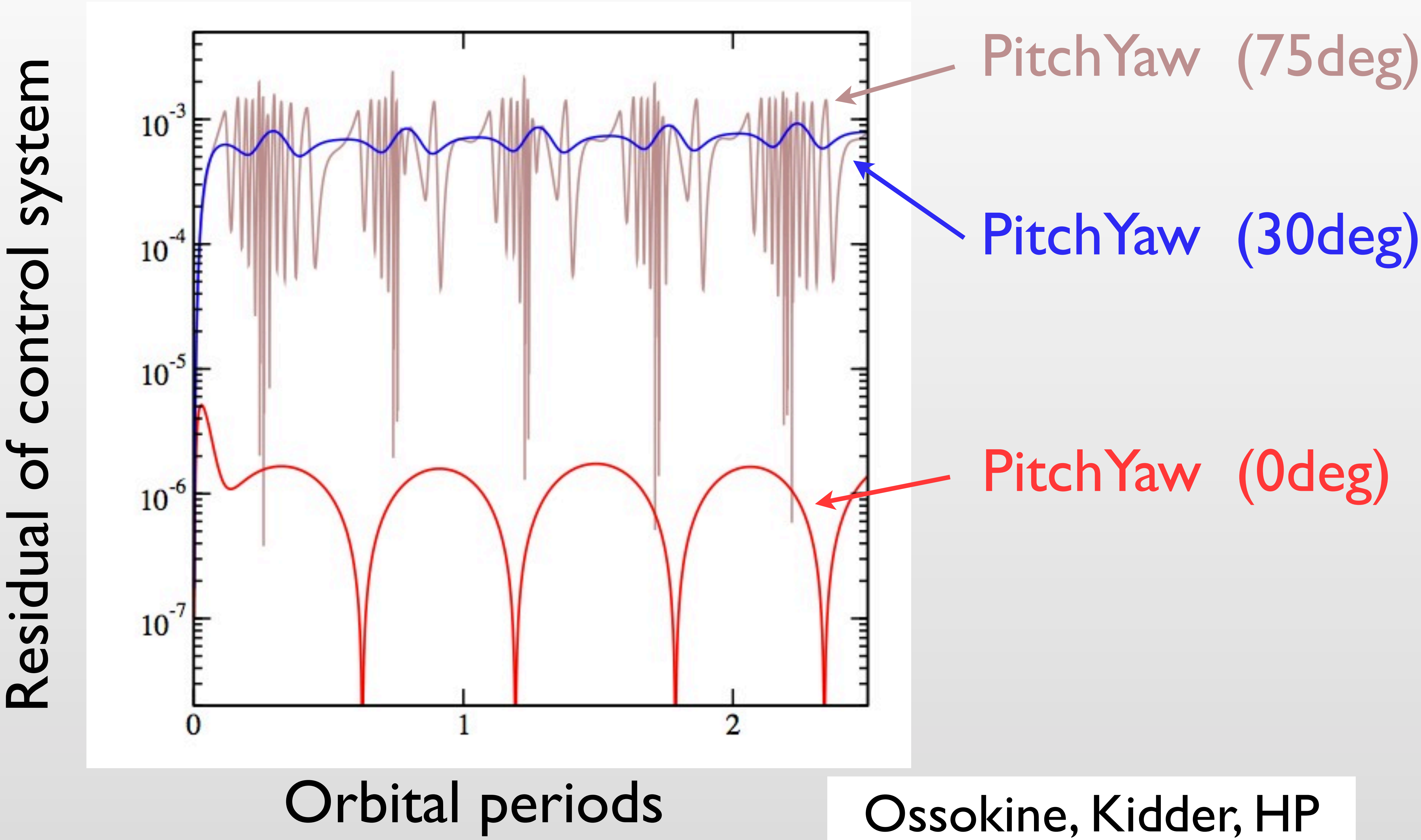
❖ Represent control-system without preferred axis

- Control instantaneous frequency $\vec{\Omega}(t)$ of grid w.r.t inertial frame
- Update rotation matrix via

$$\frac{d\mathbf{q}(t)}{dt} = \frac{1}{2}\mathbf{q}(t)[0, \vec{\Omega}(t)\,]$$

Ossokine, Kidder, HP
arxiv1304.3067

Test: inclined PN inspirals



Ossokine, Kidder, HP
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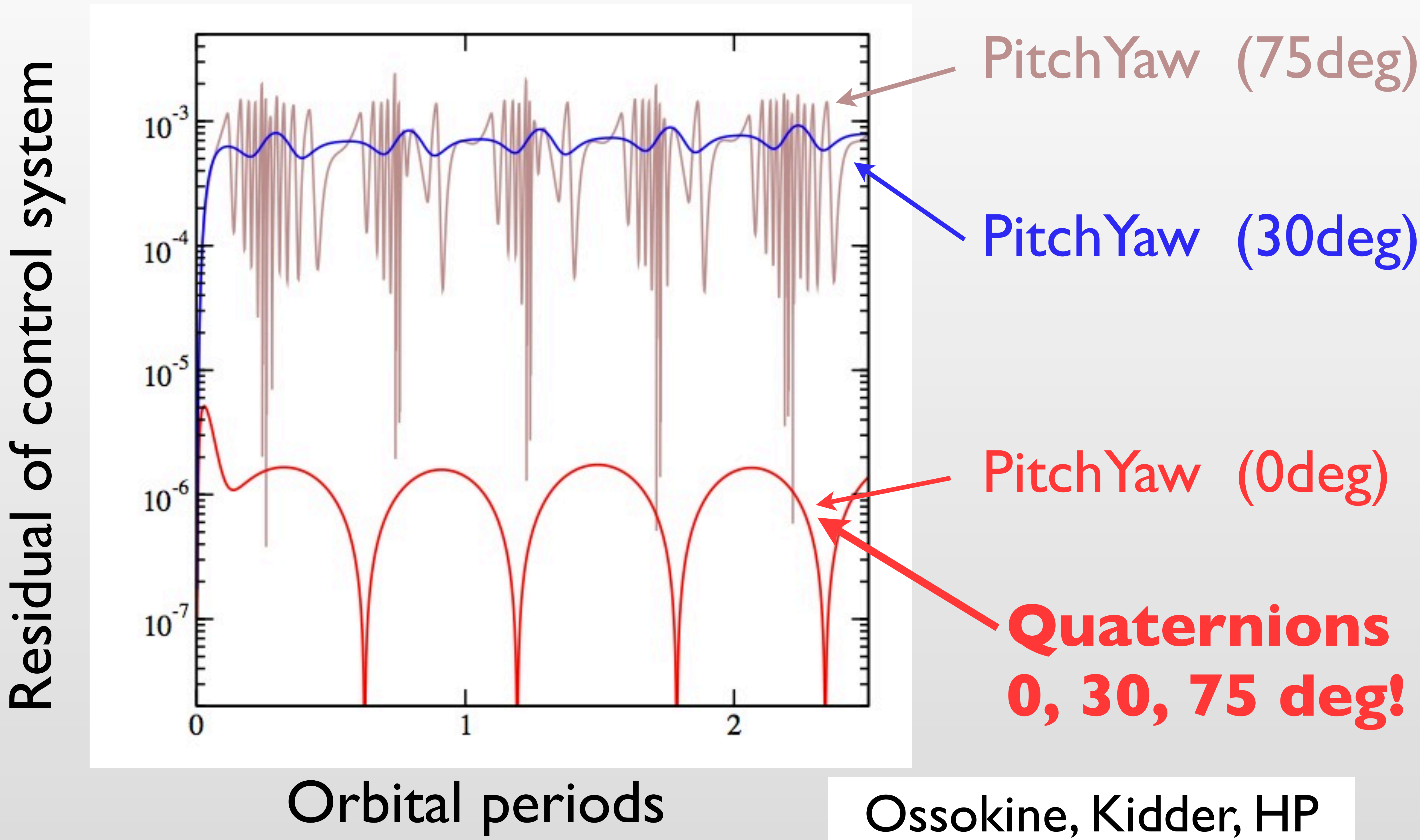


Ossokine, Kidder, HP
arxiv1304.3067

X: Merger & Ringdown

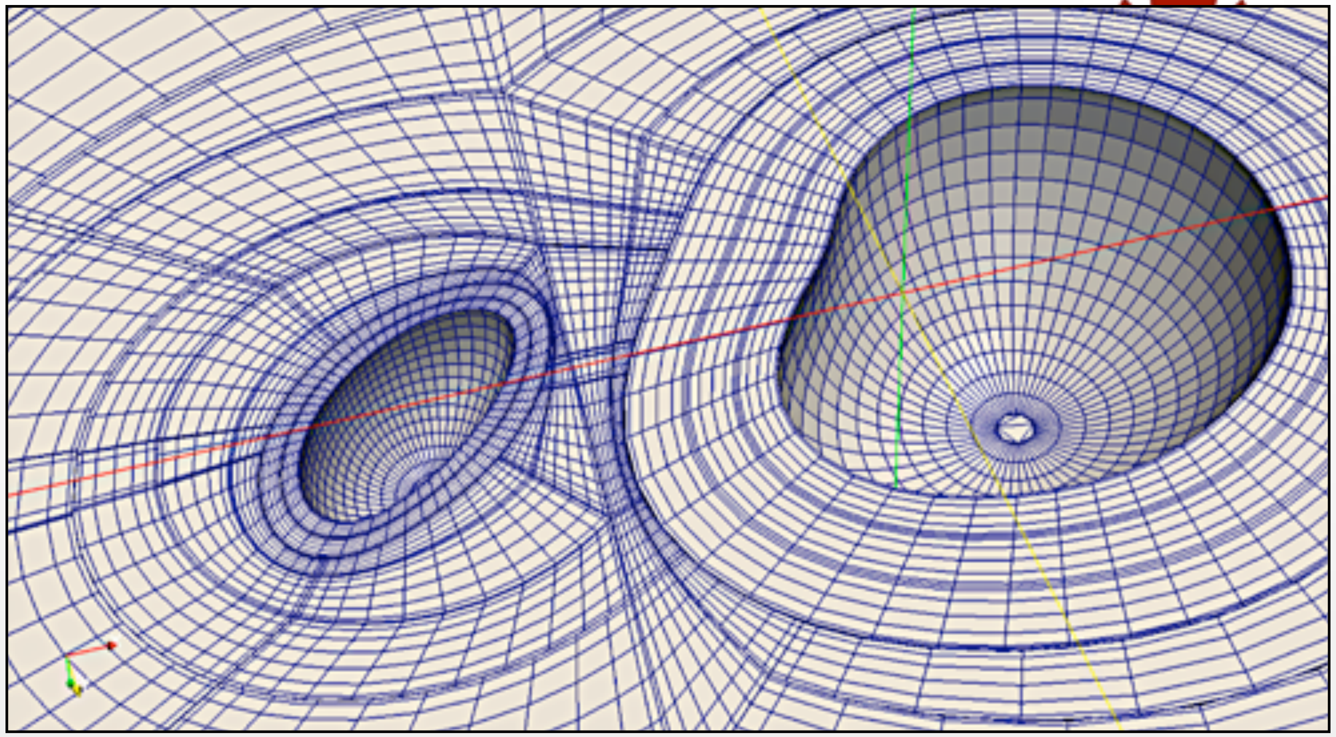
- ***Mark Scheel, Bela Szil***

Szilagyi, Lindblom, Scheel 08.
Many additions since then

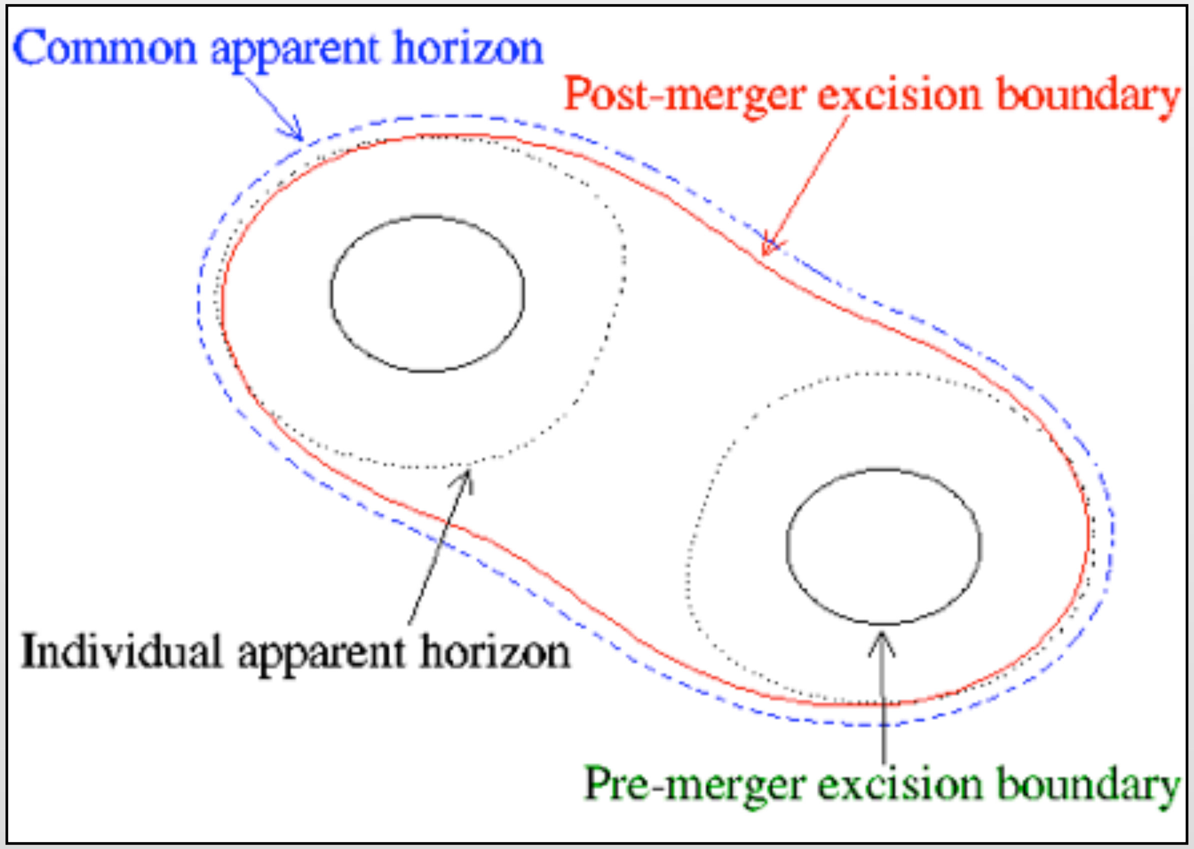
  - Hemberger ea, 1211.6079

- Close to merger
  - Switch domain-decomposition
  - Active gauge conditions
  - Adaptive Mesh Refinement

- After common horizon
  - Switch to distorted concentric shells



Bela Szilagyi

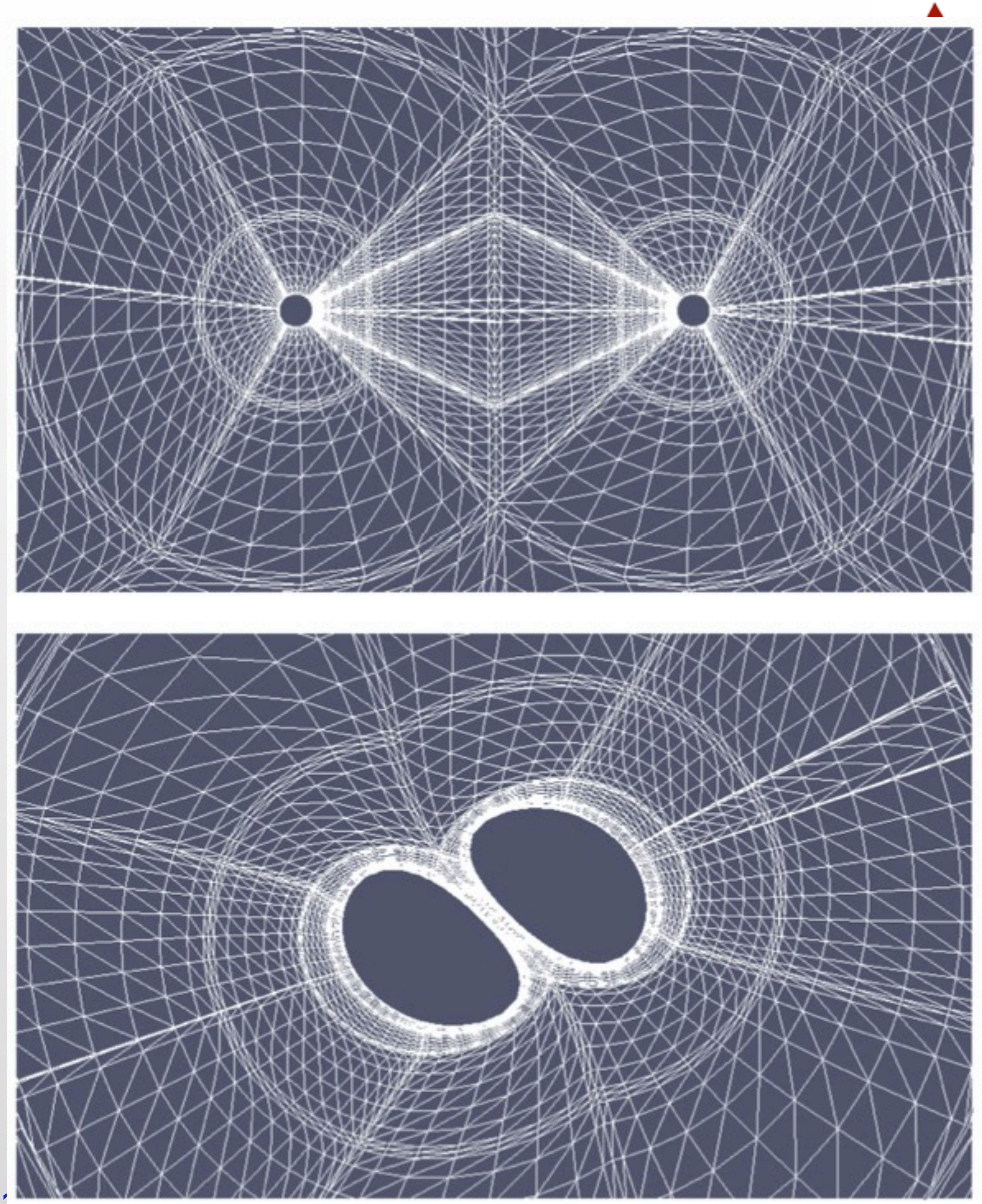


Mark Scheel

Spectral grid

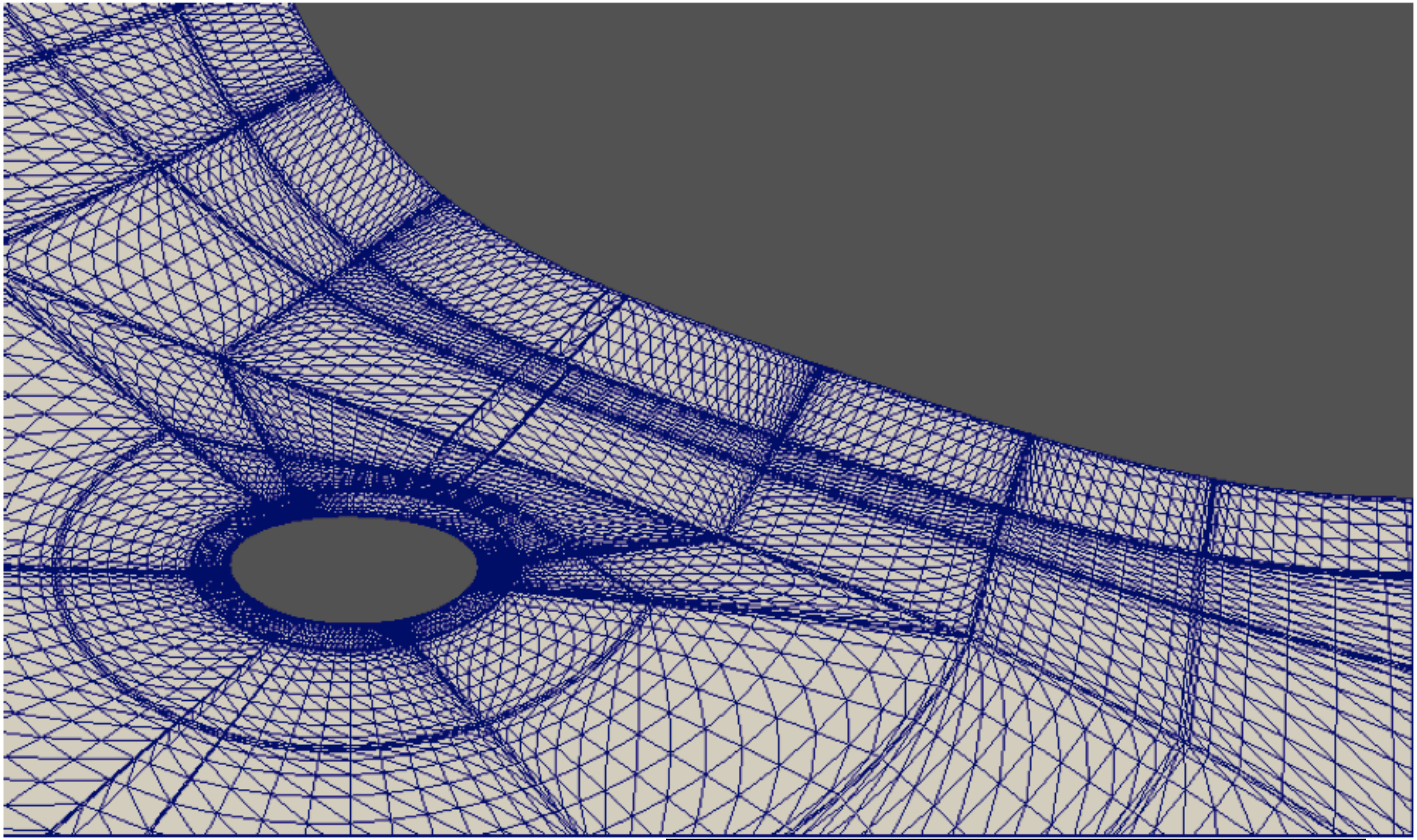
mapped

Inertial Coordinates



Hemberger, Scheel, Kidder, Szilagyi, ...
arXiv:1211.6079

Computational grid, mass-ratio 8



Hemberger, Scheel, Kidder, Szilagyi, ... arXiv:1211.6079

XI. Wave-extraction and extrapolation

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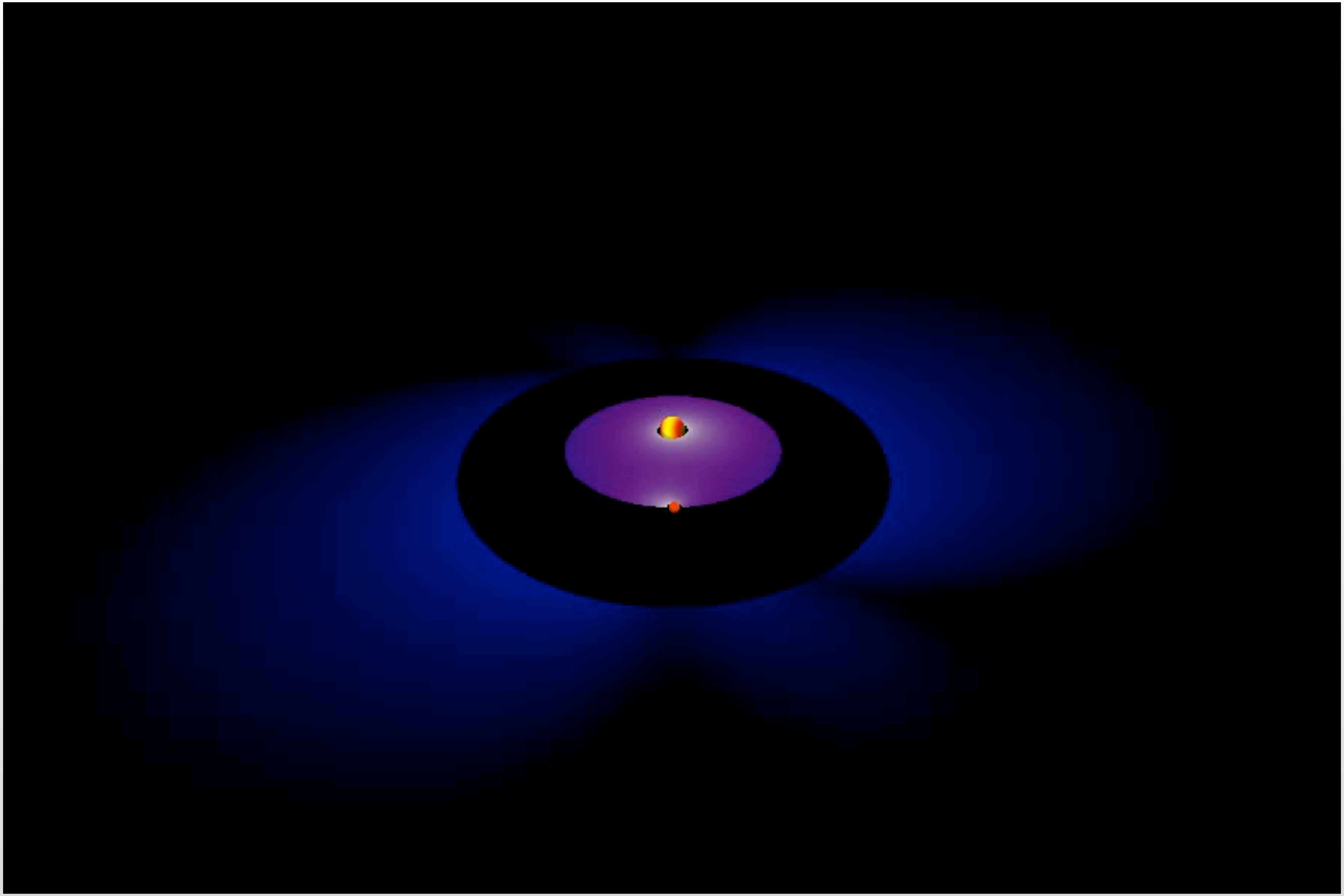
❖ Mike Boyle

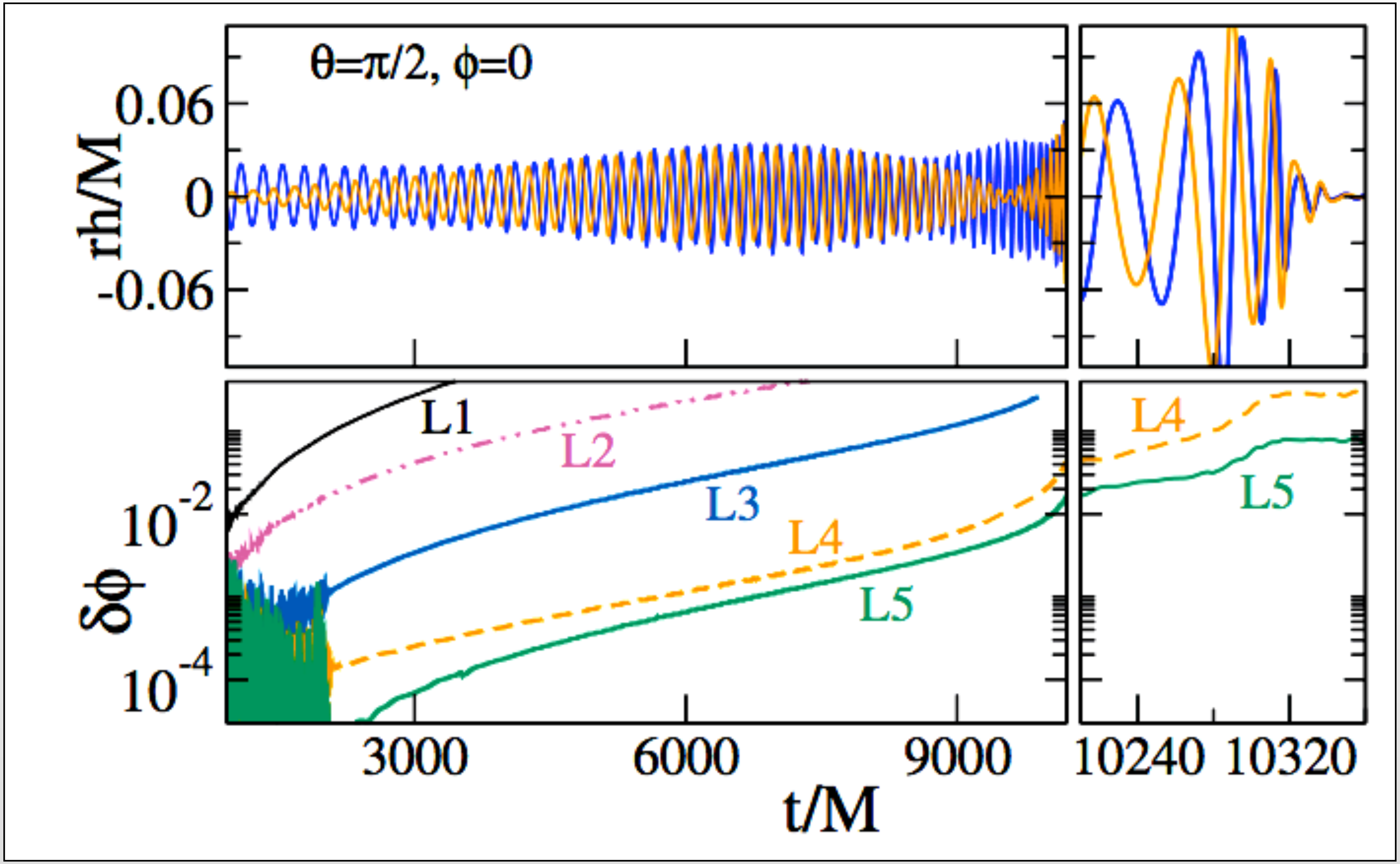
- Careful extrapolation with many consistency-checks

❖ Nick Taylor, et al (in prep):

- Cauchy-Characteristic Extraction & comparison with extrapolation



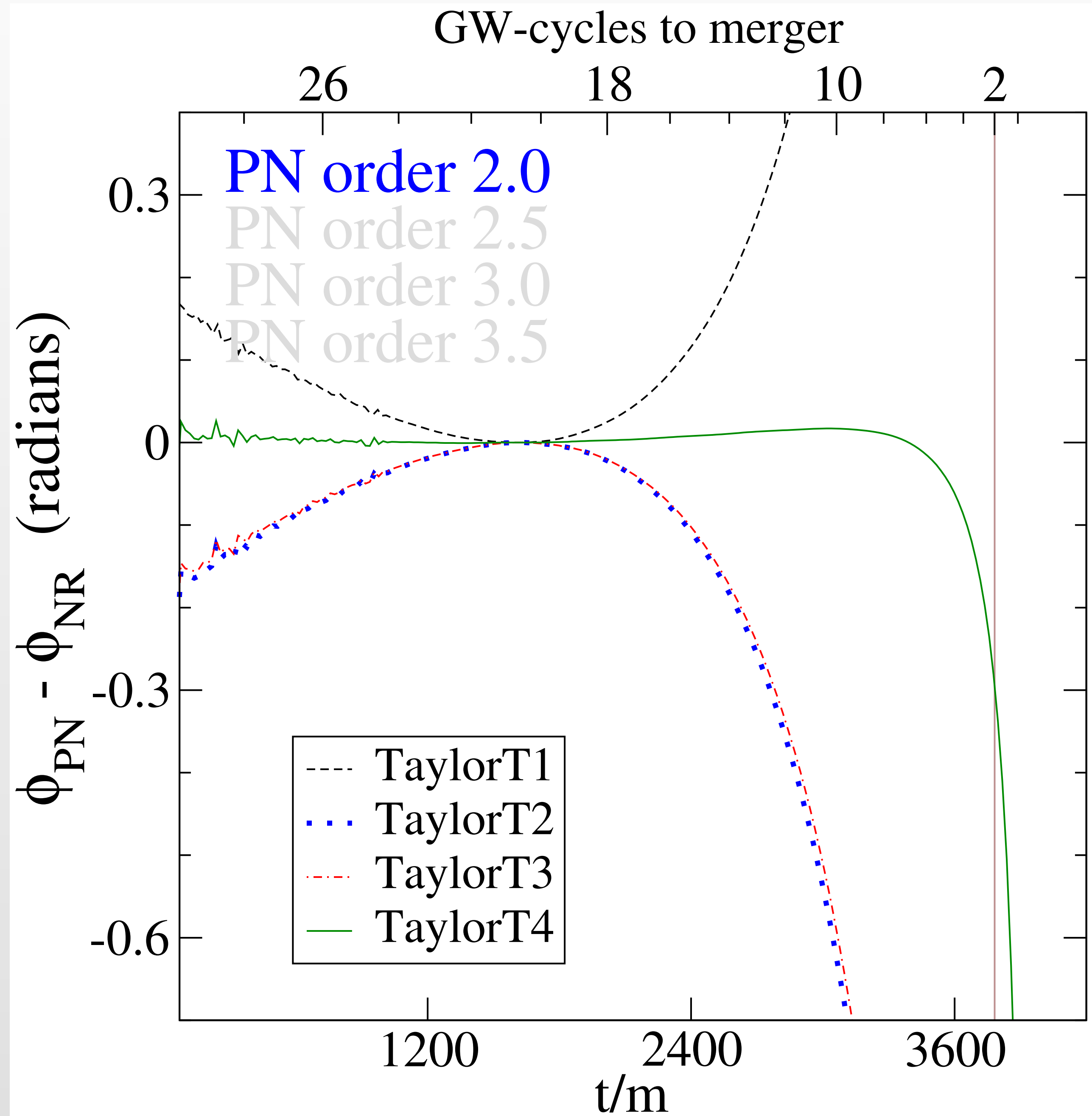
Convergence test



Mroue ea, arXiv:1304.6077

post-Newtonian -- NR comparison

- ❖ NR & PN agree!



Boyle et al 2007

post-Newtonian -- NR comparison

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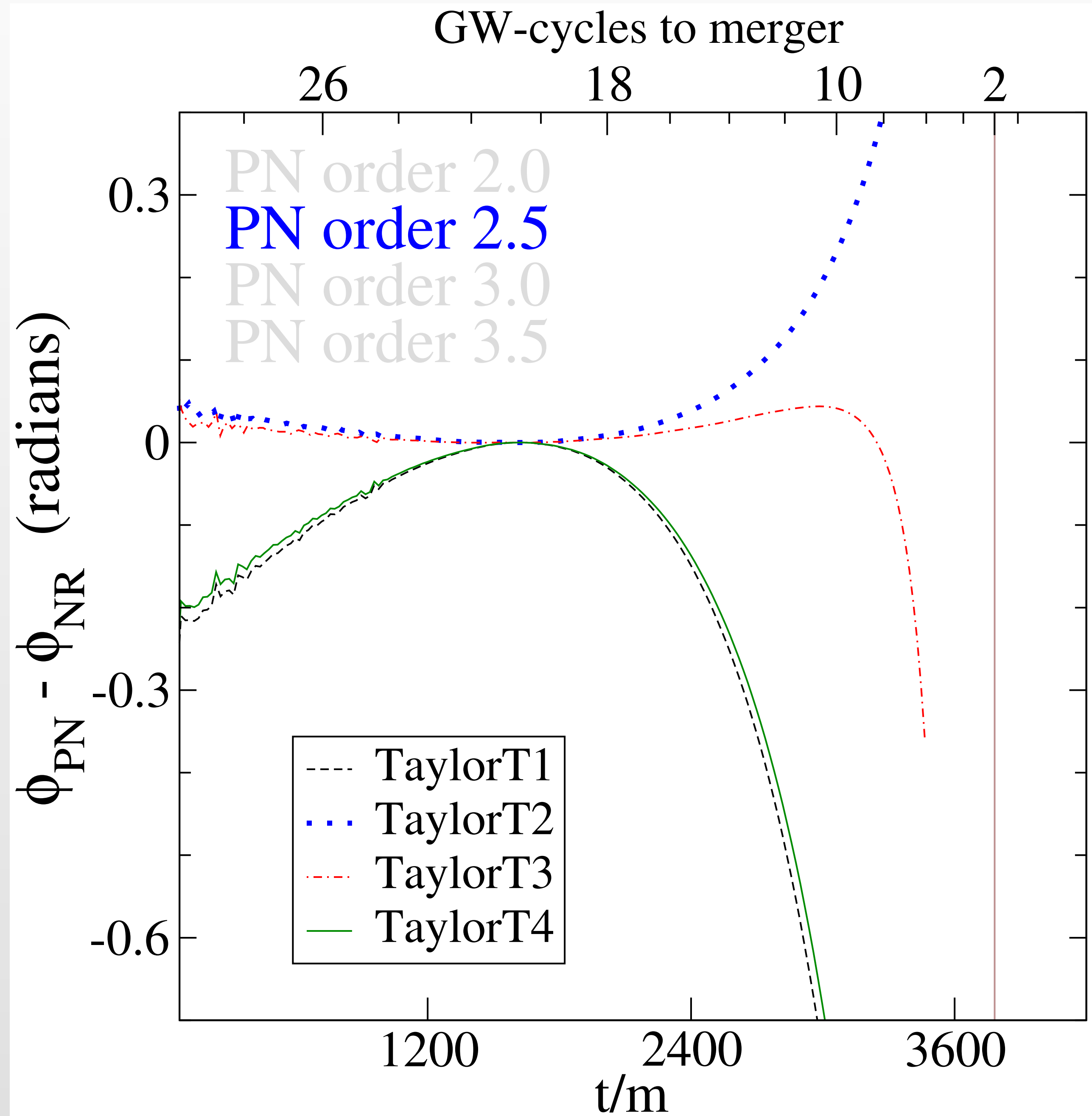

Boyle et al 2007

post-Newtonian -- NR comparison

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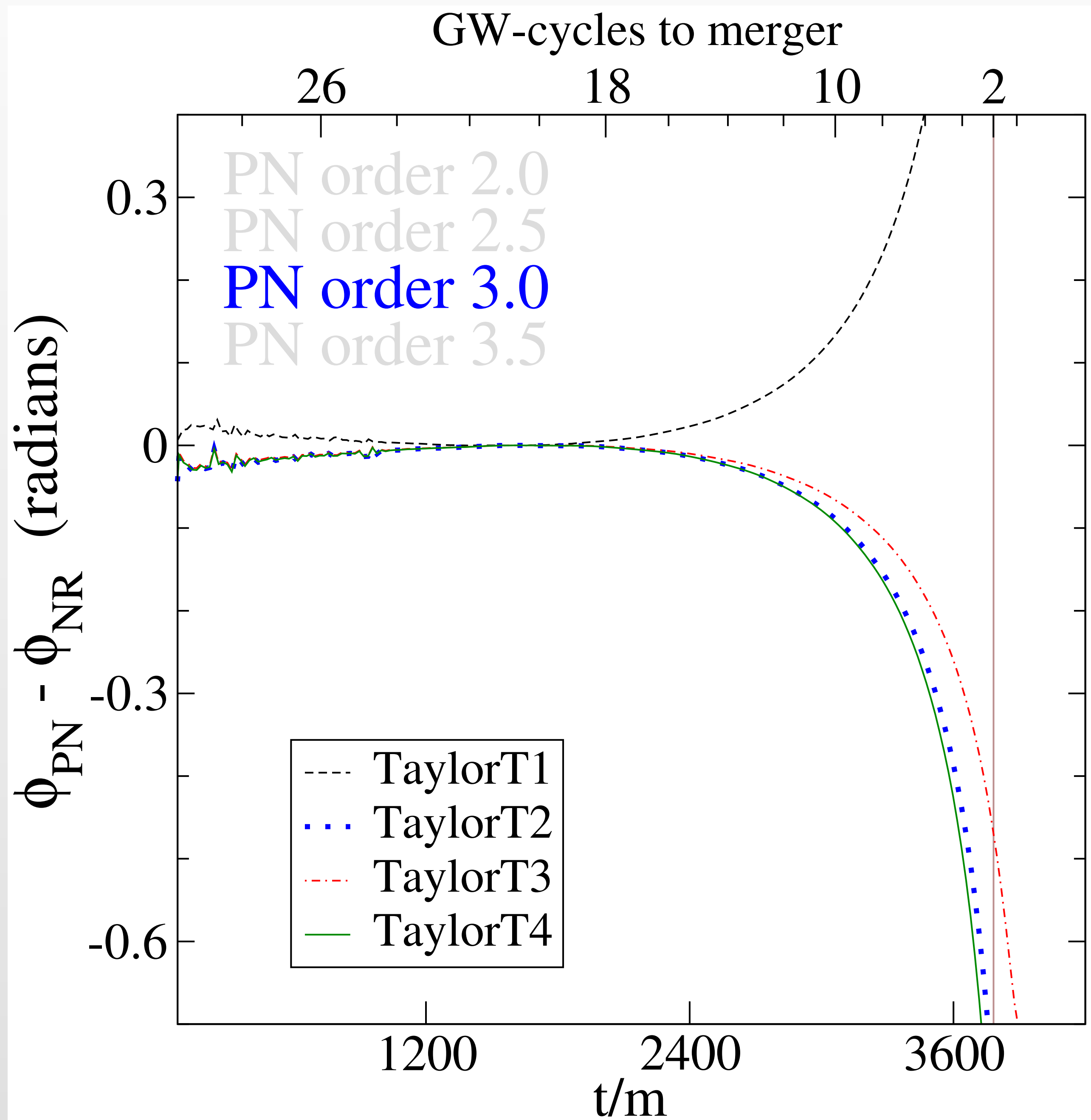

Boyle et al 2007

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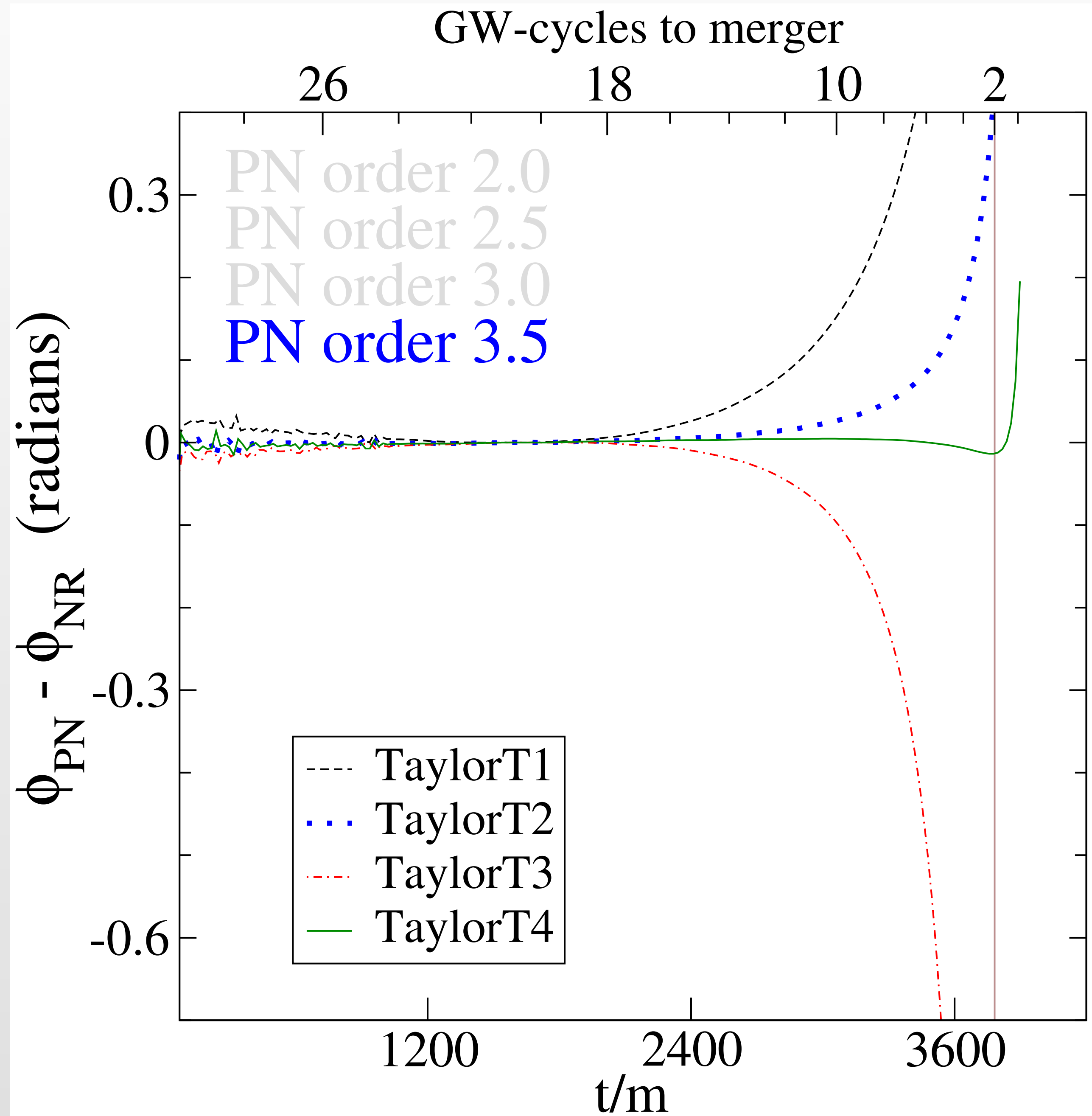
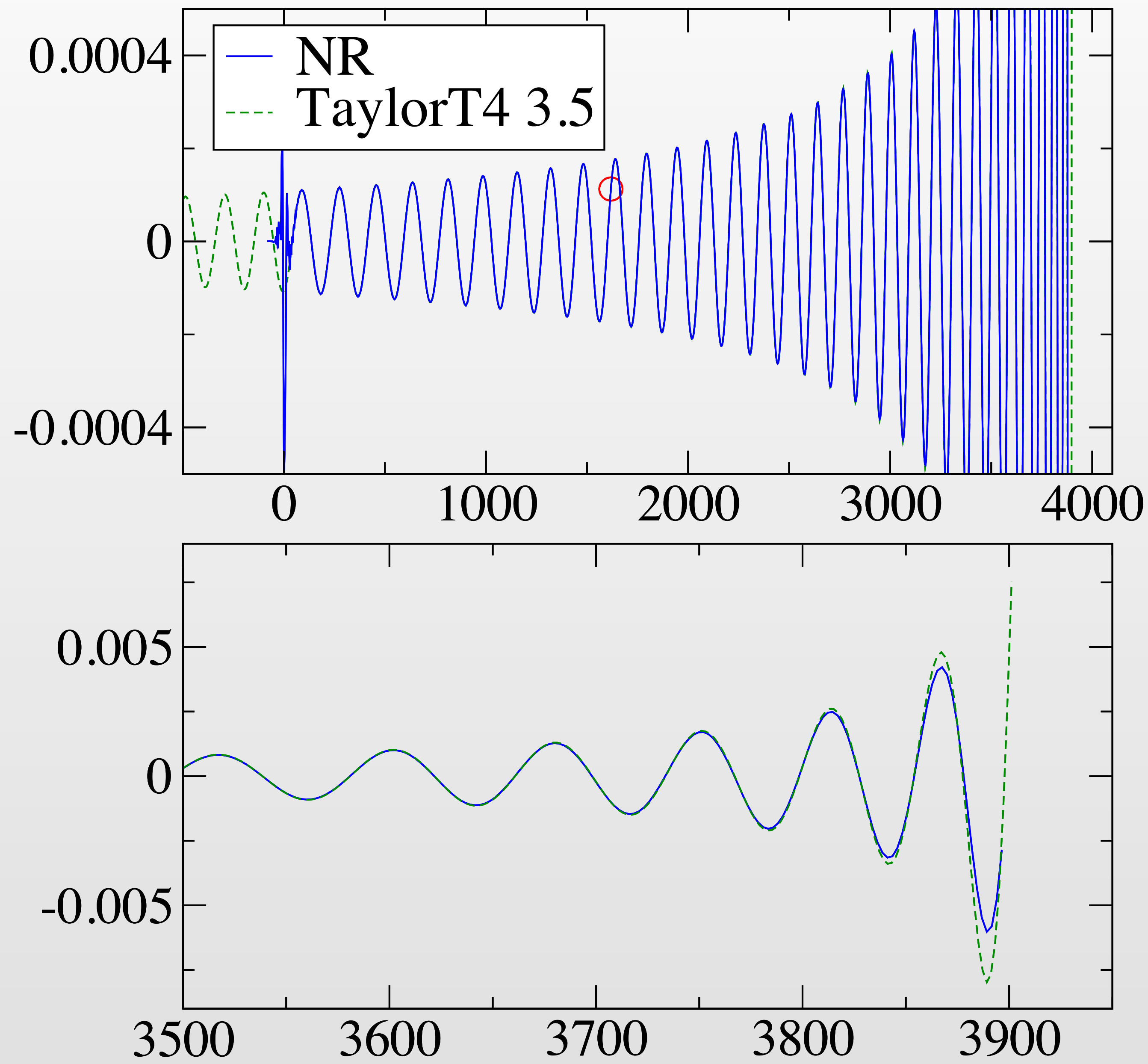


Boyle et al 2007

post-Newtonian -- NR comparison

- ❖ NR & PN agree!

- ❖ Or do they?
  - SOME versions of PN match very well
  - NO a priori knowledge which ones work (if any)



Boyle et al 2007

post-Newtonian -- NR comparison

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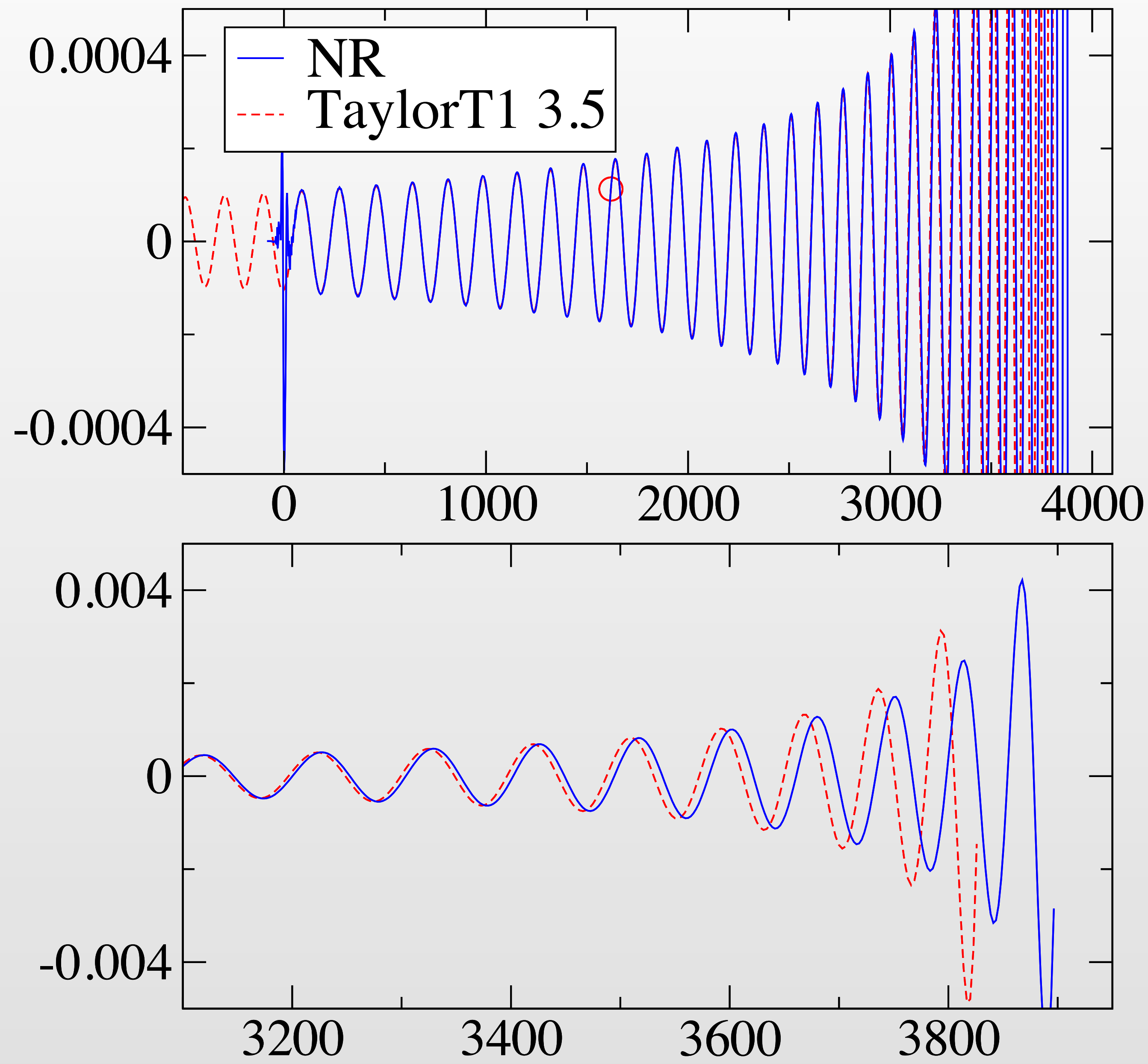


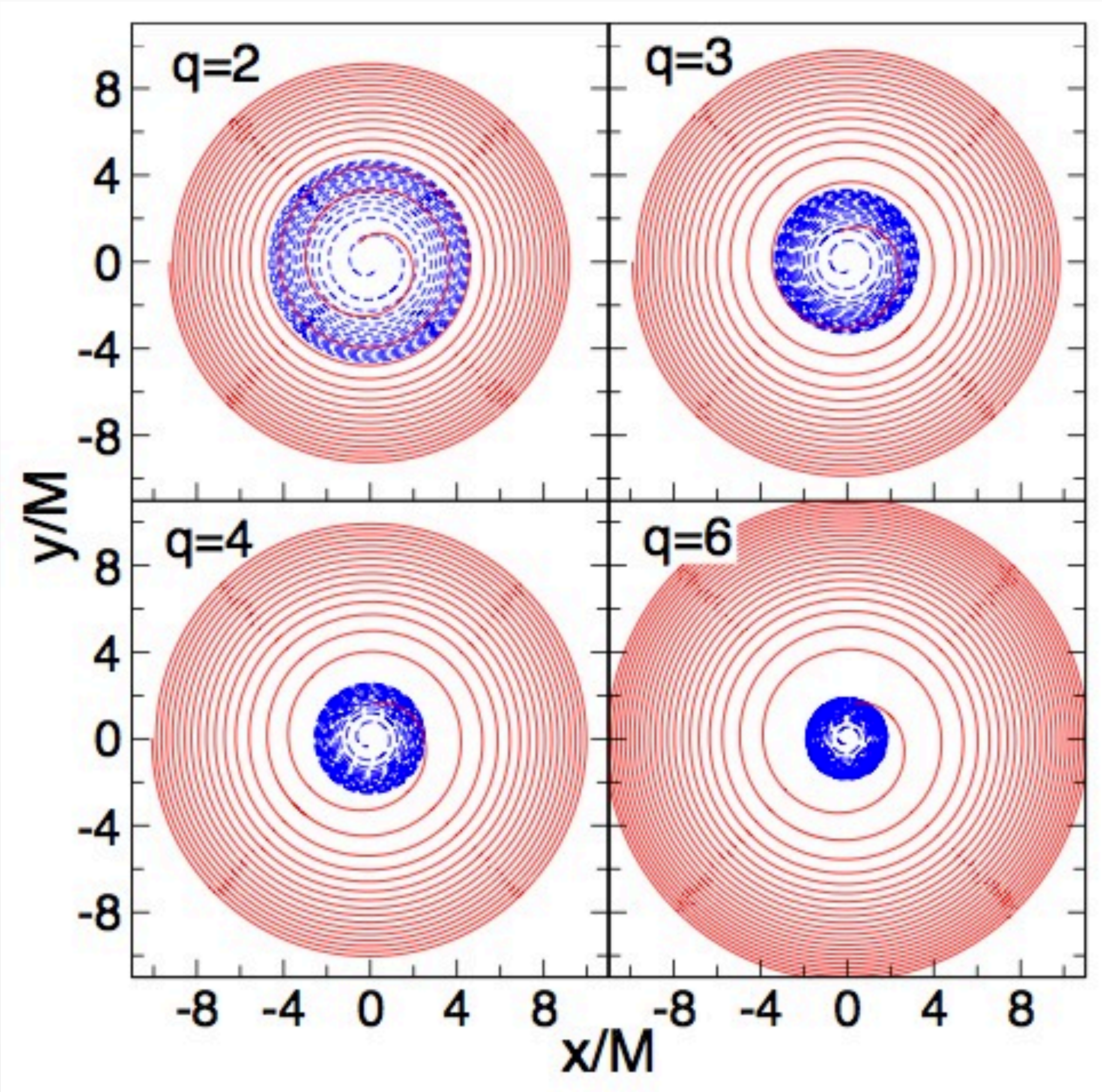
Boyle et al 2007

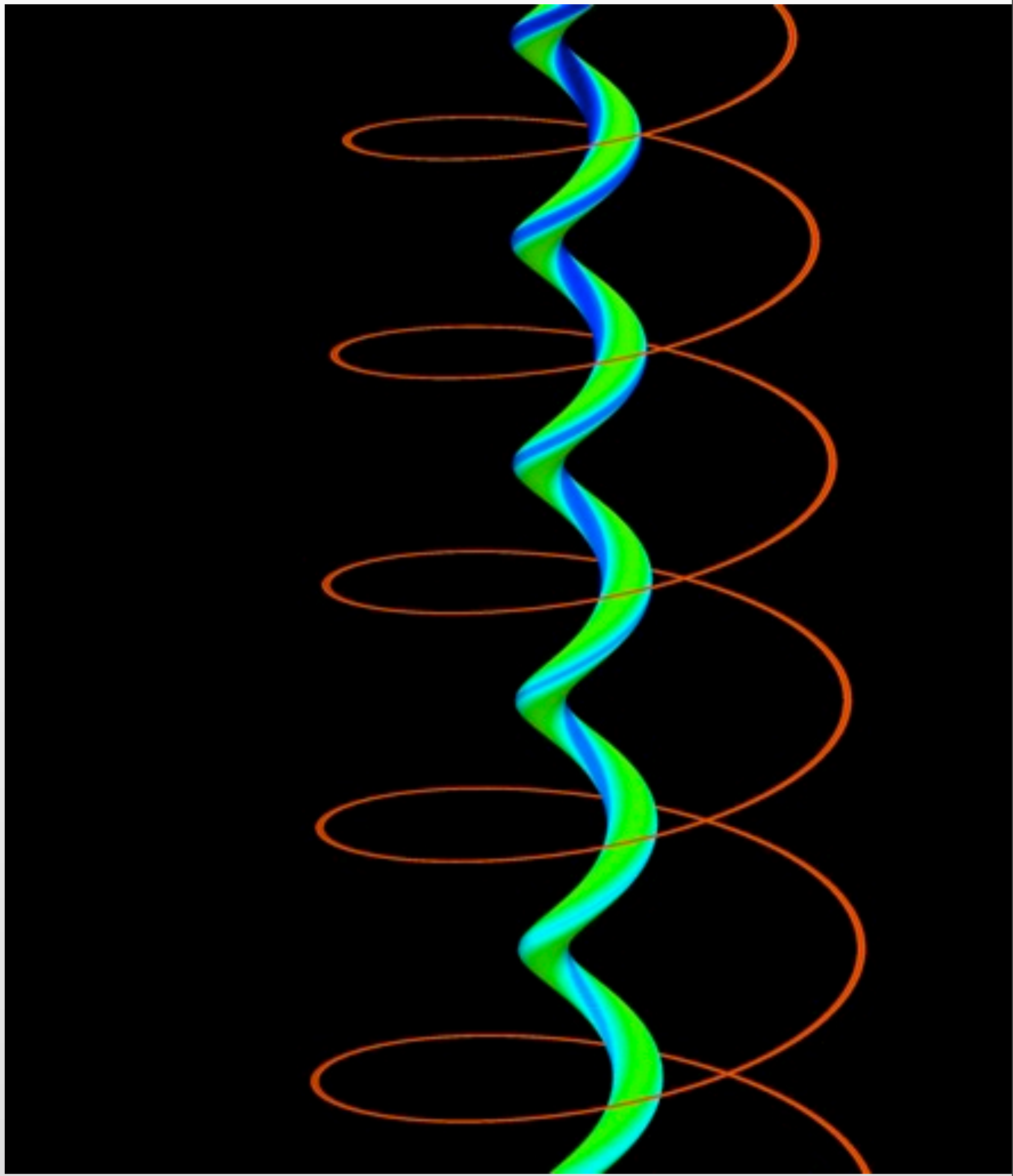
Unequal mass, non-spinning BH-BH

L. Buchman, HP,
M. Scheel, B. Szilagyi,
1206.3015

q=1,2,3,4 (15 orbits), q=6 (20 orbits)

CITA–ICAT

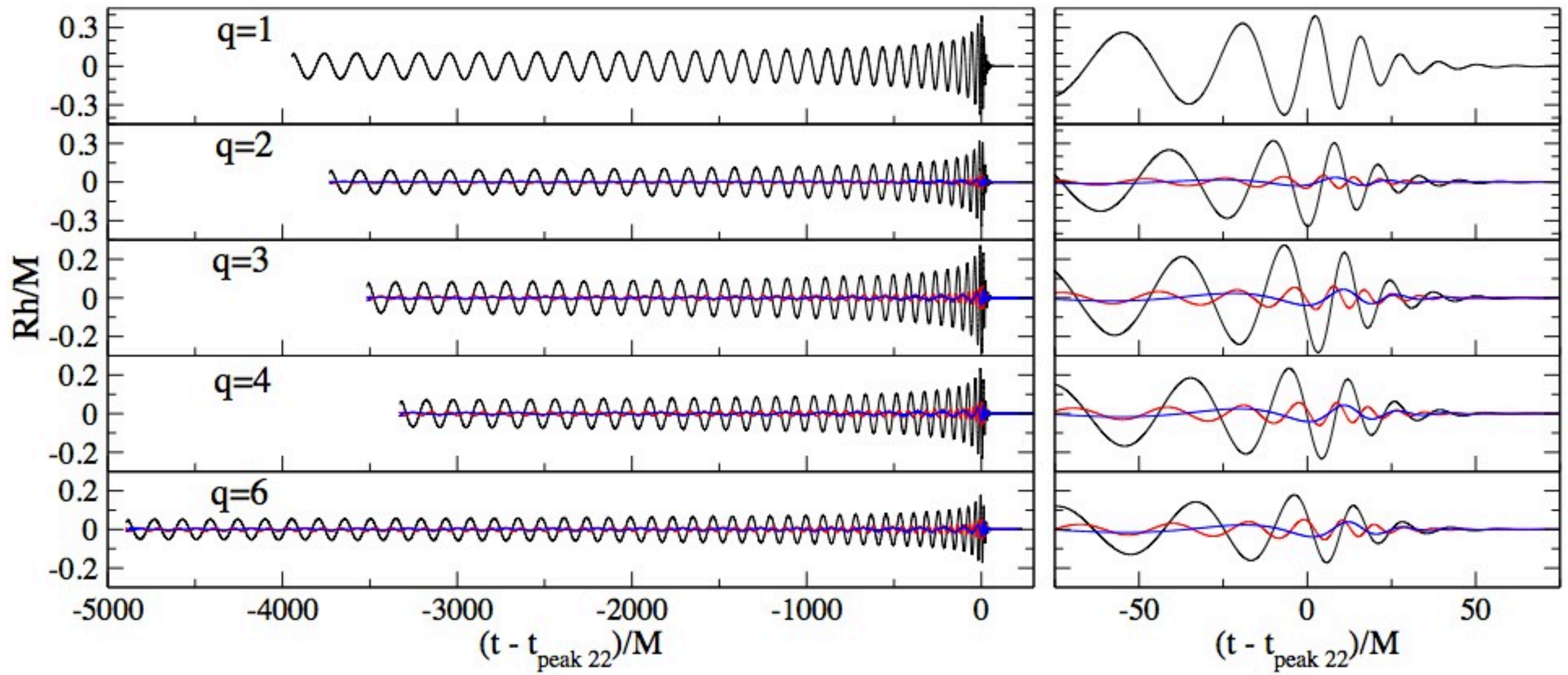




q=6 space-time diagram of AH's, colored by R

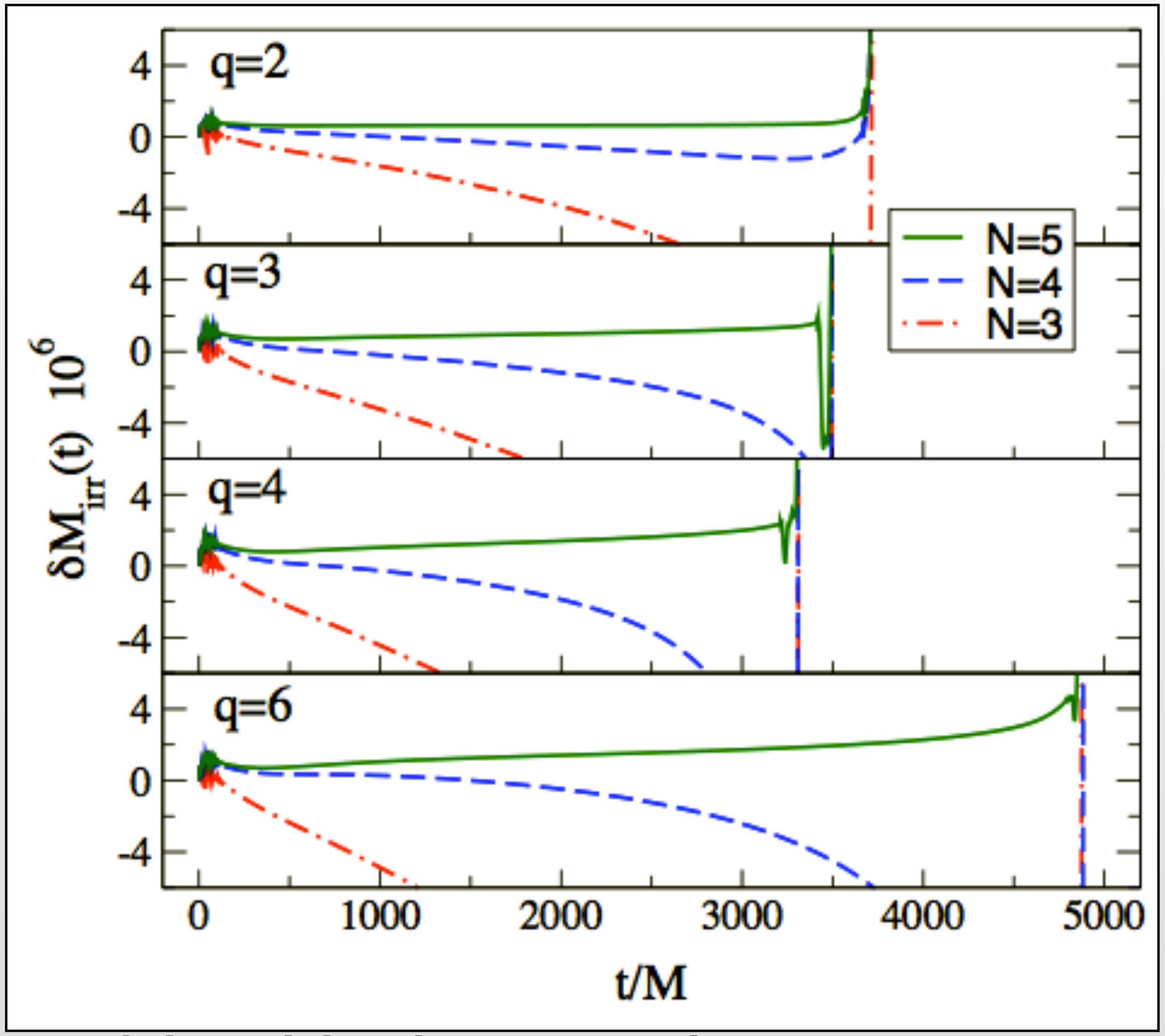
Waveforms

❖ Mass-ratios q=1,2,3,4,6. 15 orbits (20 for q=6)



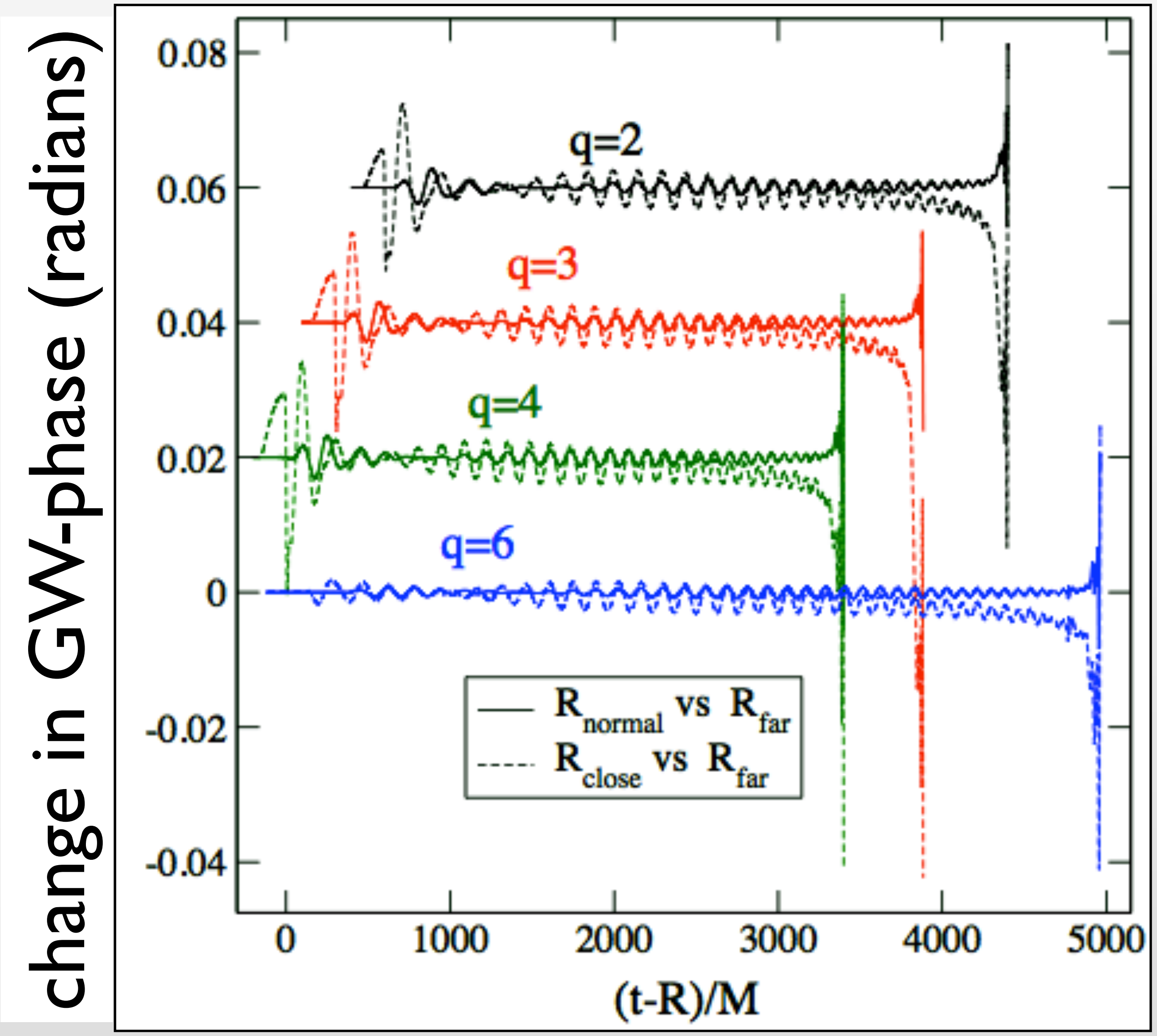
Accuracy

- Sum of irreducible masses ~1 part in 10^6



Need higher resolution to resolve tidal heating :(

- (non-)effect of artificial outer boundary

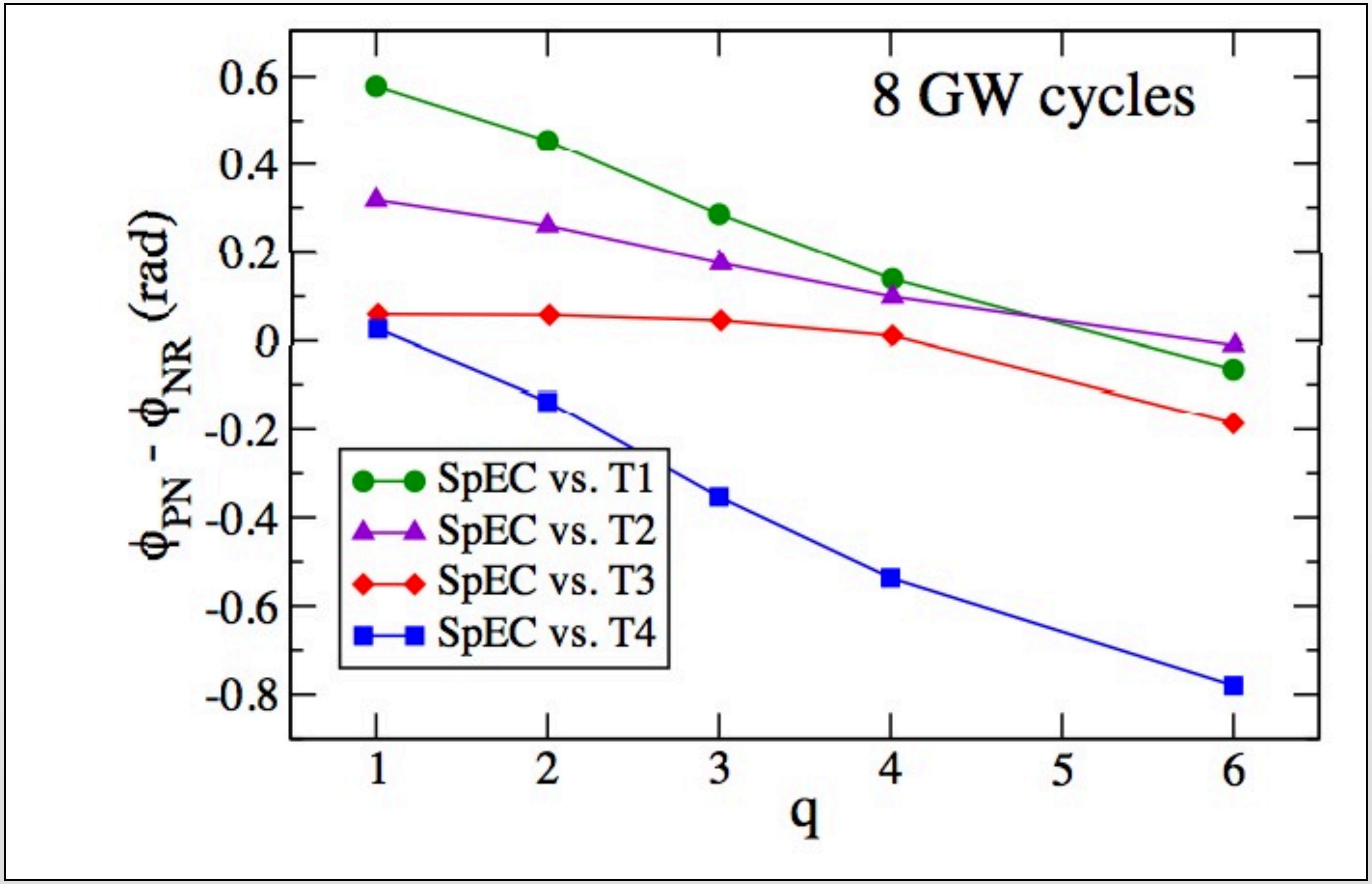


Necessary Length of Numerical Waveforms

MacDonald, Nissanke, HP, 2011
MacDonald, Mroue, HP, ... 2012

PN NR comparison

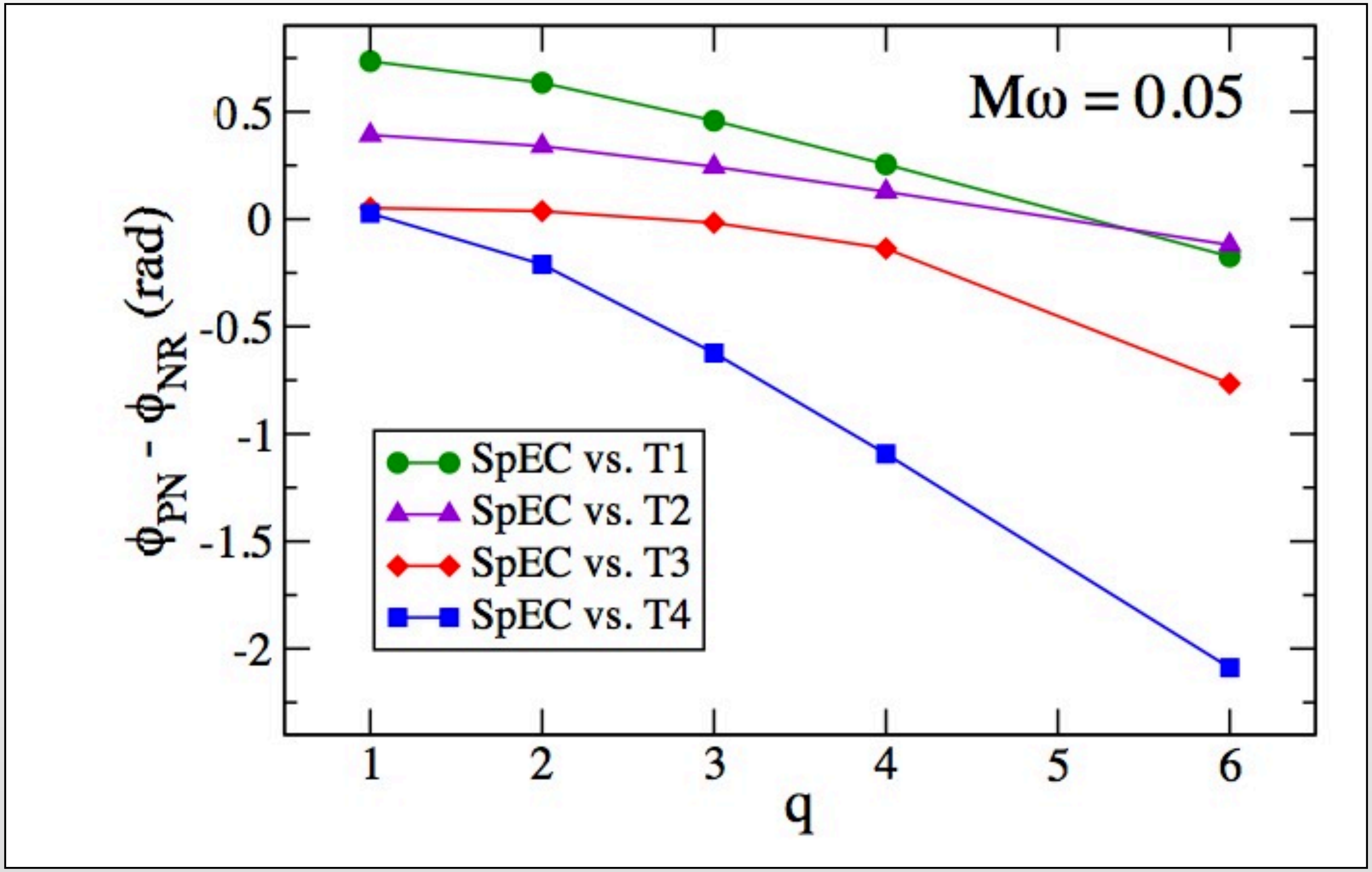
- ❖ Match at Mω=0.1, compute phase-difference over preceding 8 GW-cycles



MacDonald ea, arXiv:1210.3007

PN NR comparison

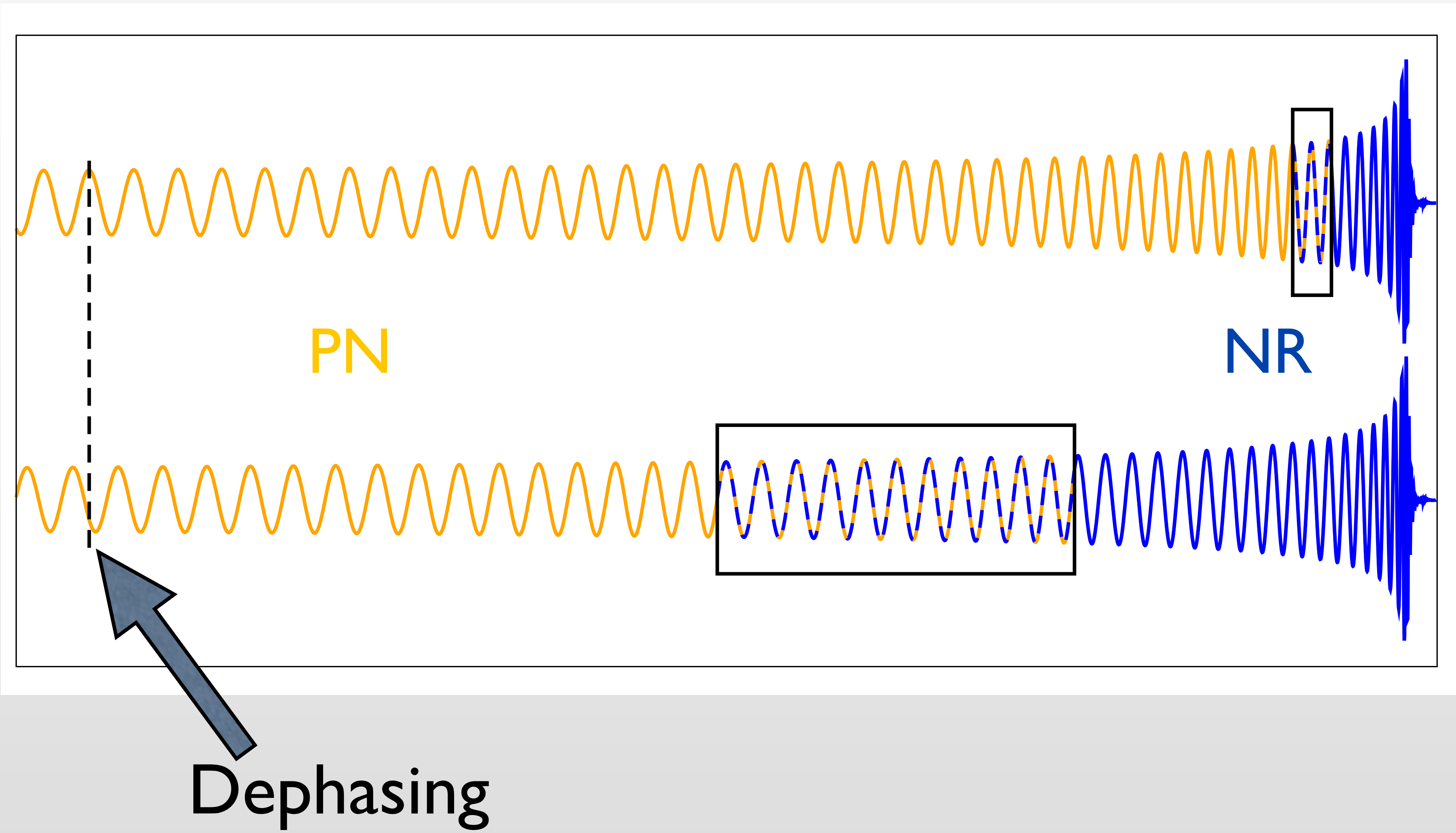
❖ Match at Mω=0.1, compute phase-difference down to Mω=0.05



MacDonald ea, arXiv:1210.3007

Length requirements for NR

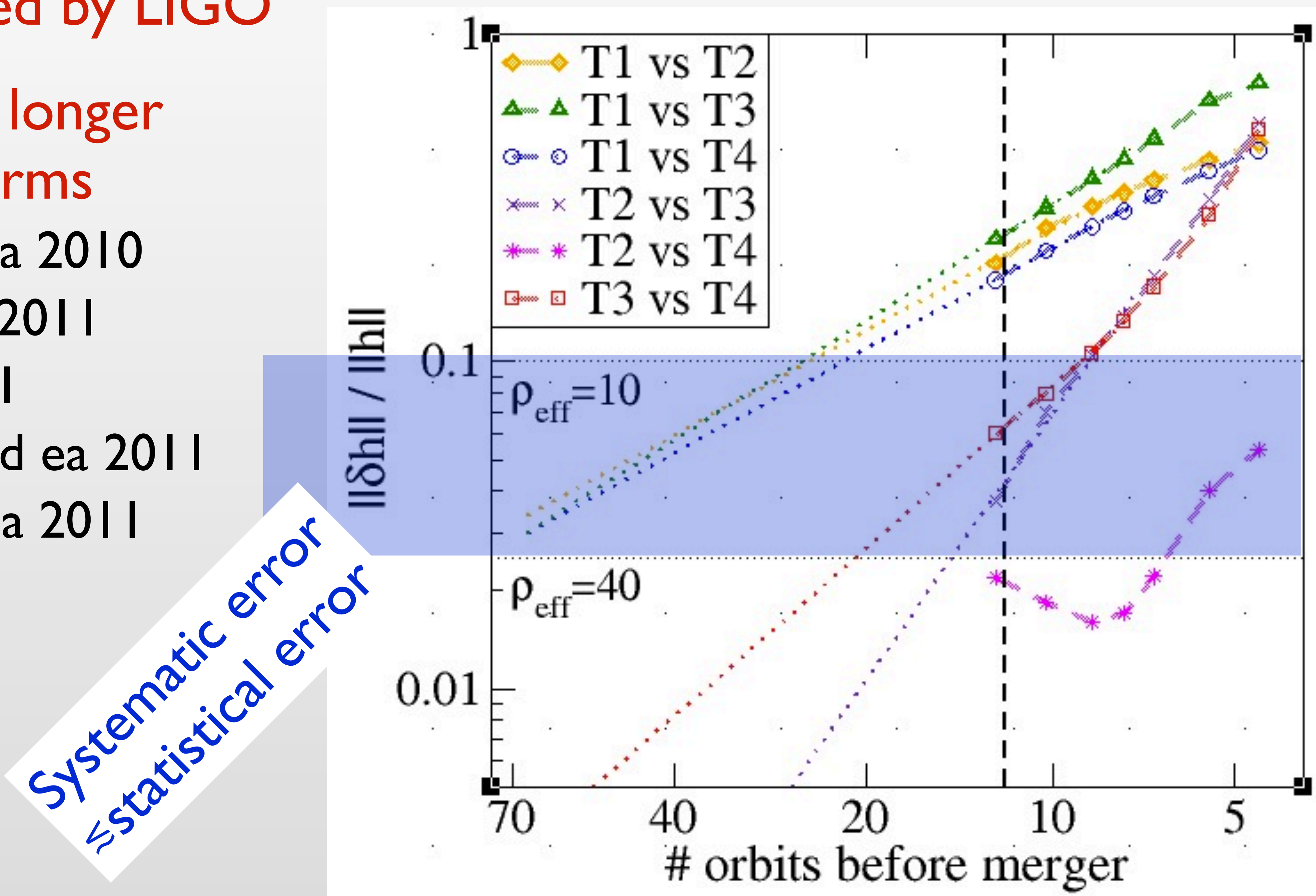
❖ Must switch to NR early enough to avoid large PN errors



Length: Parameter estimation

CITA–ICAT

- Start NR so early that different PN approximants cannot be distinguished by LIGO
- need *much* longer NR waveforms
  - Hannam ea 2010
  - Ohme ea 2011
  - Boyle 2011
  - MacDonald ea 2011
  - Damour ea 2011

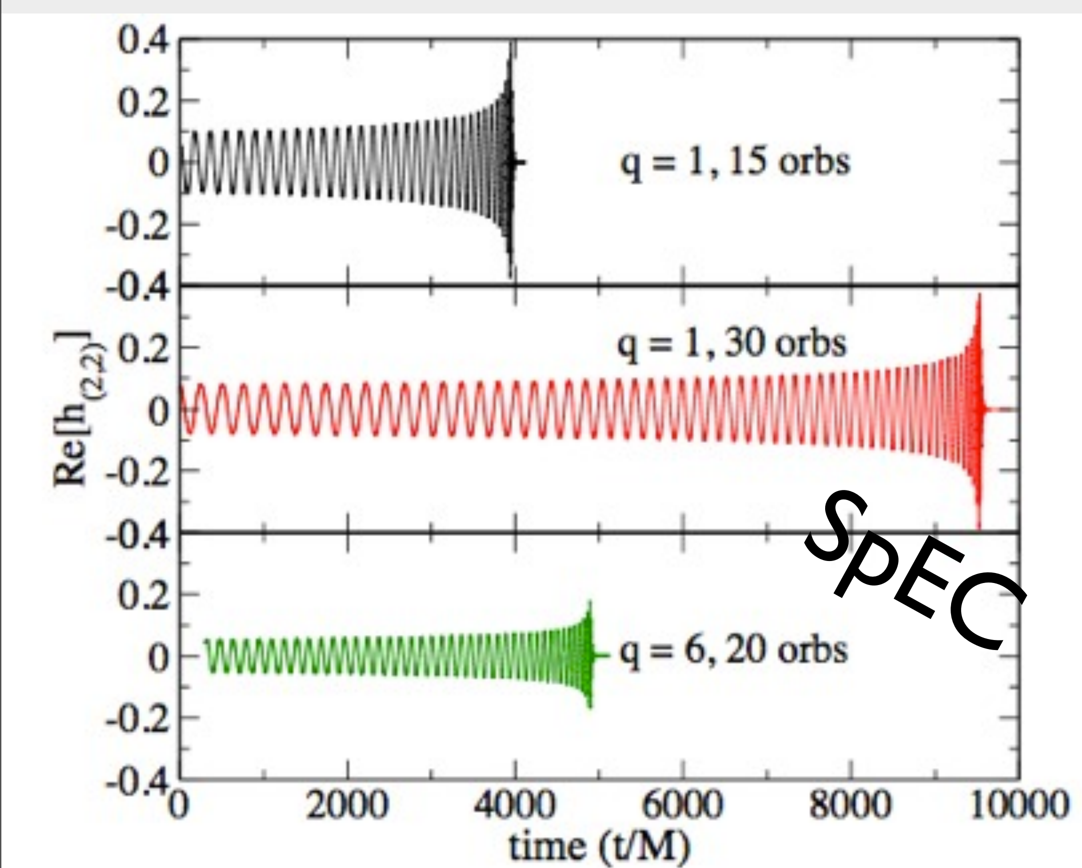


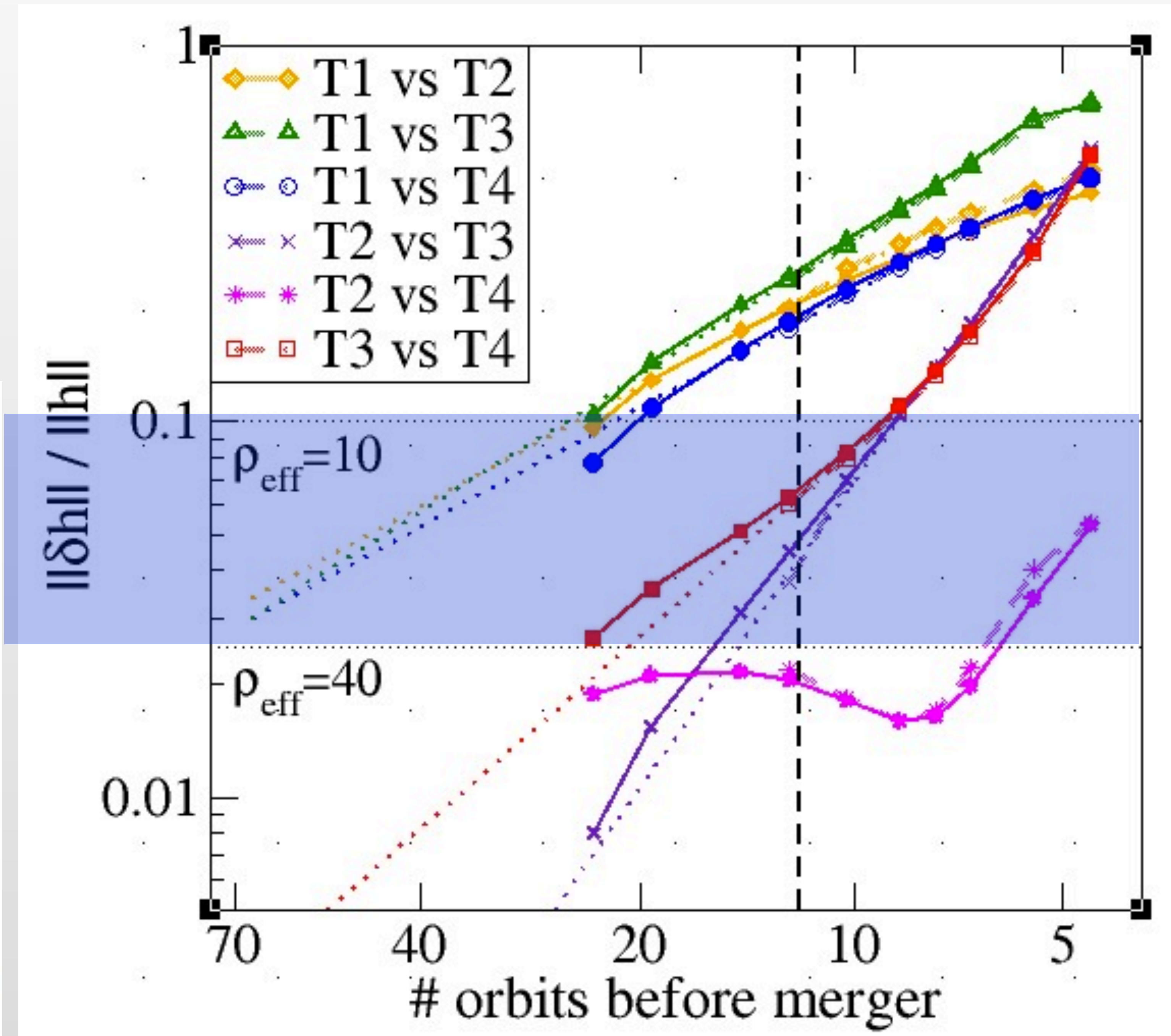
MacDonald, Nissanke, HP 2011

Length: Parameter estimation

❖ New 30 orbit equal-mass, zero spin simulation

- Confirm previous results
- Long enough for *one* choice of parameters

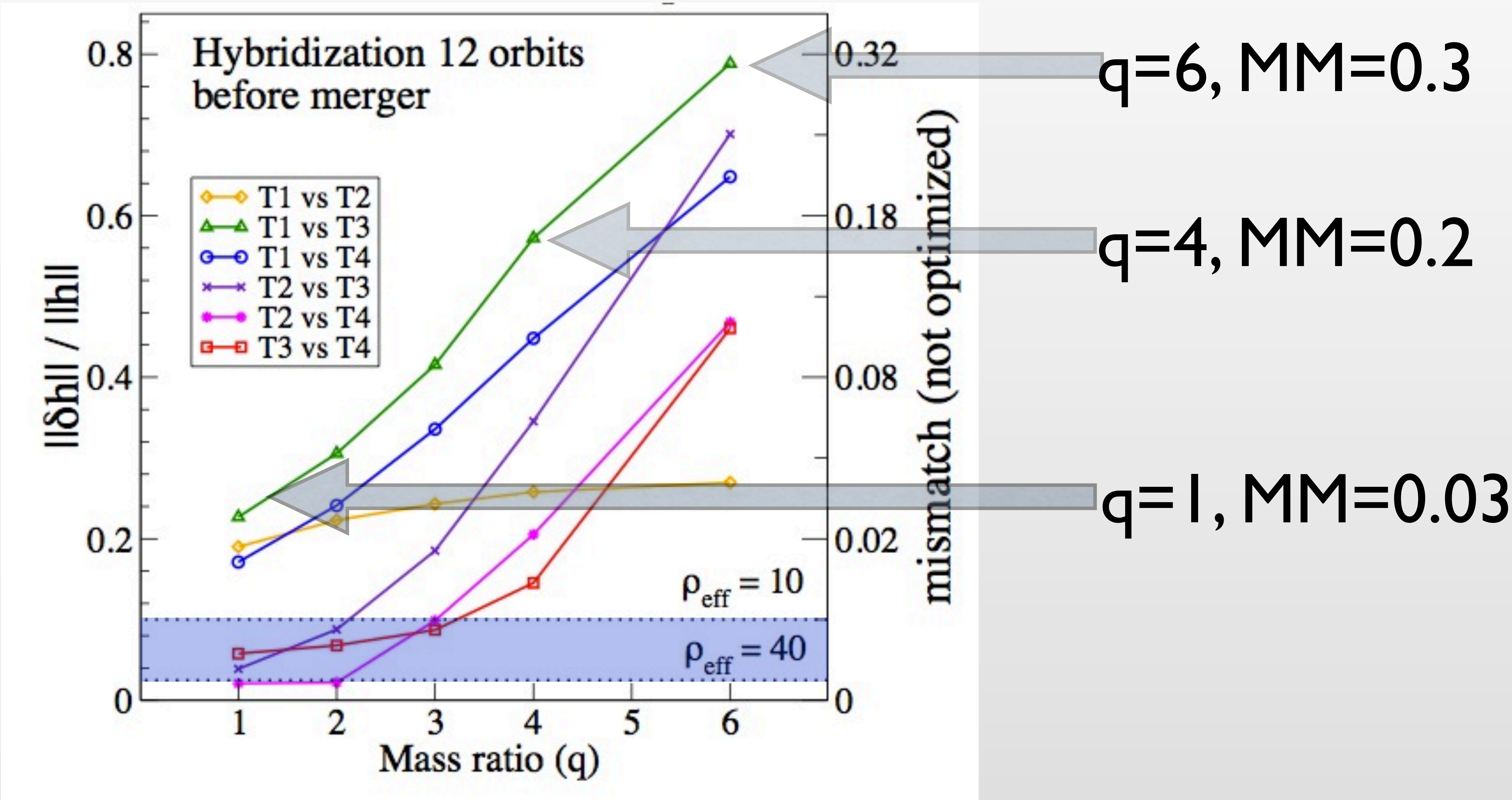




MacDonald, Mroue, HP in prep

Length-Statements depend on λ

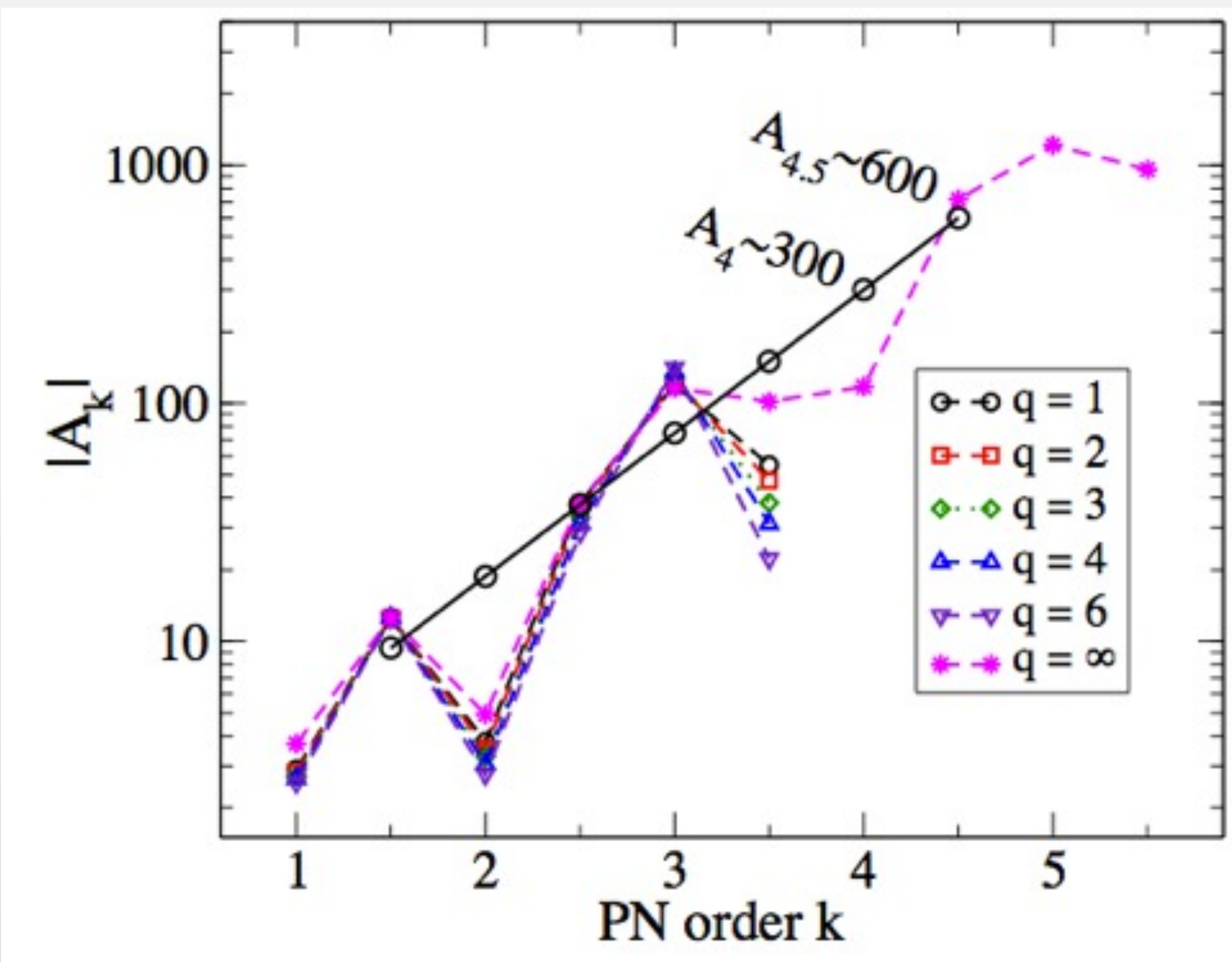
❖ Non-spinning, unequal masses

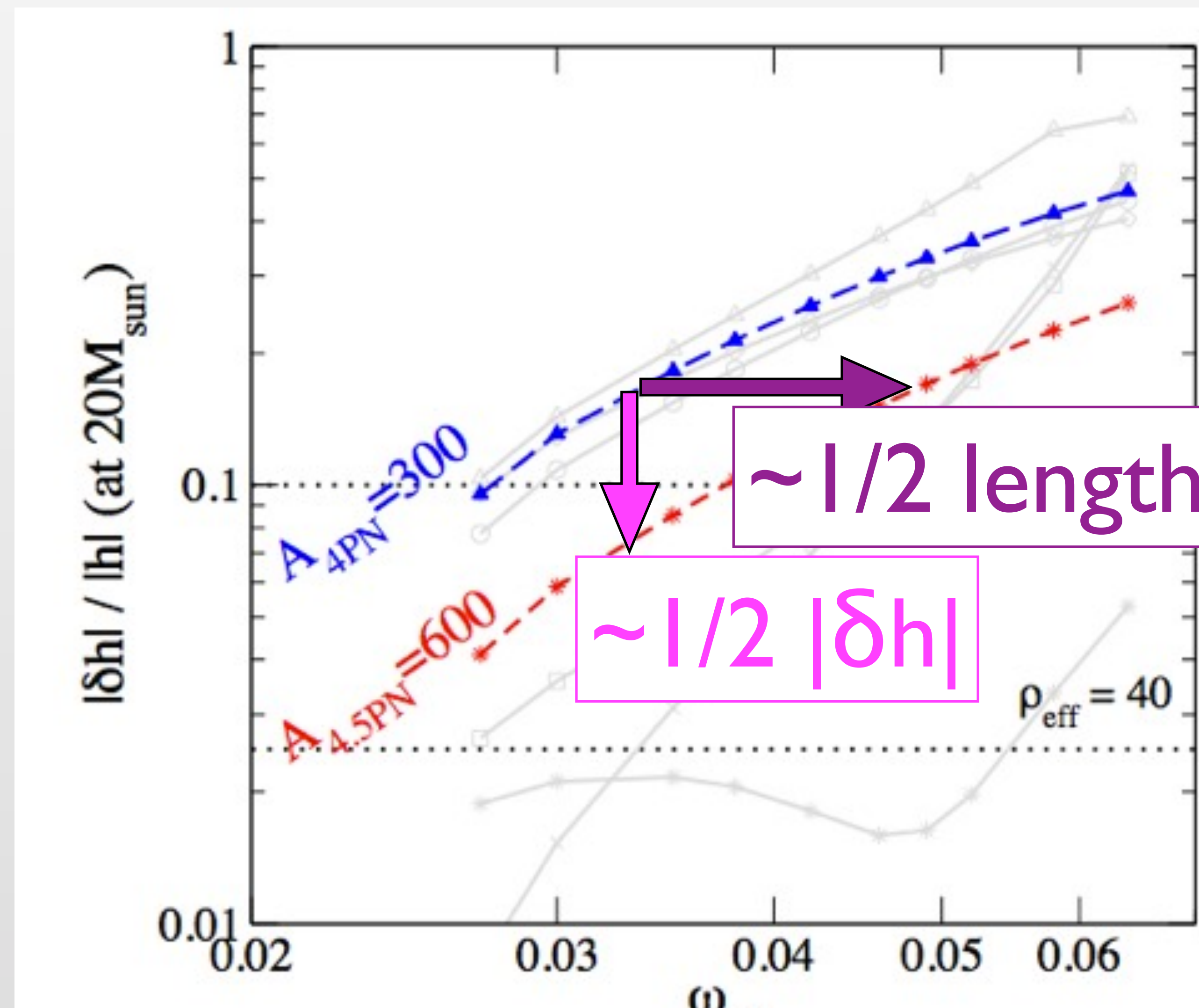


MacDonald, Mroue, HP in prep
(similar results in Ohme ea, 2011)

Estimated impact of 4-PN

- ❖ TaylorT4 phase-evolution $$\frac{dx}{dt} = \frac{64c^3\nu}{5GM}x^5\left(1+\sum_k A_k x^k\right)$$





MacDonald, Mroue, HP (in prep)

Longer NR-waveforms: Alternatives

CITA–ICAT

- ~~Option~~ 1: Longer NR?
  - Can **not** perform long enough sims

$$\frac{T}{M} \approx 5\nu^{3/5}(2\pi N)^{8/5}$$

- Option 2: Live with it
  - Ohme ea 2011: Systematic errors $\delta M/M \sim 0.1\%$, $\delta(S/M^2) \sim 0.1$
- Option 3: Wait for 4PN
  - Buys us a factor of 2
- Option 4: Relax rigor
  - Only δh *tangential* to signal-manifold causes systematic errors (➞Ilya Mandel's talk). *Give up on testing GR with orthogonal δh*
  - Fit PN or EOB to improve agreement with NR

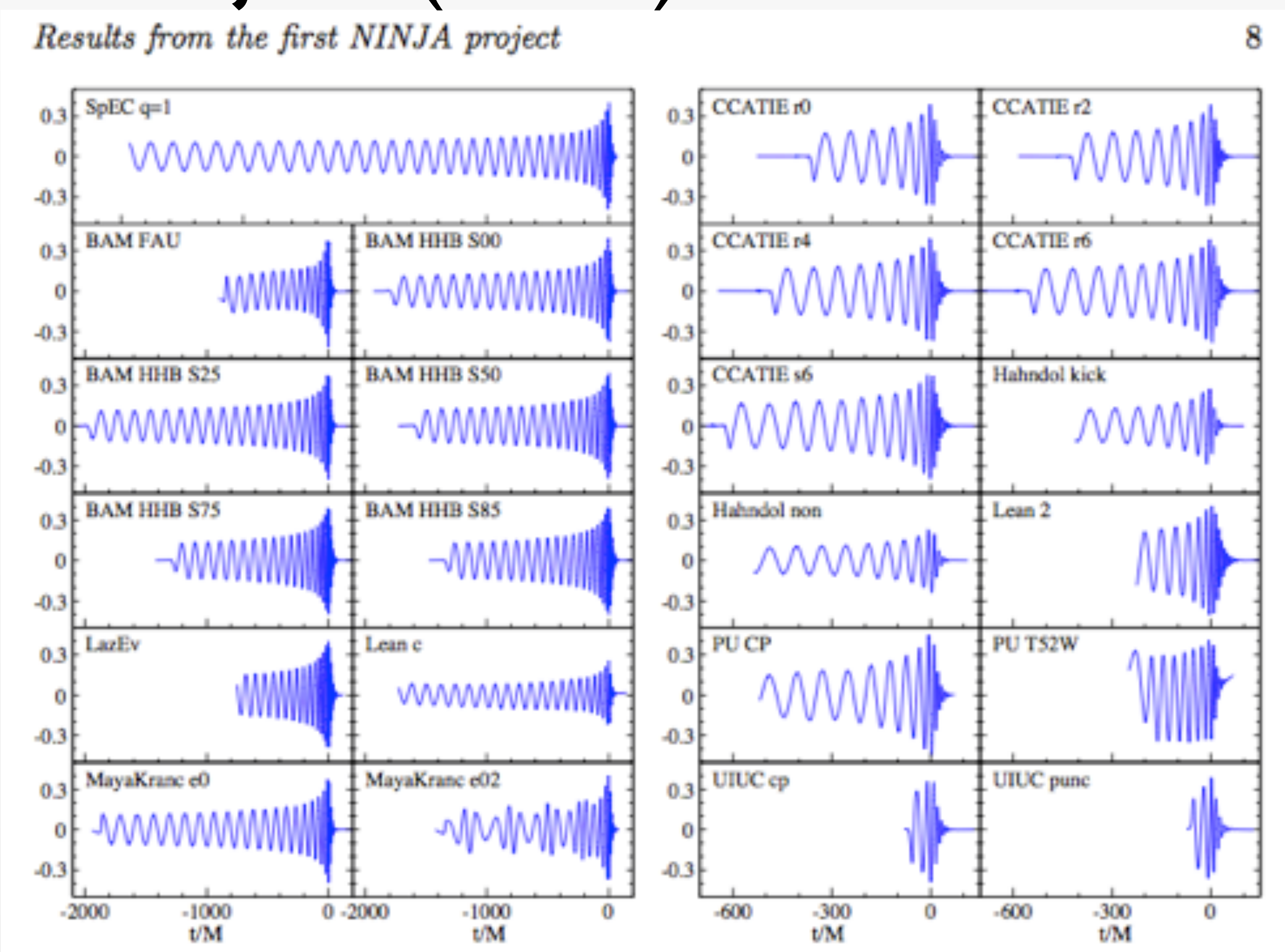
    Introduces dependence between NR and analytical waveforms, which may bias accuracy estimate of model.

Exploring Parameter Space & Precession

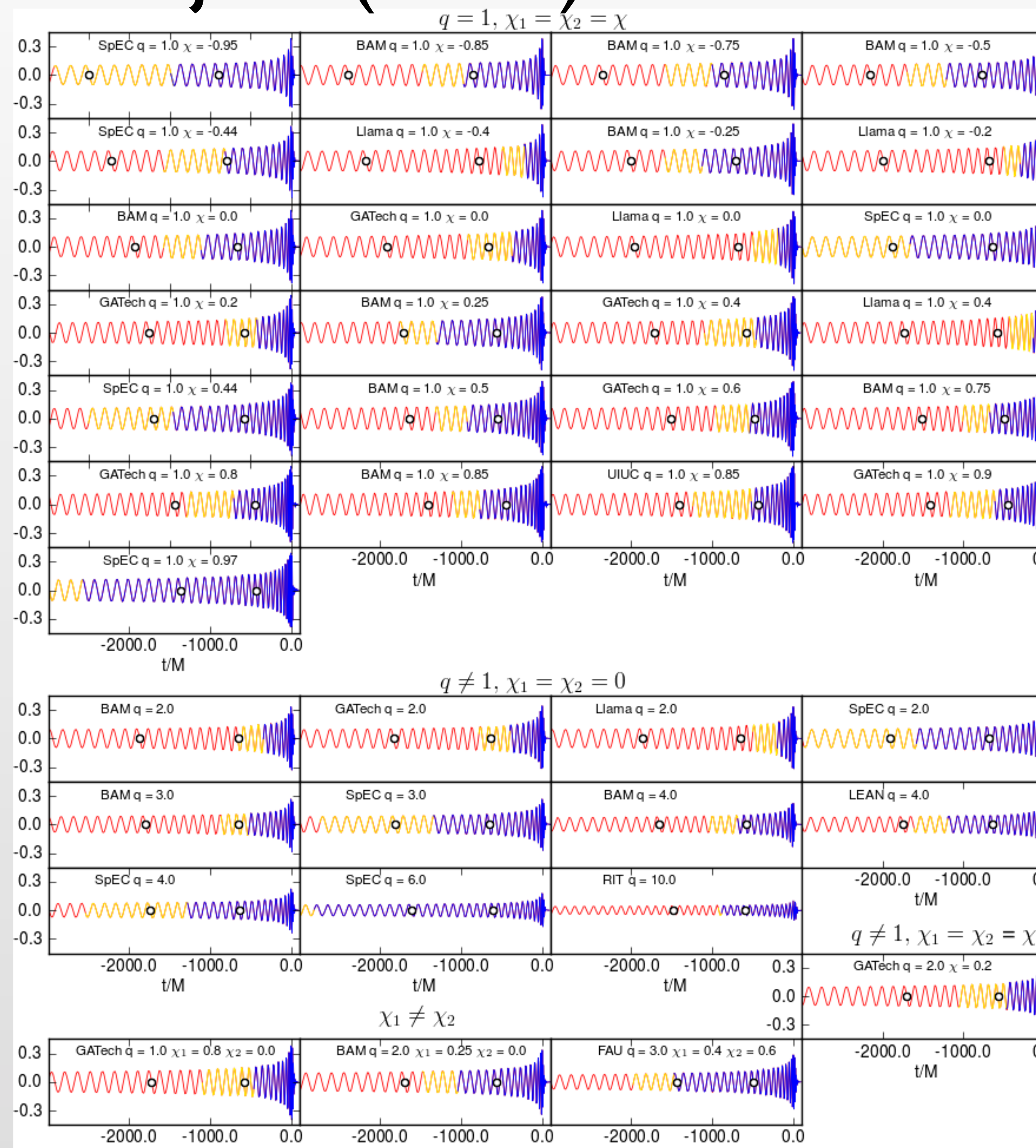
Waveform Catalog Efforts

CITA–ICAT

Ninja1 (2008)



Ninja2 (2012)



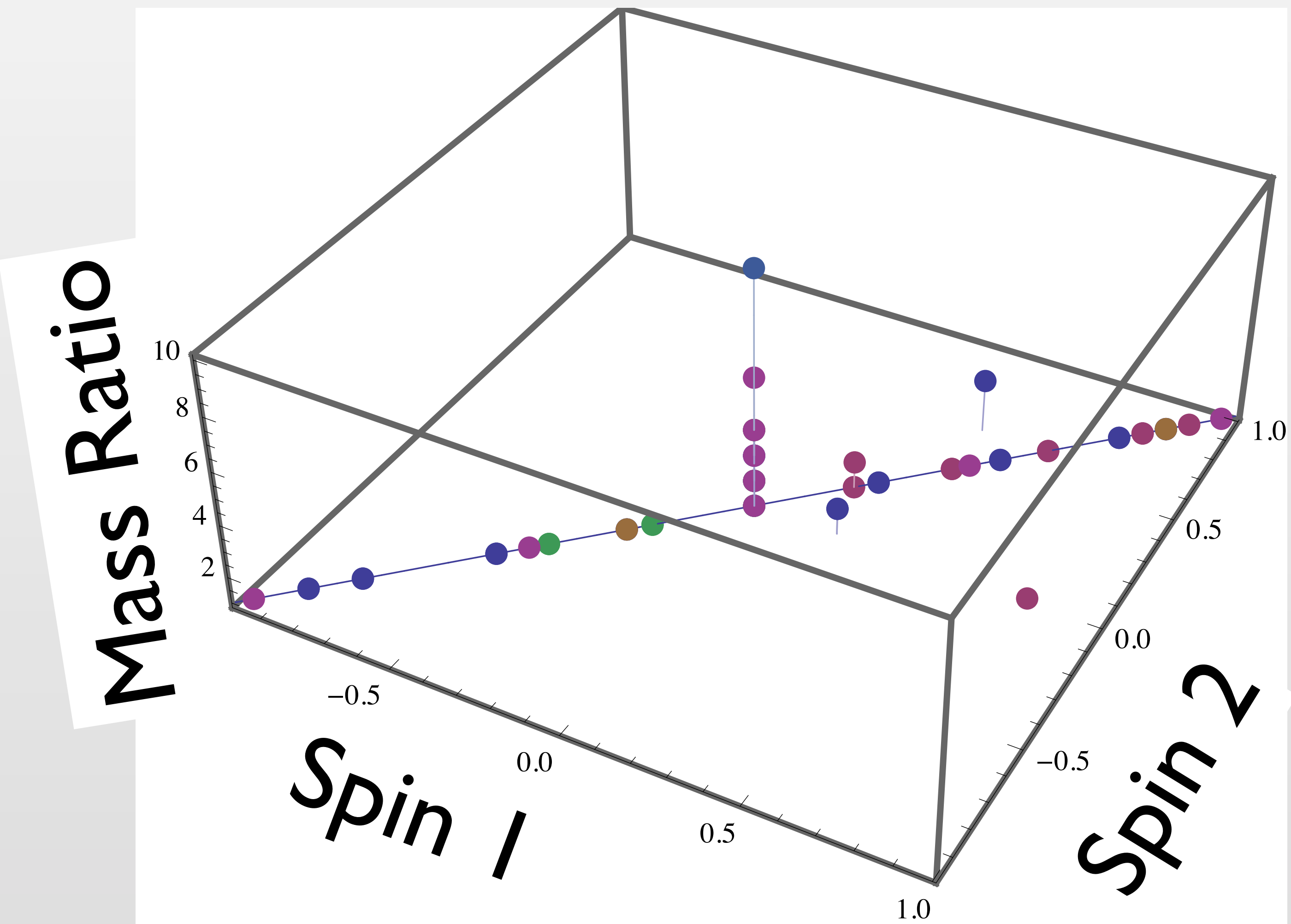
Thursday, June 27, 13

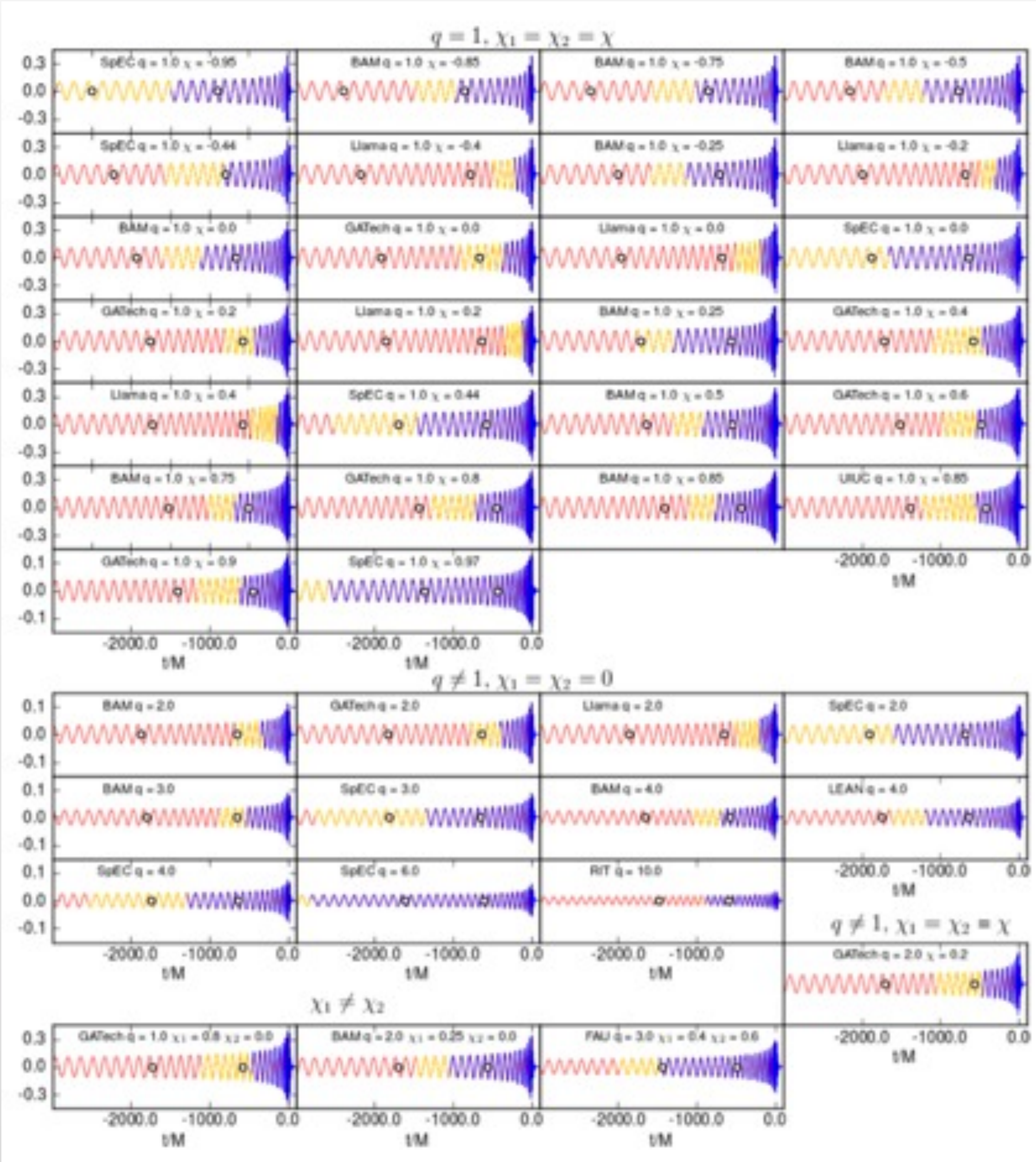
Lack of parameter space coverage

CITA–ICAT

❖ BH-BH simulations are hard

- World-wide NINJA-2 collaboration computed 40 spin-alinged systems (no precession at all)



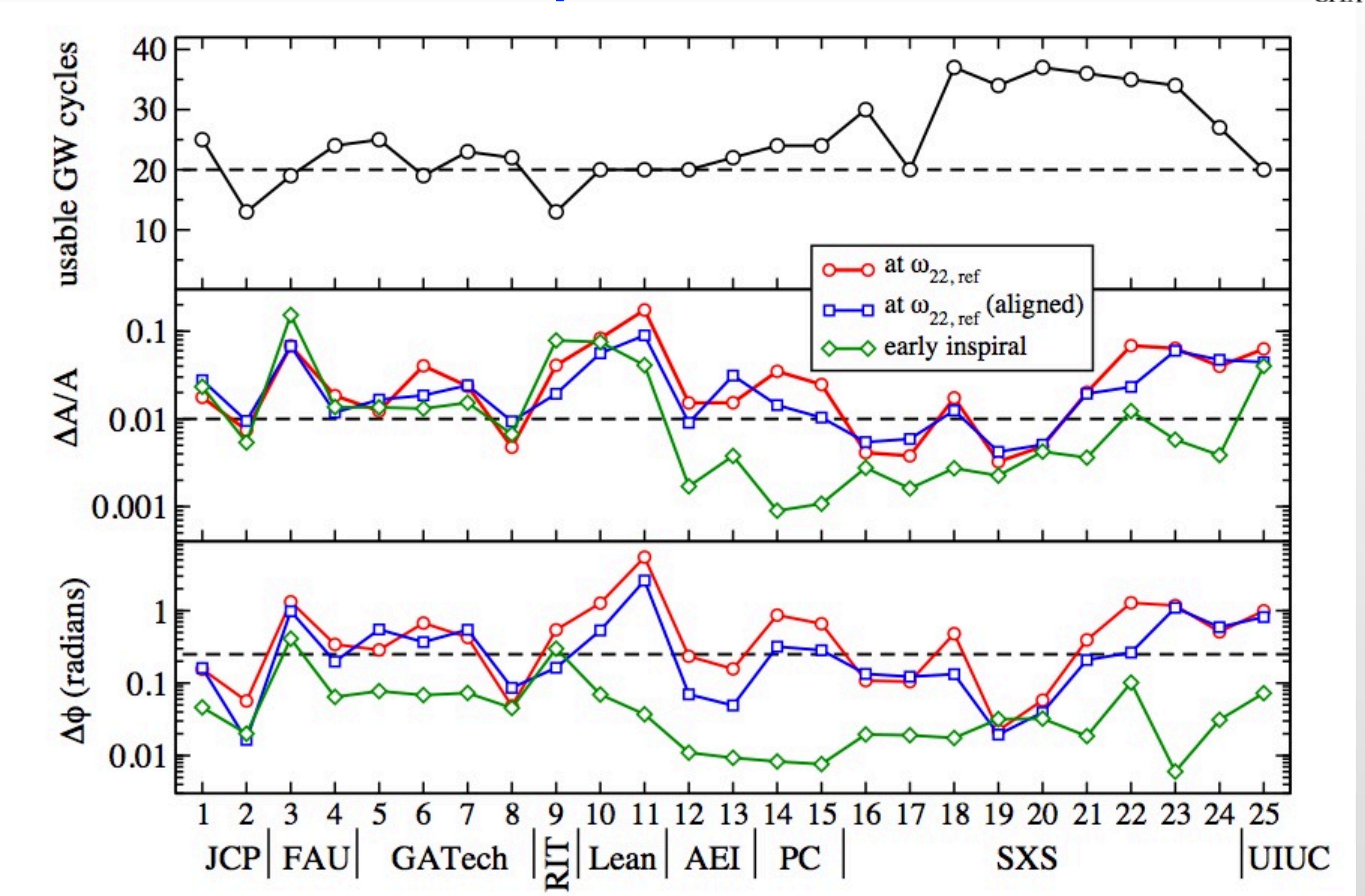


Ajith ea, 1211.5319

NR-AR

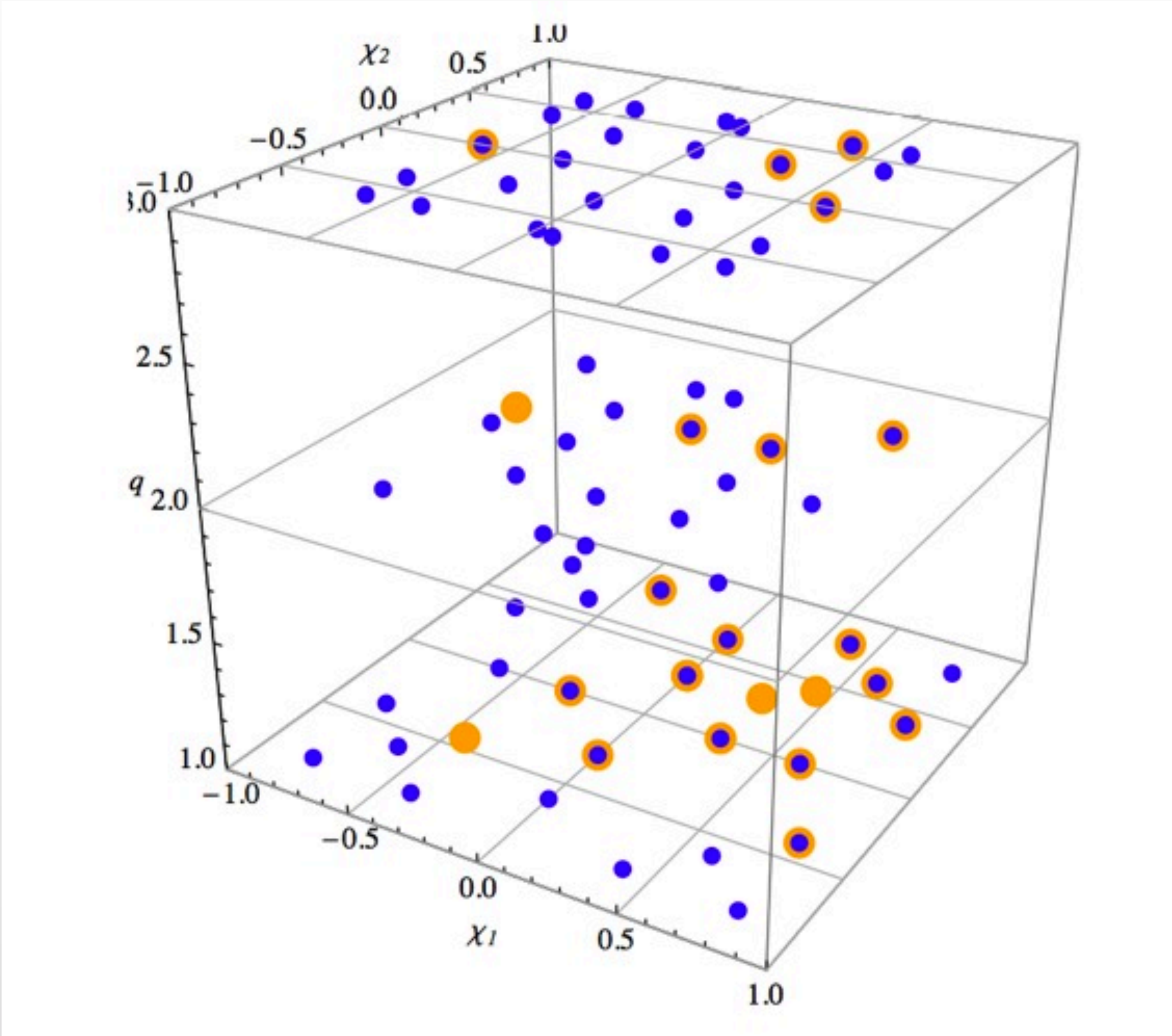
- ❖ 9 NR groups
- ❖ Very ambitious goals:
  - Significantly improved length and accuracy requirements for BSSN
  - Many mergers for SpEC/SXS
- ❖ Extensive error-analysis and cross-comparison
- ❖ Unified GW extrapolation Identified and fixed vast number of problems in various' NR groups computational approaches
  - Resolution
  - Wave-extraction
  - eccentricity removal

NR AR error analysis



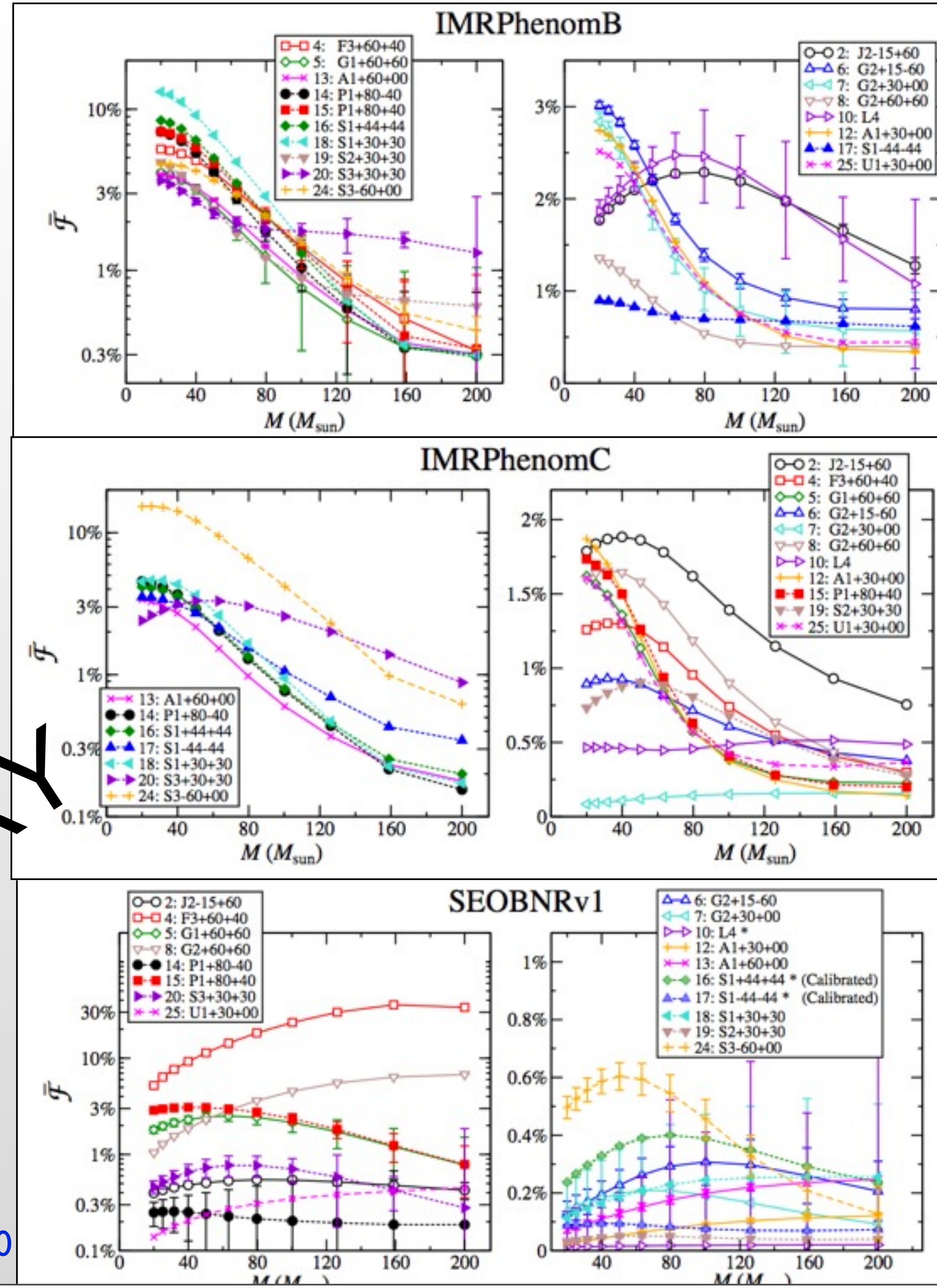
NR-AR parameter space coverage

- 25 Waveforms (orange dots)
  - 5 precessing
  - 17 spinning



NRAR: NR vs. AR comparison

- ❖ Aligned-spin waveforms:
- ❖ Comparisons with analytical models
- ❖ Overlap integration starts at f_low, NR

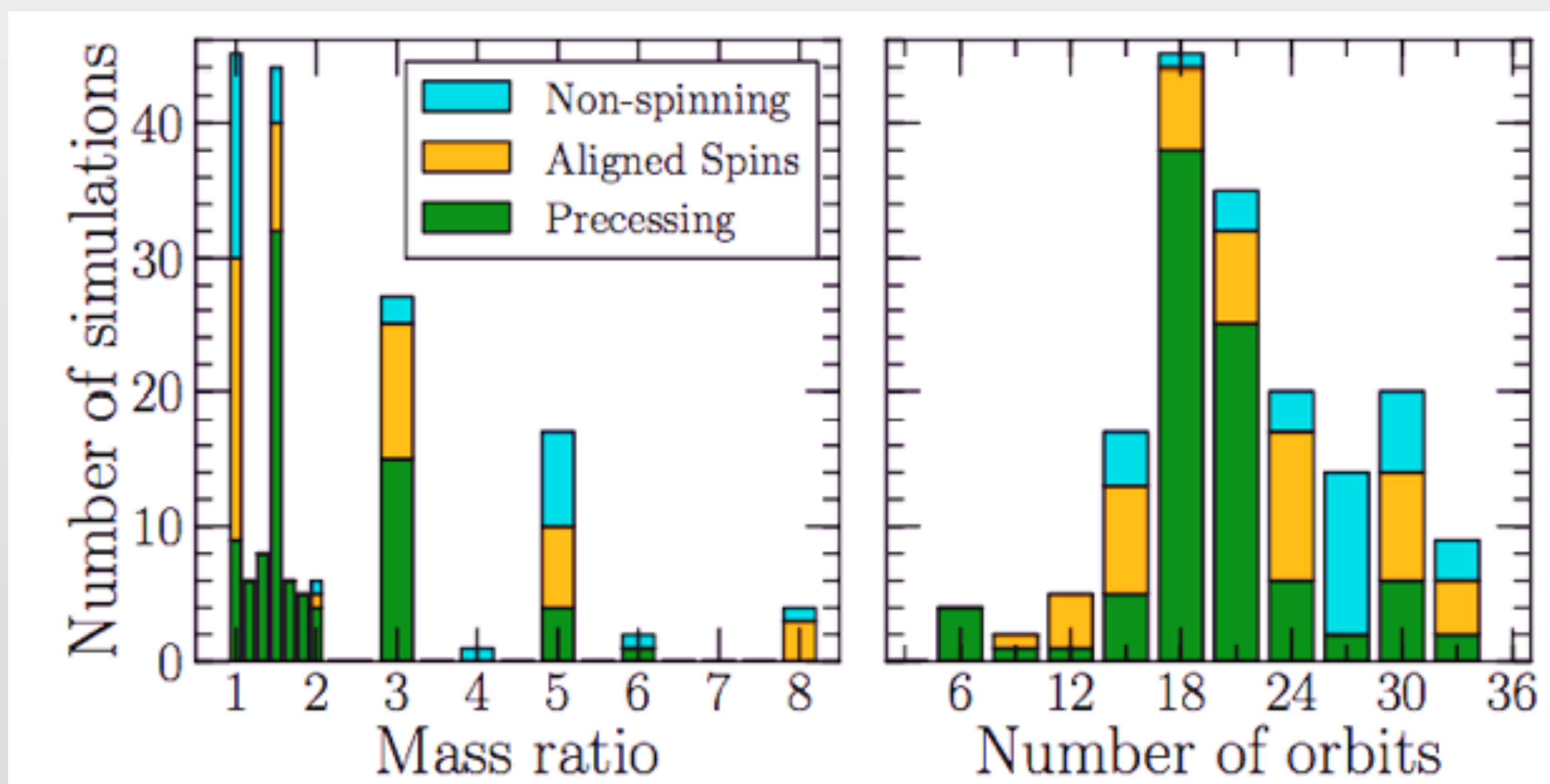


PRELIMINARY

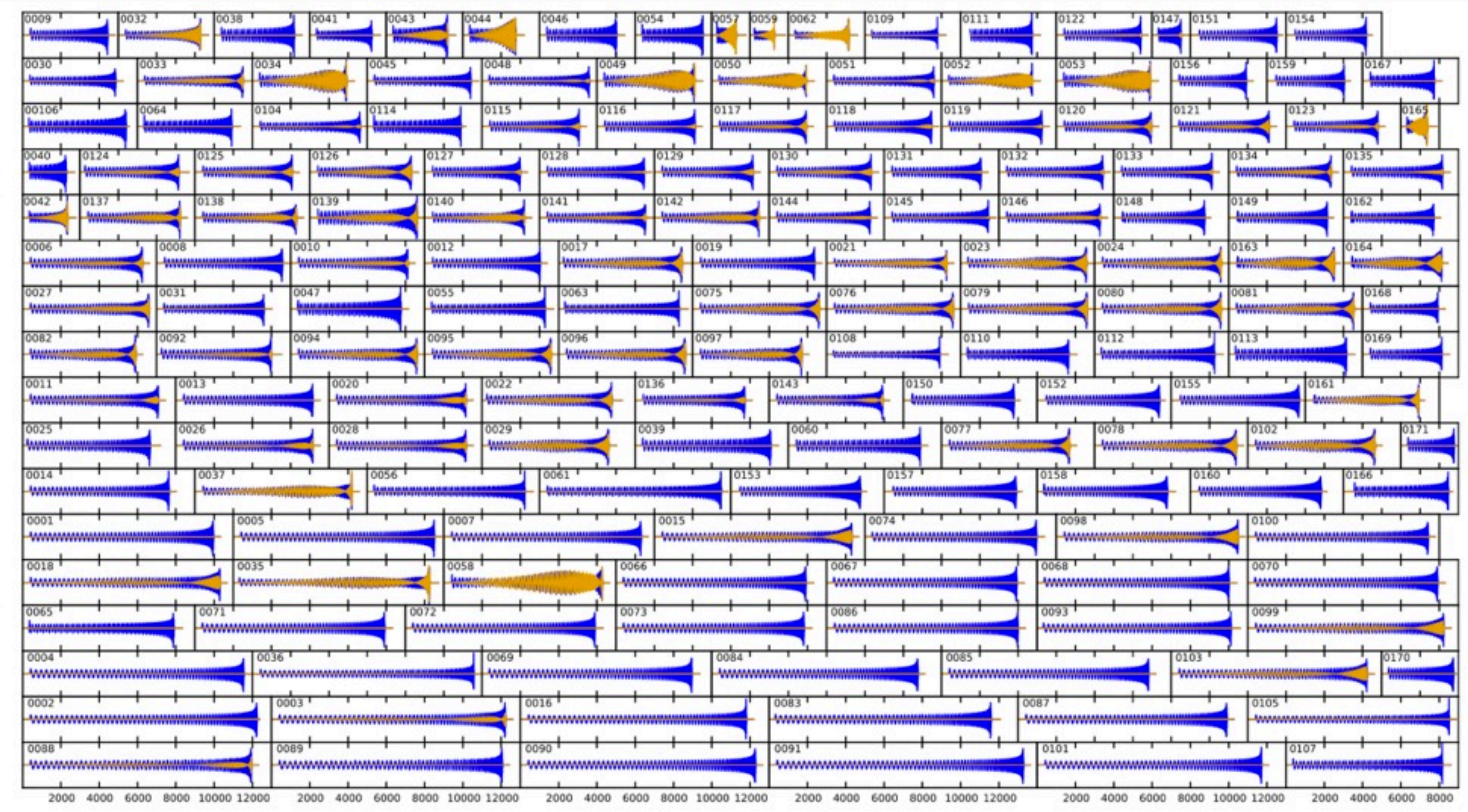
SXS waveform catalog

- 700 configurations quasi-circularized (Mroue, HP 1210.2958)
- 171 simulations completed
  - Mroue ea, arXiv:1304.6077

Abdul H. Mroué,[1] Mark A. Scheel,[2] Béla Szilágyi,[2] Harald P. Pfeiffer,[1,3] Michael Boyle,[4] Daniel A. Hemberger,[4] Lawrence E. Kidder,[4] Geoffrey Lovelace,[5,2] Serguei Ossokine,[1,6] Nicholas W. Taylor,[2] Anıl Zenginoğlu,[2] Luisa T. Buchman,[2] Tony Chu,[1] Evan Foley,[5] Matthew Giesler,[5] Robert Owen,[7] and Saul A. Teukolsky[4]



171 waveform catalog

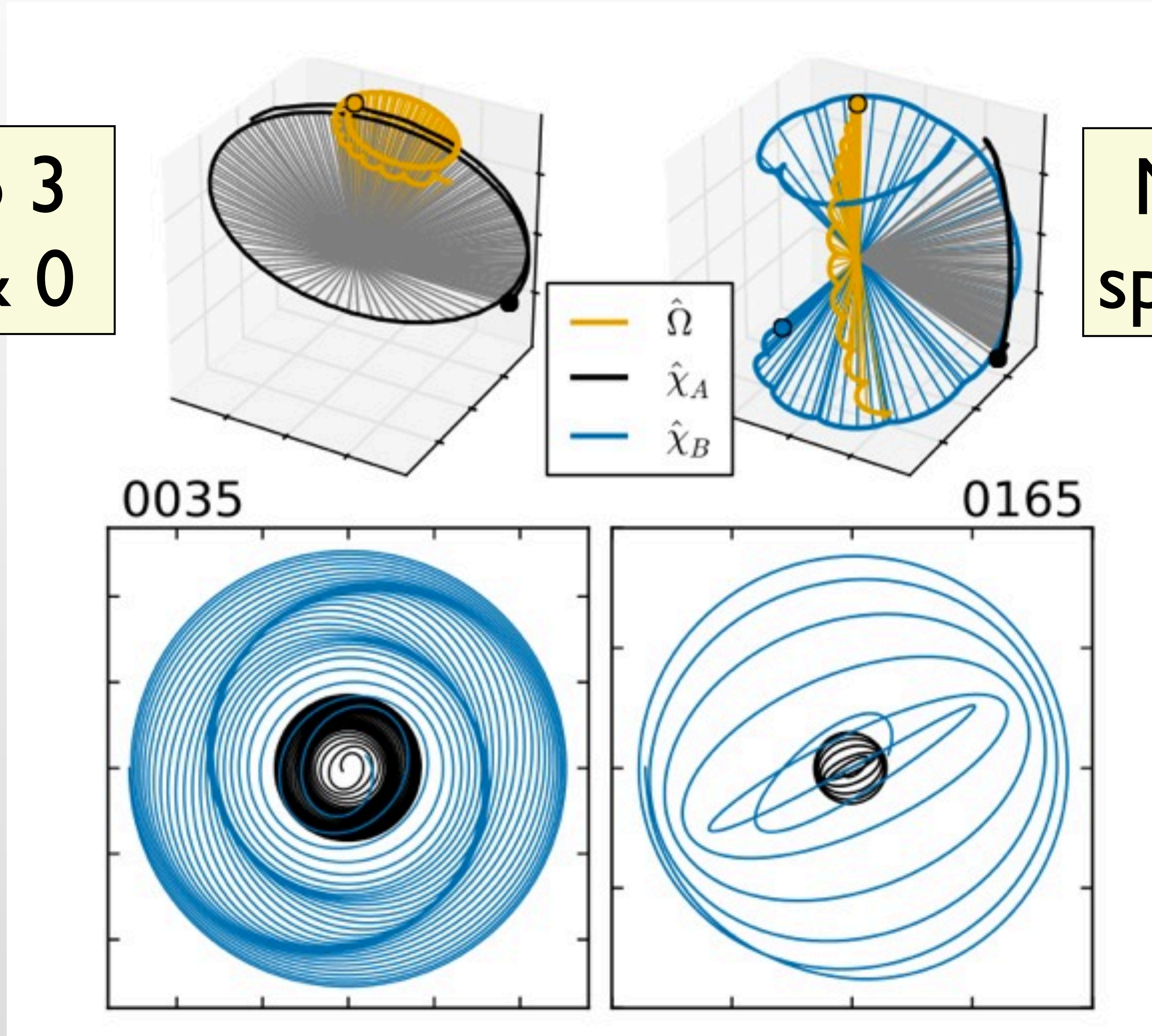


3 years, 50 Mio CPU-hours

Mroue ea, arXiv:1304.6077

Examples of precessing binaries

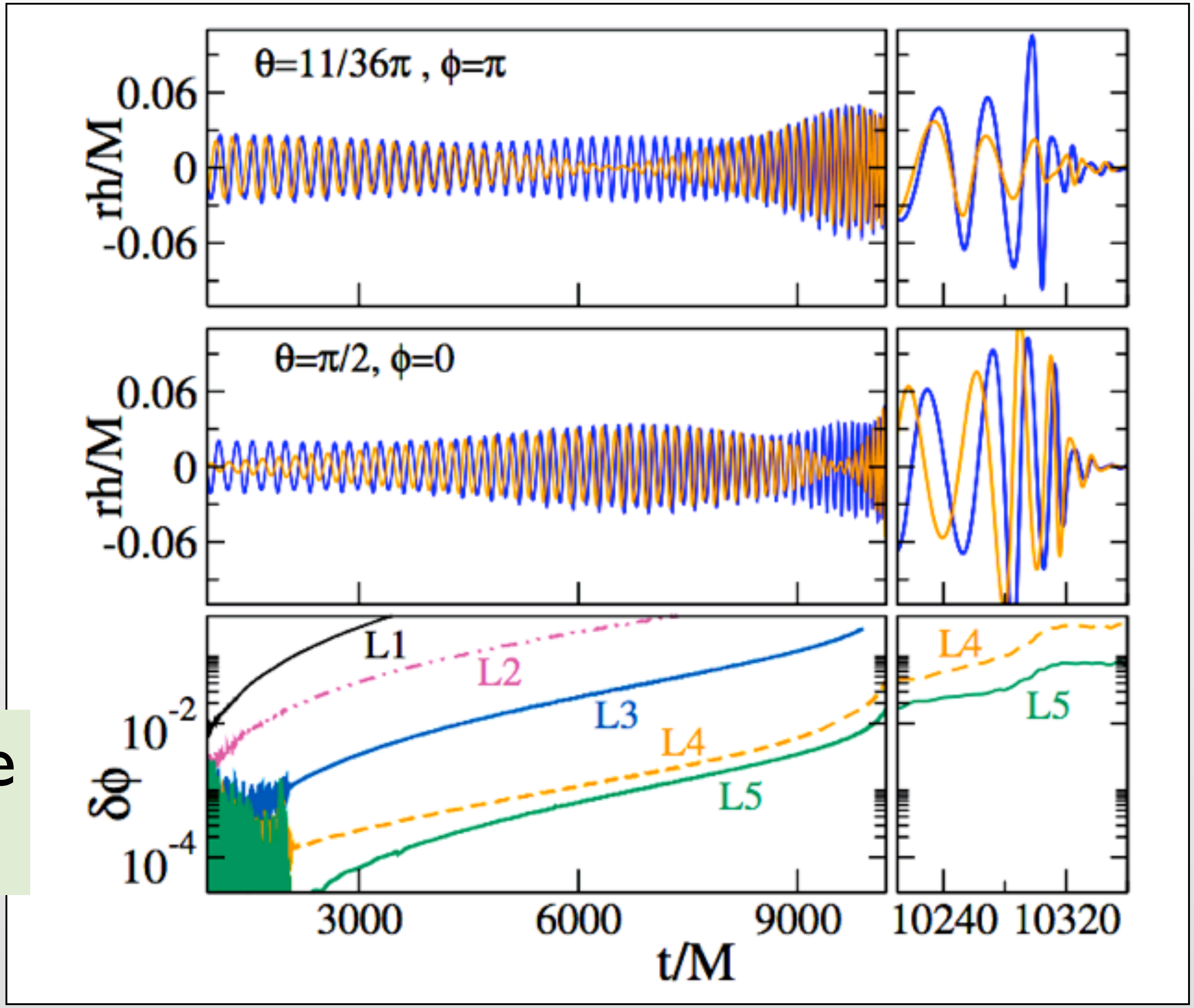
Mass-ratio 3
spins 0.5 & 0



Mass-ratio 6
spins 0.9 & 0.3

Mroue ea, arXiv:1304.6077

Orientation-dependence of waveform



Convergence test!

Mroue ea, arXiv:1304.6077

Uses

Non-spinning BH-BH

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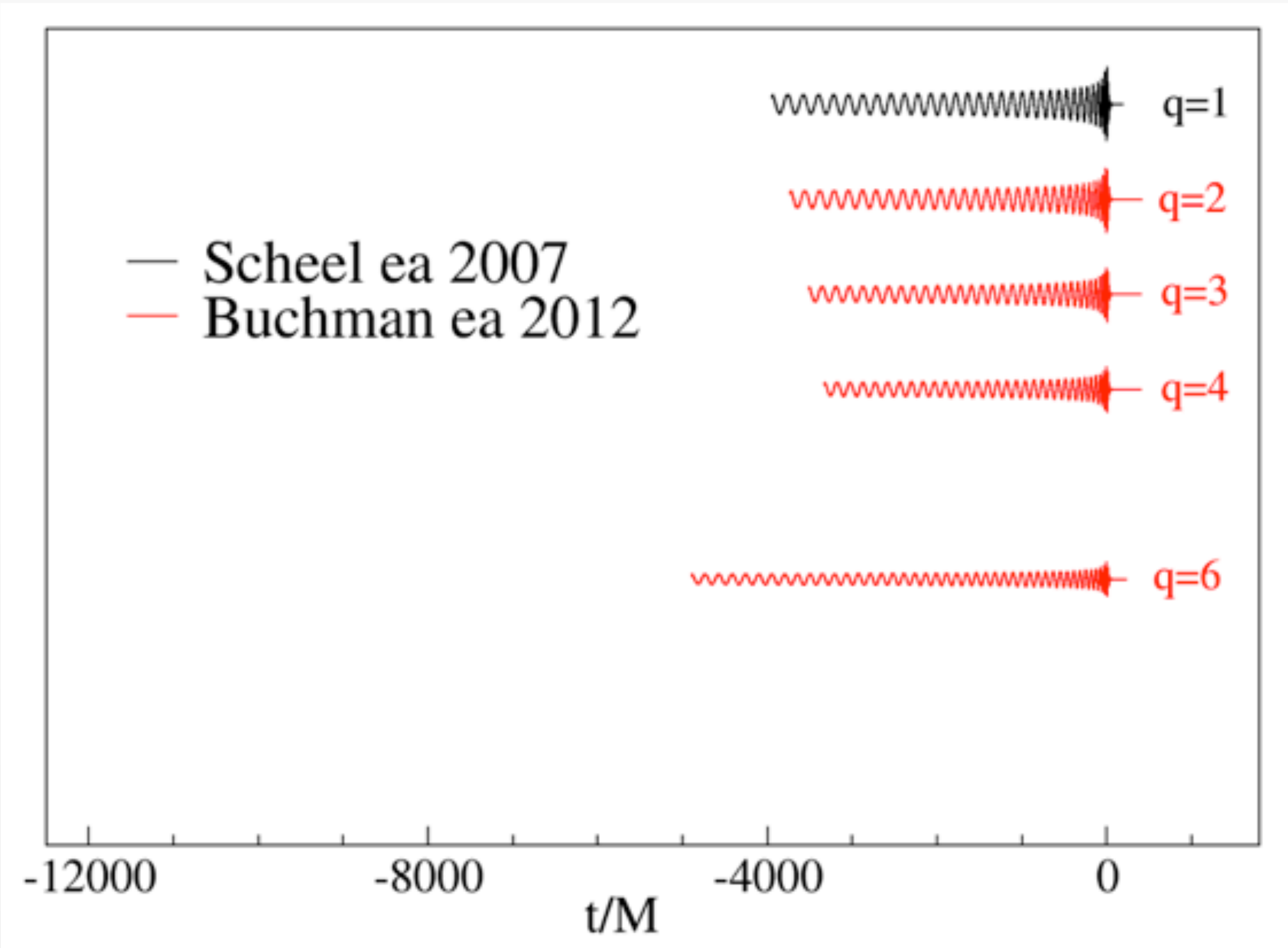
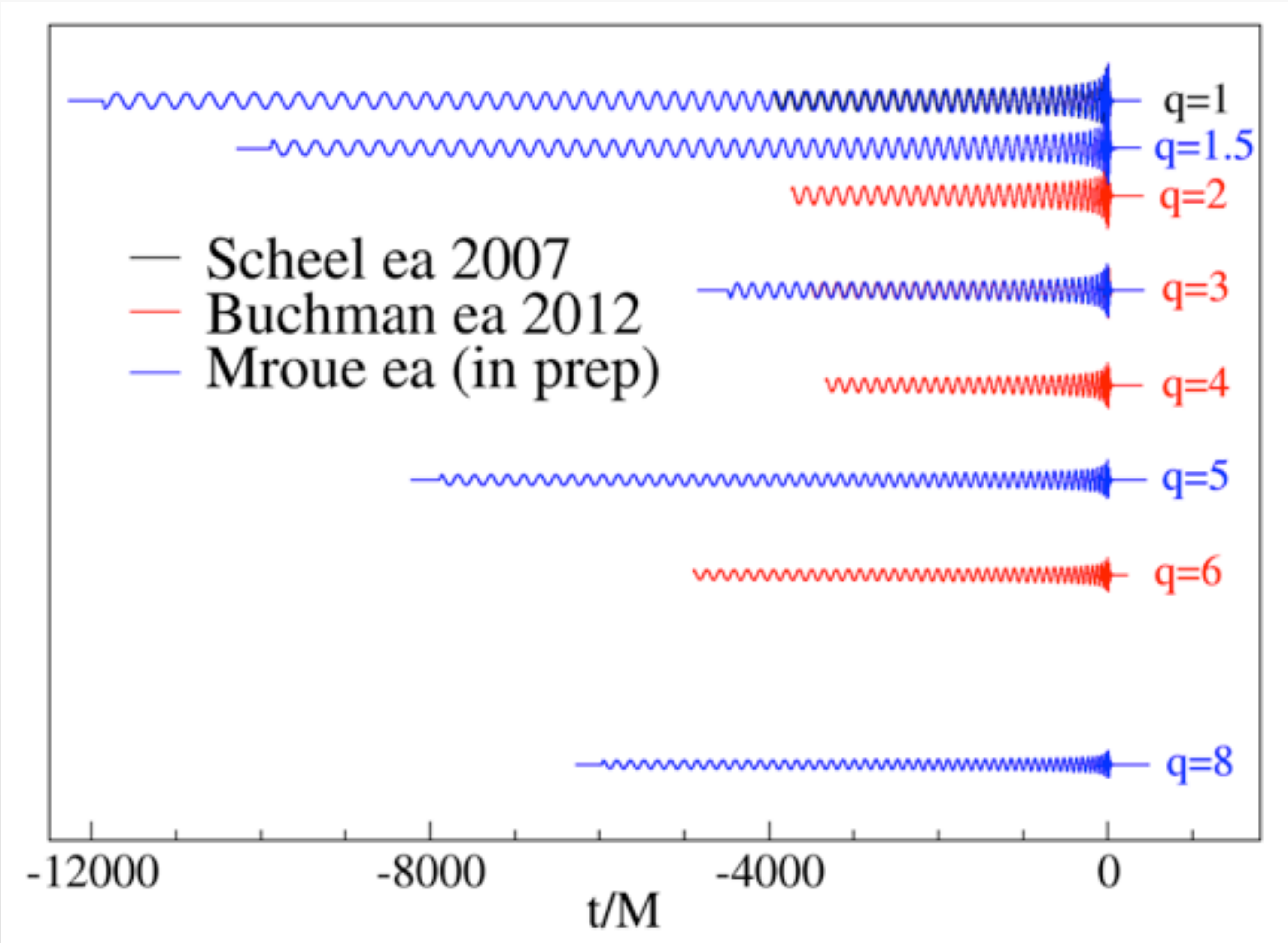


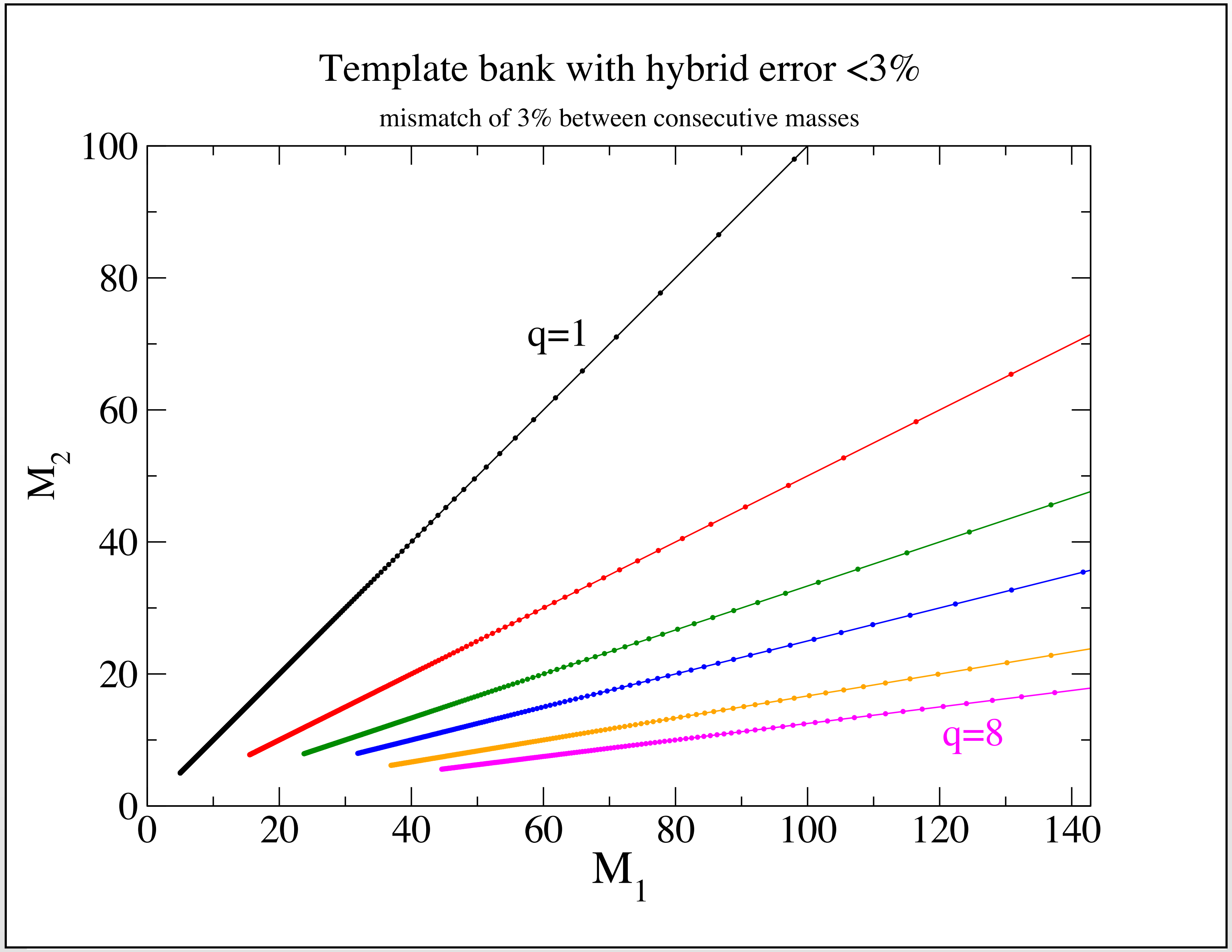
- Basis for EOBNR

Non-spinning BH-BH



- Basis for EOBNR
- test & improve EOBNR with more and longer waveforms

Non-spinning BH-BH

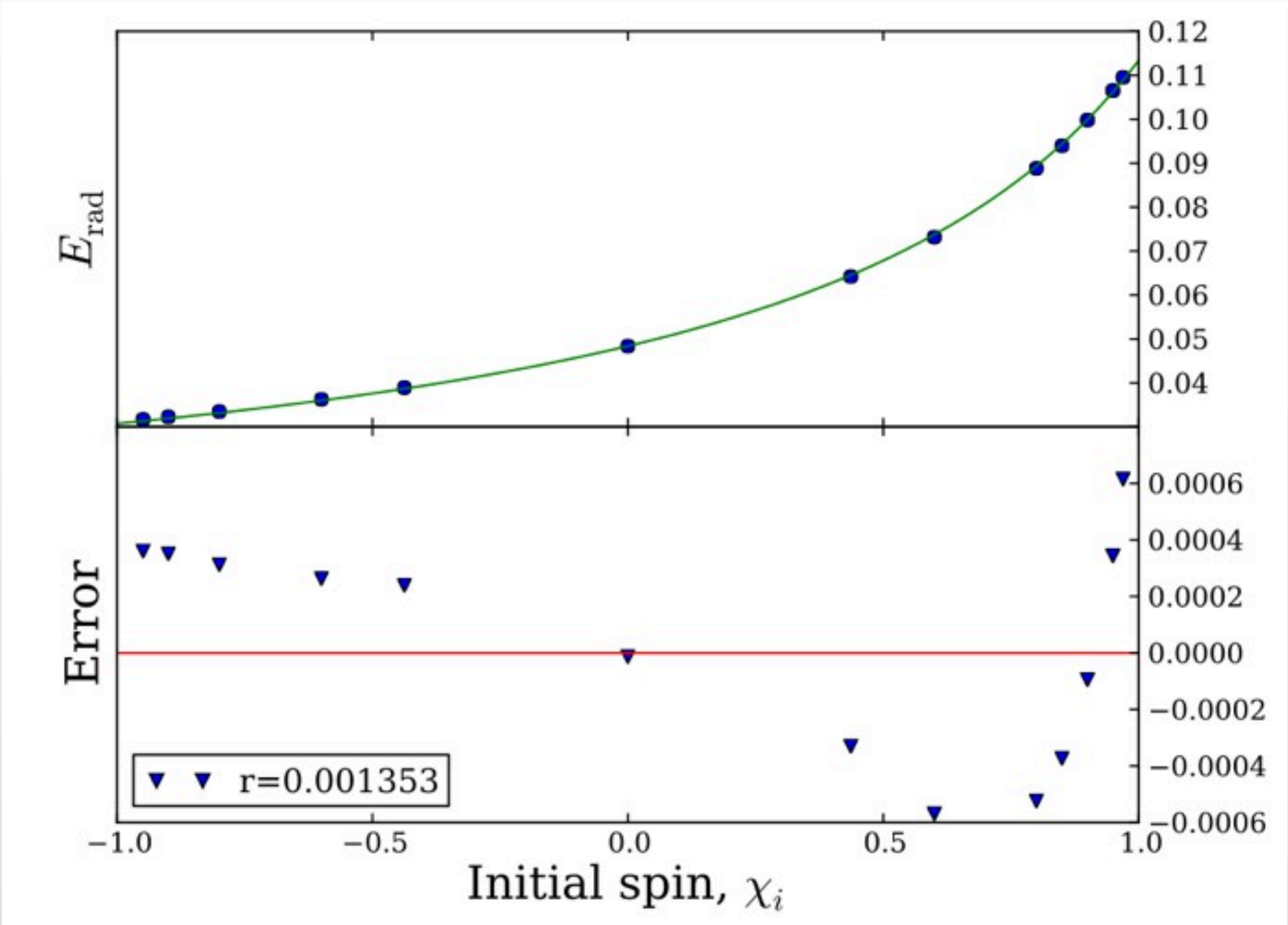


Ilana MacDonald (CITA)

- ❖ Basis for EOBNR
- ❖ test & improve EOBNR with more and longer waveforms
- ❖ construct NR-only template banks
  - Brown, Cannon, Kumar, MacDonald, HP + SXS
  - *H8: Prayush Kumar*

Spinning black holes

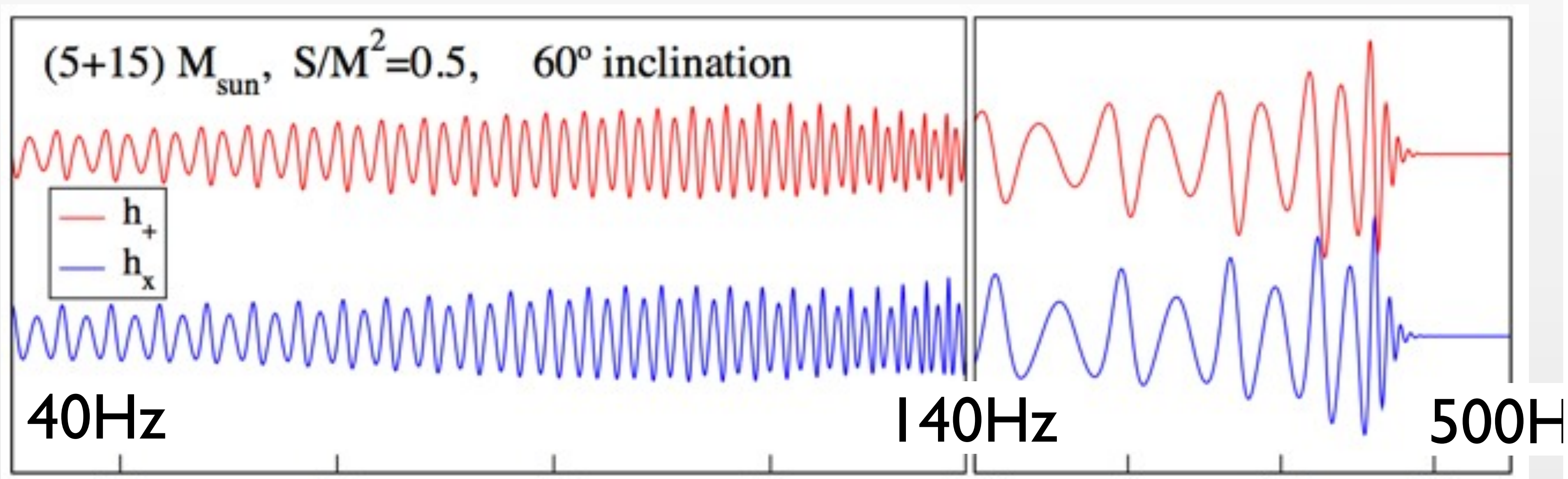
- Test & refine formulae for remnant properties
  - Dan Hemberger et al in prep
- Aligned Spin EOB-models
  - *C10: A. Taracchini*

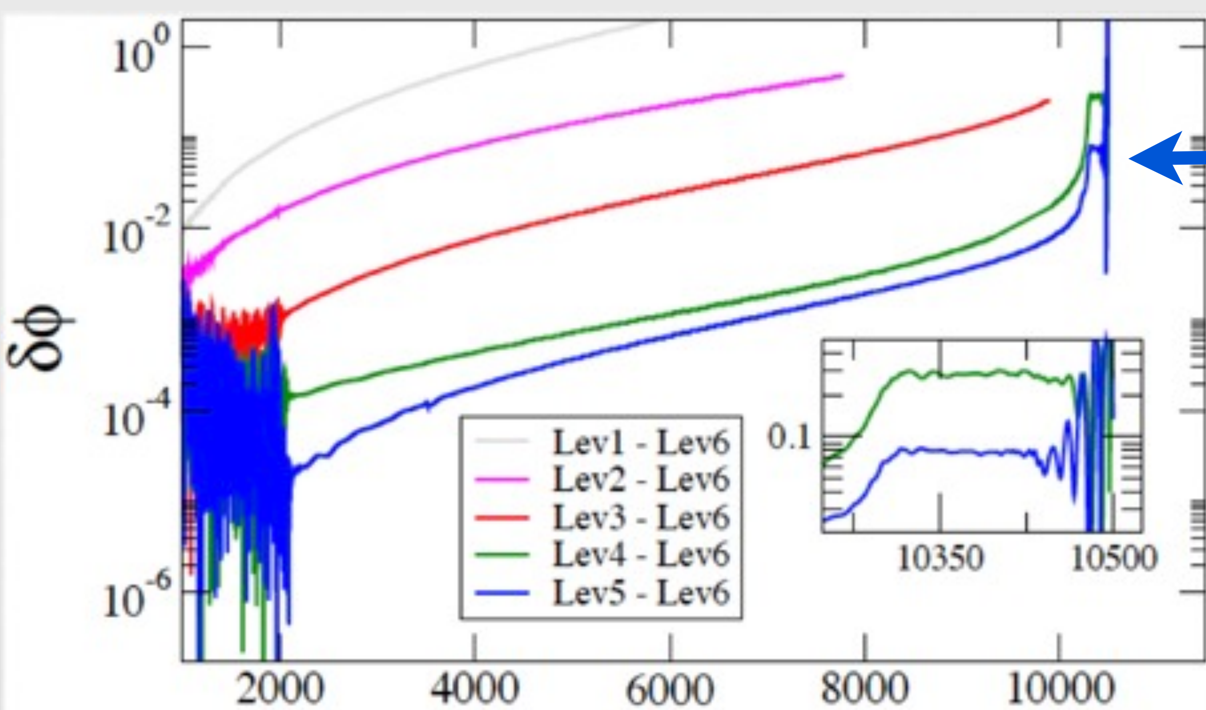


q=1, equal & aligned spins (courtesy Dan Hemberger)

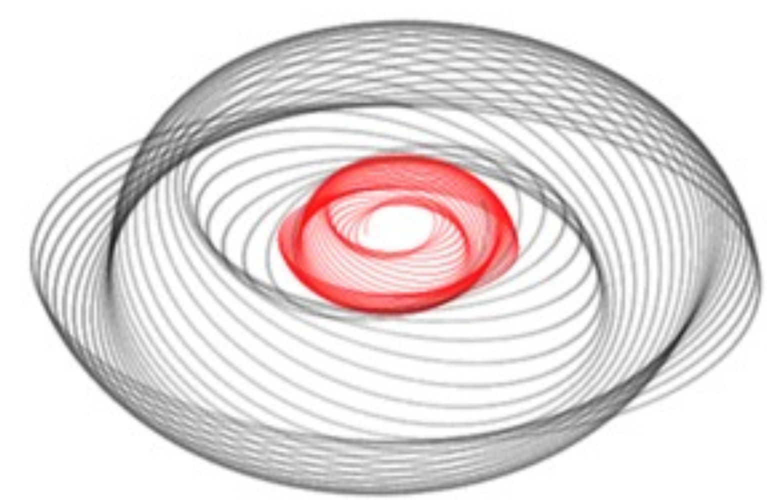
Very long waveforms

- q=3, χ_A=0.5, 33 orbits



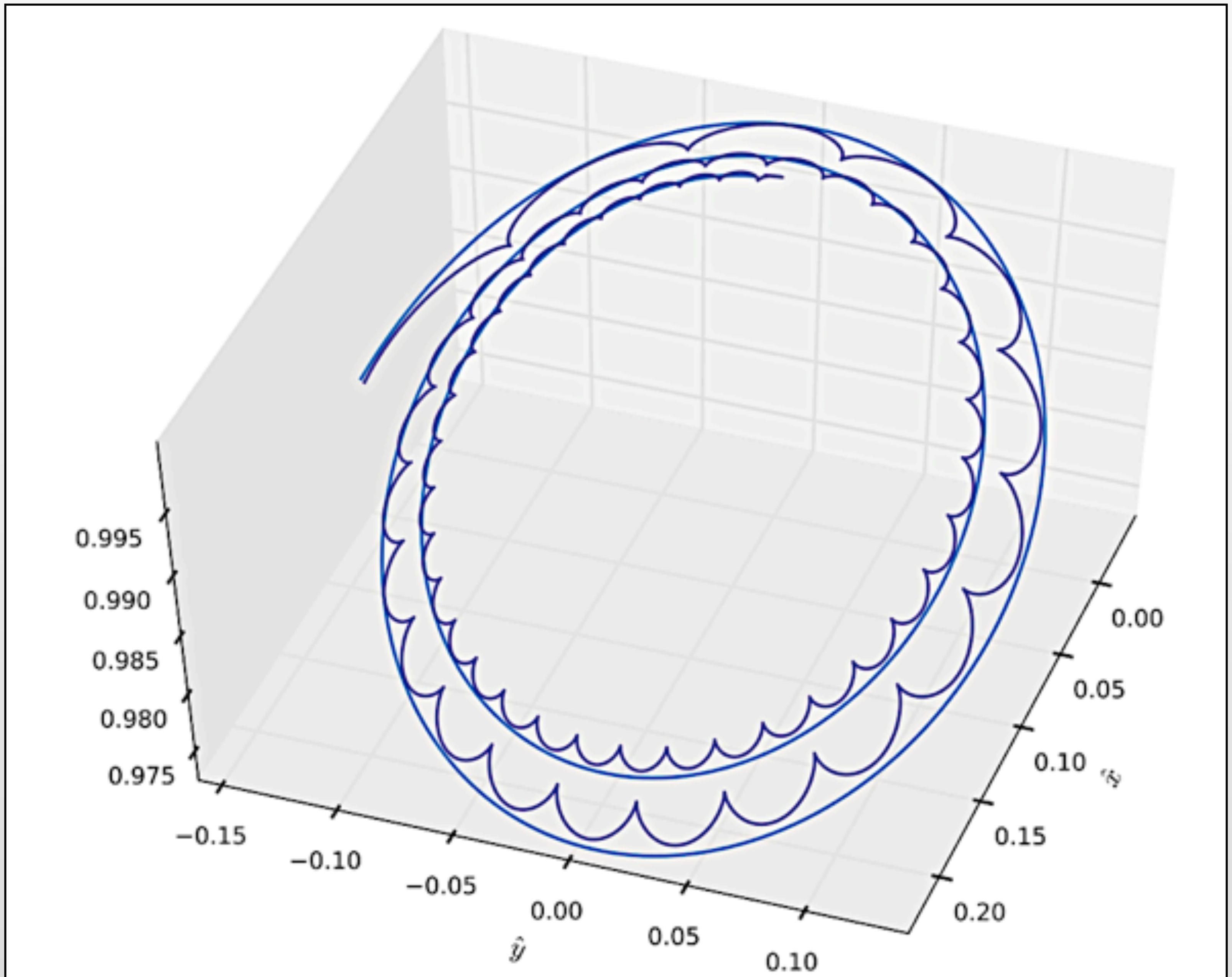


Cumulative phase-error *smaller* than 15-orbit non-spinning simulation (!)



Very long, precessing waveforms

$$\hat{L}_{\rm PN} \text{ vs. } \hat{L}_{\rm NR}$$



$q=1.5$ $\chi_A=0.5$

2 precession cycles

work by Sergei Ossokine (CITA)

C10: Serguei Ossokine

Modest precession

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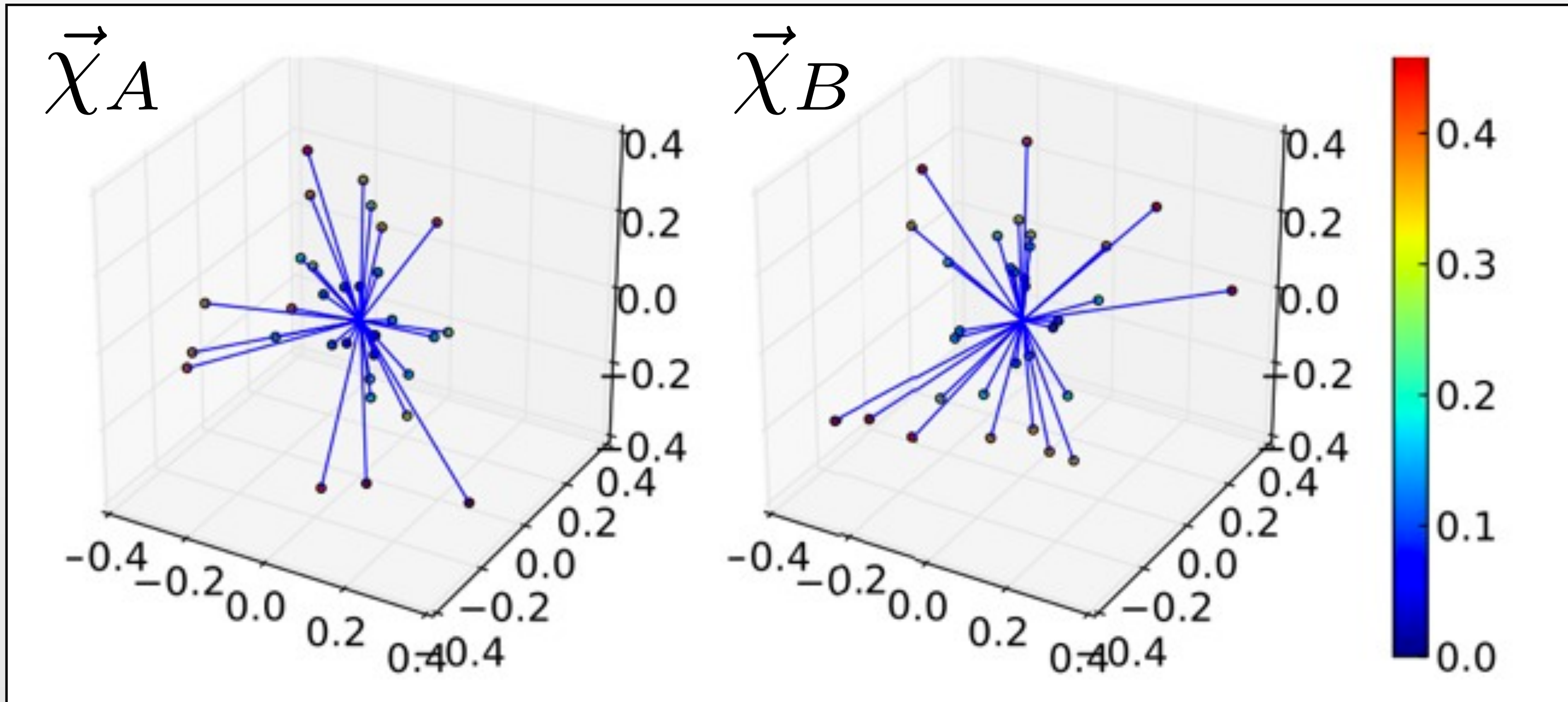
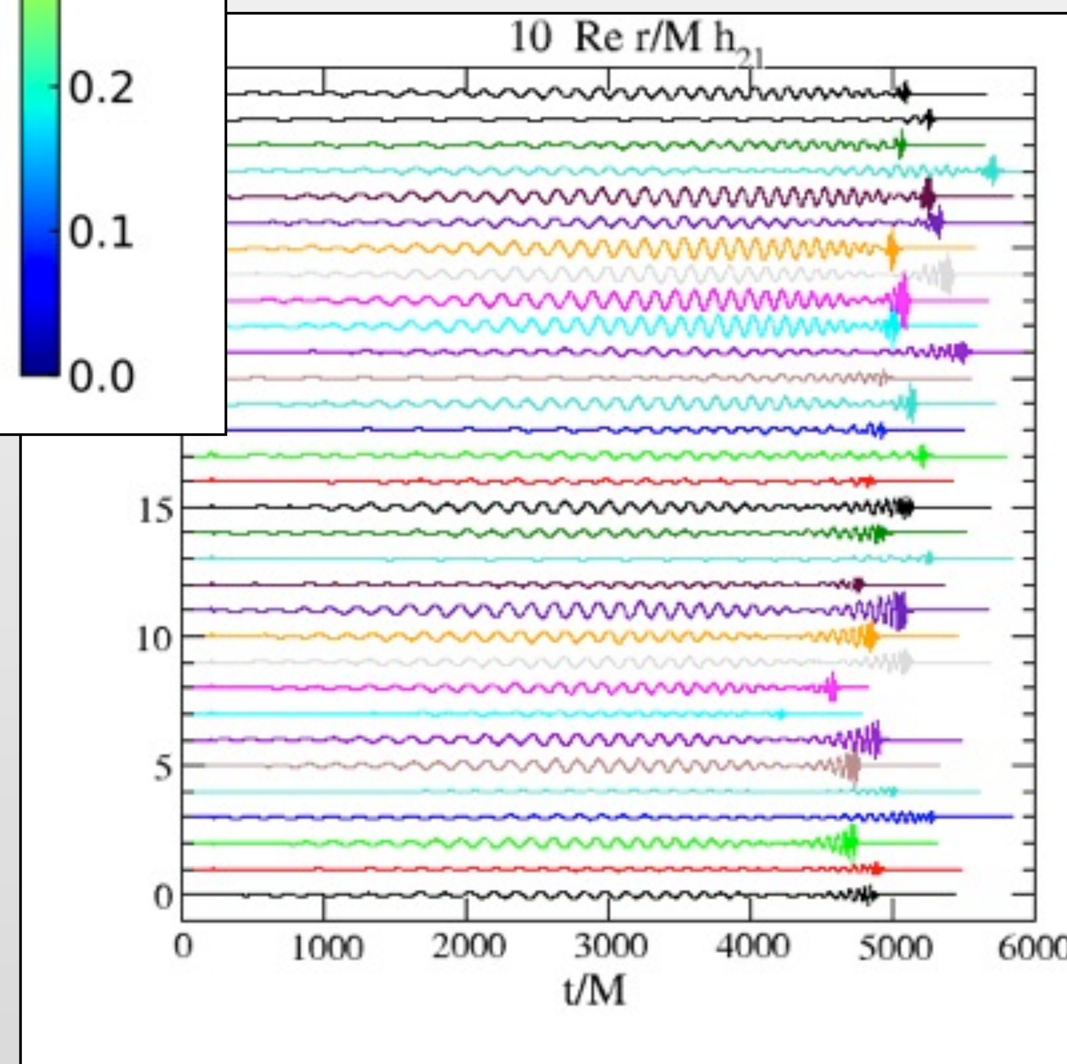
❖ 32 runs with random mass-ratio 1<q<2, random spins χ<0.5





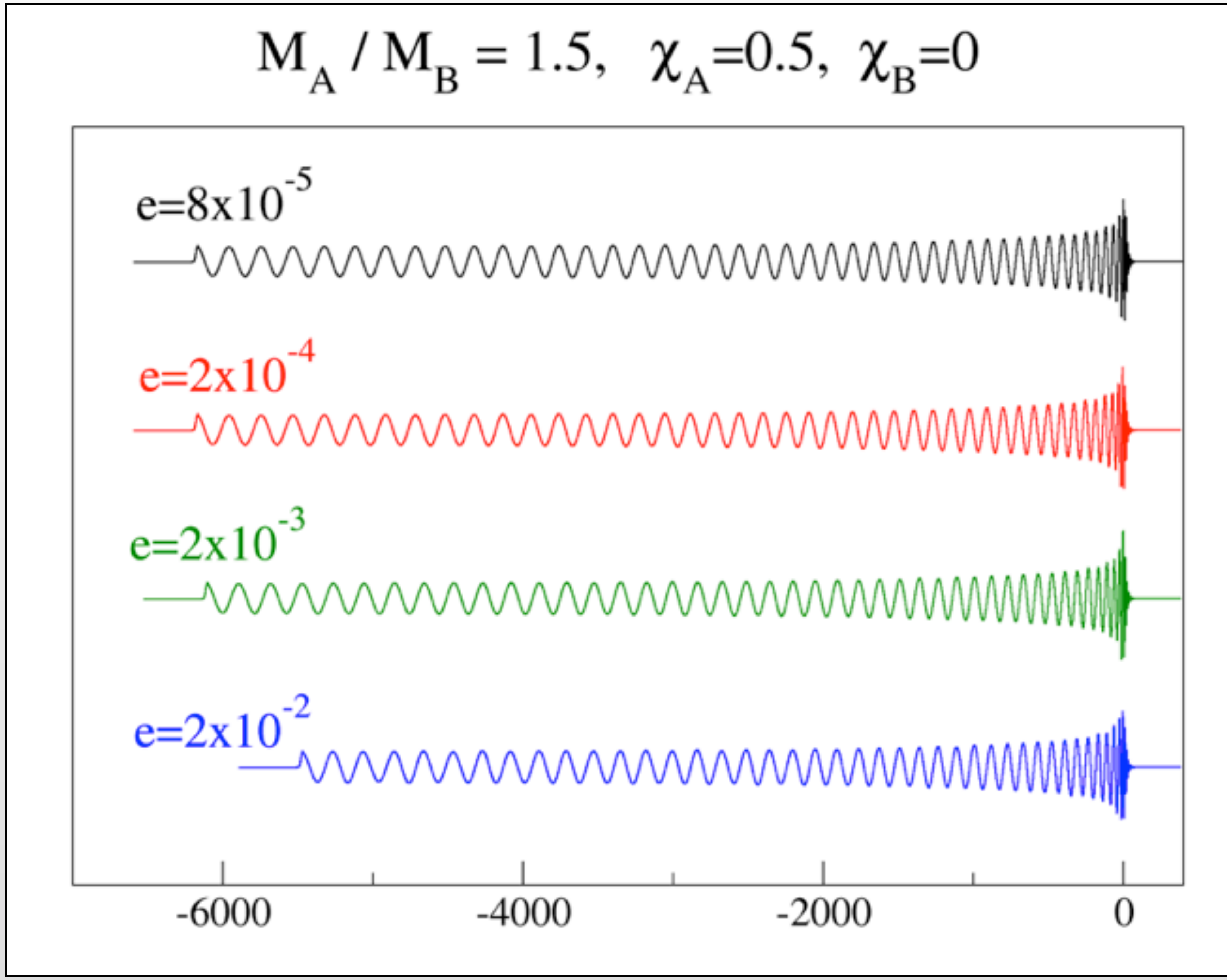
❖ Test impact on searches with non-precessing templates

❖ Spin-Trends in precessing PN

Varying eccentricity

CITA–ICAT

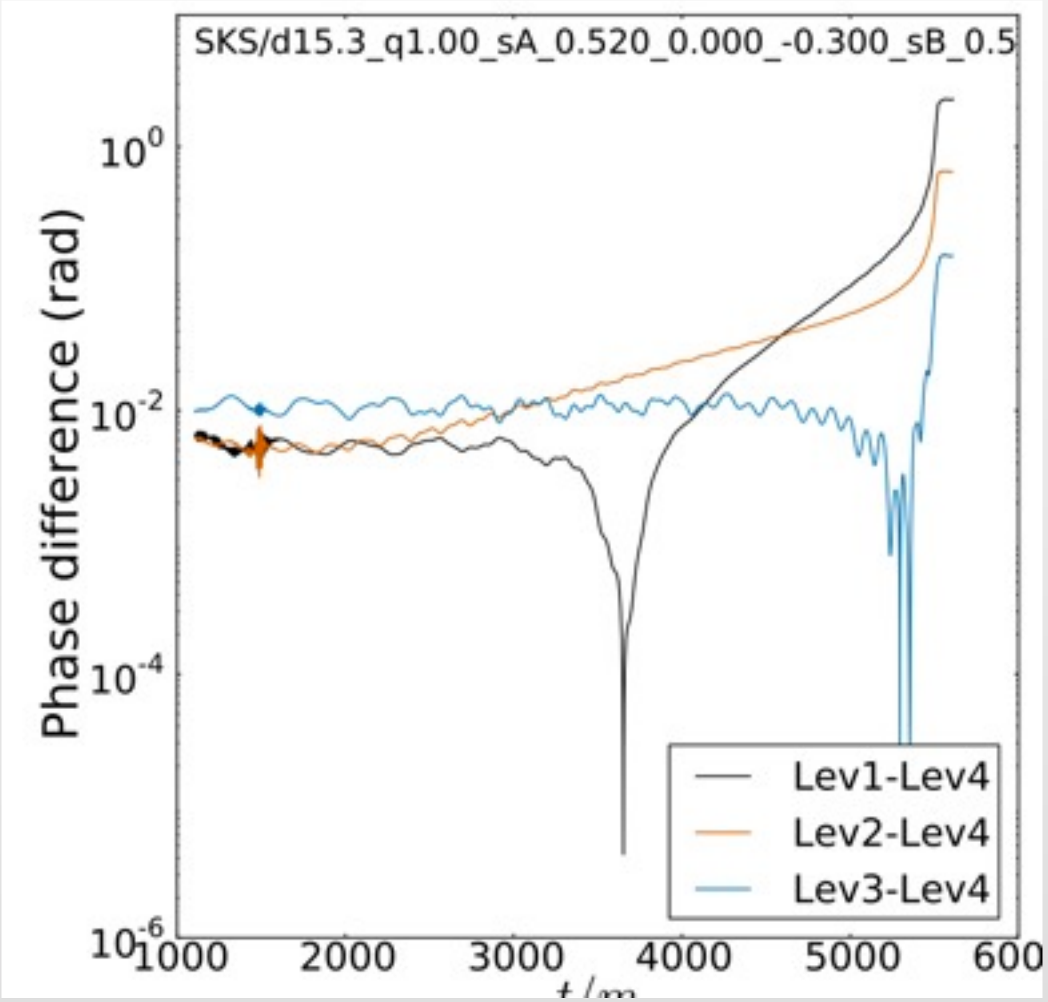
- Investigate when $e \neq 0$ becomes noticeable
- Periastron Advance
  - *C10: Tanja Hinderer*



Caveats

Convergence of some runs

- Spectral Adaptive-Mesh-Refinement
  - instrumental for mergers & efficiency.
- Not as well understood as "old" code
  - some runs affected
- Decided to keep the waveforms anyway



Higher modes, GW extrapolation

❖ Higher modes extracted

- BUT: much less experience and testing of higher modes compared to (2,2) mode.

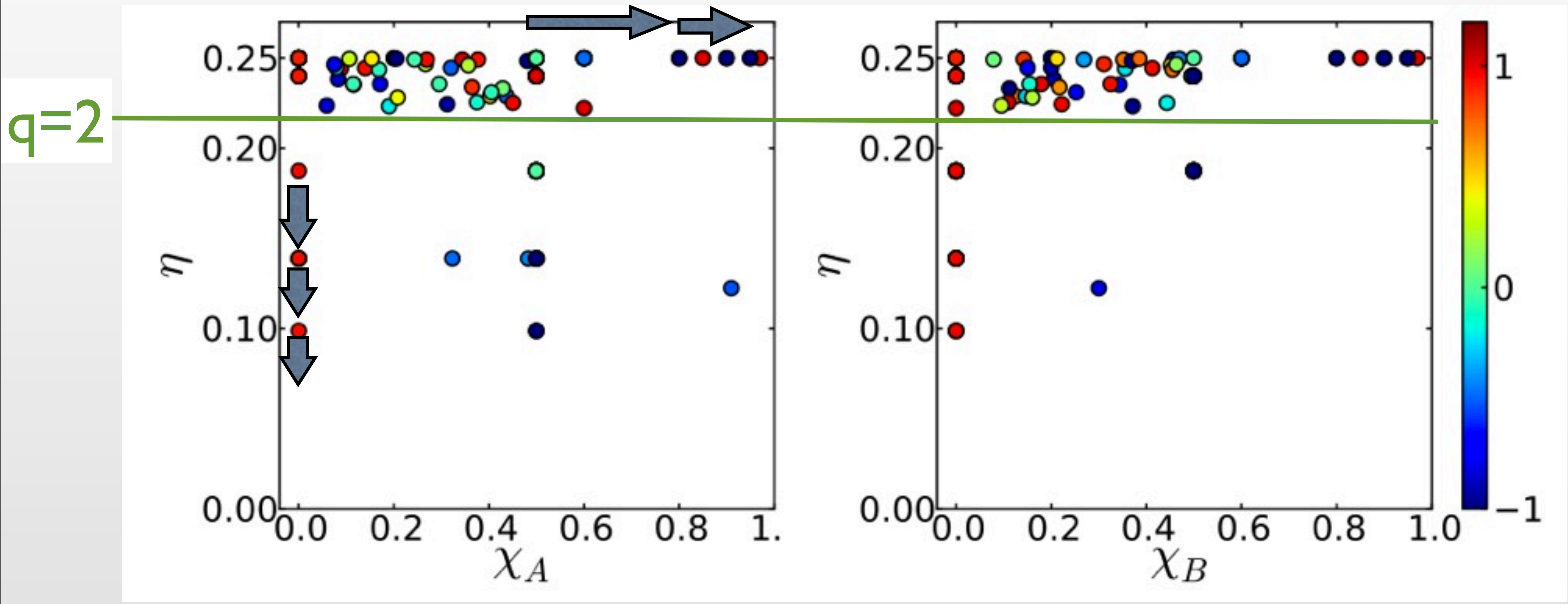
❖ Waveform extrapolation works fine

- BUT: our experience based on non-precessing waveforms. Problems may arise.

Challenges

Expanding parameter space coverage

❖ Most spinning runs at q<2



❖ So far, pushing parameters was *always* difficult

- Each arrow 1-2years hard work