Capital Requirements and the Taxpayer Put

Dilip B. Madan
Robert H. Smith School of Business

Tata Institute of Fundamental Research

International Center for Theoretical Sciences

School in Mathematical Finance
January 16-27 2012, Mumbai, India

Motivation

- Commenting on the recent financial crisis led to the remark that the ability to trade electronically in thousands of assets in numerous markets simultaneously at an ever increasing frequency was a privileged access to a service with dangerous social consequences if one is caught short.

- These considerations led to a demand for regulatory capital requirements for at least the larger hedge funds.

- Yet I felt we lacked a theoretical foundation for the determination of such capital requirements.

- They are in practice determined in a competitive market place for access to trading platforms where anticipated commissions on trades lure counterparties to lower capital standards.

- However, the question remains as to what is the right level.

Introduction

- I begin with a recognition of the virtues of free market capitalism, reviewing for our benefit its theoretical foundations that are now very well understood.

- We see the role played by linear pricing rules in separating out the activities of consumption and production.

- I then introduce the non-linearities arising from limited liability.

- The world has become very adept at leveraging the benefits of limited liability with companies inside companies inside yet other companies, with each one limiting the liability of its parent.

- Ultimately we end up in the world of SPV's or special purpose vehicles delivering limited liability corporate status to predefined cash flow streams, leaving no recourse if and when the stream dries up.

- The question then arises as to how to recover the first best results of linear pricing rules in the presence of limited liability run amok.

- The result is our proposed theory for capital requirements, externally set, at levels that mitigate the adverse risk incentives introduced by limited liability.

Market Efficiency

- We have theorems indicating the assumptions needed for the equivalence of competitive equilibria and Pareto optima (see for example, Arrow and Hahn (1971)).

- They are:

  - Contexts of full information for all participants, making individual optimization rational.

  - complete markets for all items of interest.

  - The absence of external effects, so that we have the possibility of separation of all effects at a local level.

- We reference Arrow (1964) and Allen and Gale (1994) in this regard.

- It has long been recognized that these assumptions are grossly unrealistic and hence this paradigm has been naturally called into question by numerous scholars.

- Yet the system of markets remains the best we have found for the allocation of resources in modern economies.

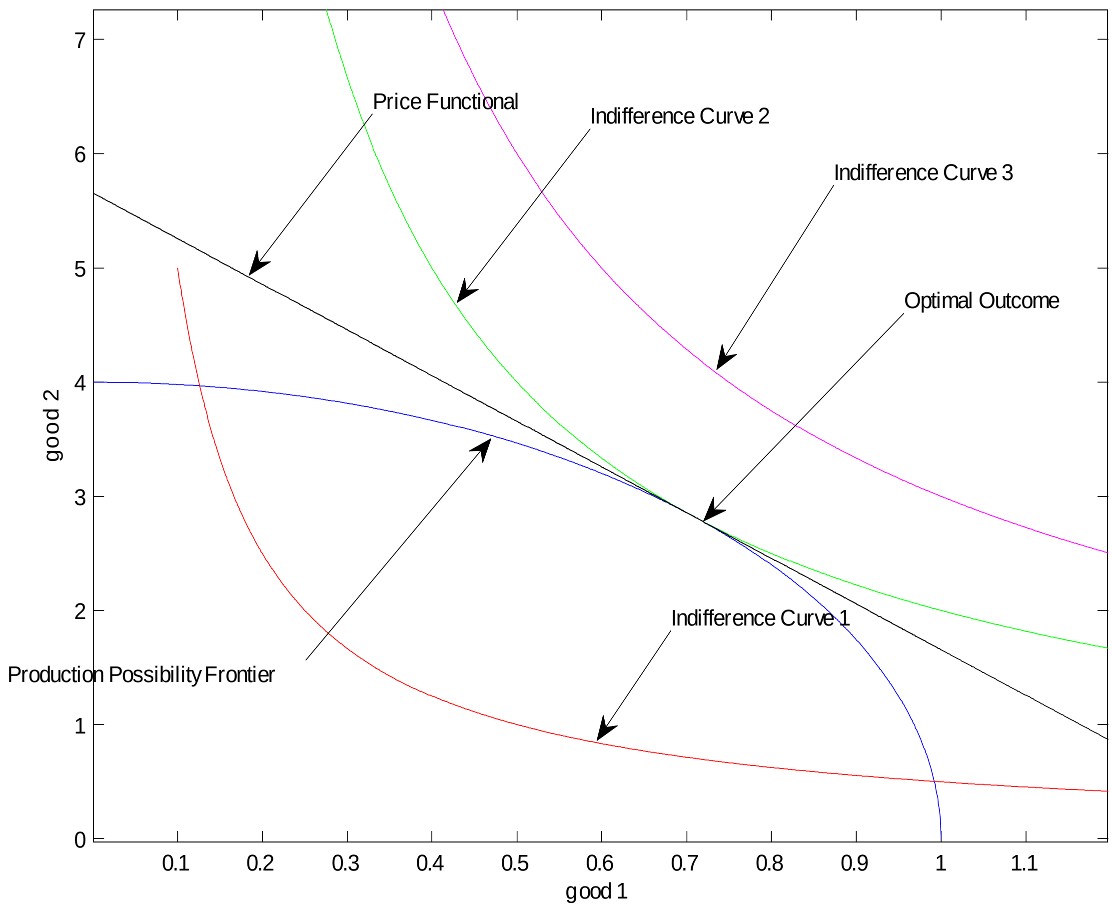


Figure 1: Separating Robinson Crusoe the consumer from Robinson Crusoe the producer.

Some specific issues

- If we keep the model of profit maximization for firms, we know that in the presence of uncertainty and incomplete markets it is unclear how to realize this objective.

- Let us suppose that tentatively we have agreement on a pricing kernel with respect to which asset valuations may be conducted and let us call this measure Q.

- We may also for reasons of focus restrict attention to a one period model with the firm's objective being the maximization of $E^Q[X]$ where X is some attainable cash flow.

- This gives us a typical linear pricing rule that is central to the separation arguments built into the folk theorems.

Hedge Fund Cash Flow Model

- We now wish to recognize that X could be the result of a balanced long short strategy adopted by a hedge fund in financial markets set up with a limited liability rule and a capital requirement.

- The log return to the long portfolio is X_L while for the short log return we have X_S and for balance we have a unit investment in both.

- The cash flow accessed is therefore

$$X = \exp(X_L) - \exp(X_S).$$

- We shall model X as the difference of two exponential Lévy models.

Limited Liability Effects

- The fund has to post capital to take on this position and we suppose that this is C. The final cash flow at a period interest rate of r is then

$$X + C(e^r - 1).$$

- The cash flow or profit for the limited liability fund, however, is

$$(X + Ce^r)^+ - C,$$

as it gets to put the losses back into the system to parties that are unclear and unspecified.

- What is clear is that it holds the put option.

- The firm's objective of maximizing its profit is then to maximize

$$e^{-r} E^Q \left[(X + Ce^r)^+ \right] - C.$$

Some Consequences

- The hedge fund's objective is to maximize a the value of a call option on the spread between assets and liabilities with strike $-Ce^r$ less the initial capital posted.

- The firm cannot be the one that chooses the capital C as it would choose the lowest allowable value of zero.

- In practice it is other counterparties in the market that determine the capital required for the positions being held.

- The capital or strike of the embedded option must be set by the party delivering the put and in the new regulatory environments being envisaged this is the government or its regulatory nominee acting on behalf of all stakeholders to whom the losses if any would be put.

Risk Distortions

- It is also known that the option to put losses back to the economy built into limited liability contracts distorts risk choices and provides incentives for increased volatility (Gollier, Koehl, Rochet (1997)).

- There are in fact many other dimensions to risk beyond volatility and we investigate these effects in the context of a model permitting a variety of risk characterizations, including variations in skewness, kurtosis separately on the asset and liability side along with variations in correlation between assets and liabilities.

- We propose rules for capital requirements that are in principle capable of controlling perverse private sector risk incentives once capital requirements are sufficiently well administered in a limited liability context.

Related Literature

- In order to address the interactions between capital requirements, socially acceptable risk choices and private risk incentives in the presence of limited liability we need
  - a precise model for capital requirements, and here we follow the recent work of Cherny and Madan (2009),
  - a description of balance sheet risks permitting random liabilities and assets for which we can perform equity value computations
    * This leads us to pricing spread options in a Lévy process framework and here we follow the recent work of Hurd and Zhou (2009).

- With these two components in place we analyze required interactions and report on
  - how well set capital requirements can counteract the perverse risk incentives introduced by limited liability.

- We do this without the necessity of introducing risk averse managers, as is done for example in Golliers, Koehl and Rochet (1997).

Acceptable risks

- We will contrast the market objective of equity value maximization with the primary economic objective that in complete markets was written earlier as maximizing
$$E^Q[X].$$

- This objective is well understood as it amounts to accepting all trades with a positive alpha as judged by the measure Q.

- In fact we know that this translates to expected returns, $E^P[X]$, being in excess of the covariation of returns with $\frac{dQ}{dP}$, the density of Q with respect to P where $E^Q[X] = E^P\left[X\frac{dQ}{dP}\right]$.

- The latter computation may be viewed as pricing the risk being undertaken.

Acceptable Risk Cones

- Cherny and Madan (2009) argue that this is too simple an objective that accepts all trades in what is a very large cone of acceptability defined by the half space generated by the density $\frac{dQ}{dP}$.

- This position merely reflects the view that with our general lack of knowledge about events and their probabilities one does not know the measure Q and the theory of acceptable risks merely asks that to be conservative we require instead that $E^{Q}[X] \geq 0$ for all $Q \in \mathcal{M}$, for some convex set of measures $\mathcal{M}$.

- This wider and more conservative definition of acceptability goes back to Artzner, Delbaen, Eber and Heath (1998),

  - was further studied in numerous papers including Carr, Geman and Madan (2001), Jaschke and Küchler (2001) from the perspective of generalizing the absence of arbitrage,

  - Madan (2004) from the viewpoint of financial equilibria in the presence of a lender of last resort,

  - Cherny and Madan (2009) who provide examples of effectively computable sets of acceptable risks using the theory of concave distortions also studied in Föllmer and Schied (2004).

- Here we propose to employ the $minmaxvar$ distortion at a prespecified stress level introduced in Cherny and Madan (2009) to define acceptable risks and capital requirements.

- Our concept of acceptable risks may also be seen as a generalization of the Gain Loss ratio and its associated Omega measure of risk (Shadwick and Keating (2002)).

Base Measure Considerations

- The original definition of acceptability used for a base measure the physical, statistical or so called true measure.

- From the perspective of risks acceptable to general economy at large this is not appropriate.

- Note that we seek to generalize the traditional first best to a more conservative selection.

- This observation teaches us that a positive expectation that fails to earn requisite compensation for risks taken is not acceptable to general economy.

- Hence the base measure must be one of the risk neutral measures, when there are many. Perhaps we choose one that is close to the P measure.

- We advocate defining acceptability in terms of a convex set of measures yielding a positive expectation, but these measures are absolutely continuous with respect to a base risk neutral measure.

- The base risk neutral measure should reflect minimal levels of risk premia.

Acceptability Using Distortions

- For a practical example of acceptable risks we follow Cherny and Madan (2009) and require that X be acceptable at some stress level for a particular stress function like $minmaxvar$.

- The stress function employed is a family of concave distribution functions from the unit interval to the unit interval parameterized by a single parameter γ termed the stress level that increases the degree of concavity as the stress level is increased.

- In the case of $minmaxvar$ the stress function is
$$\Psi^{\gamma}(u) = 1 - (1 - u^{\frac{1}{1+\gamma}})^{1+\gamma}.$$

- One verifies that the derivative of Ψ^{γ} decreases from infinity at zero toward zero at unity.

- For a risk X with base risk neutral distribution function $F_X(x)$ the risk is acceptable at level γ provided

$$\int_{-\infty}^{\infty} x d\Psi^{\gamma}\left(F_X(x)\right) \geq 0,$$

or equivalently

$$\int_{-\infty}^{\infty} x \left(\Psi^{\gamma}\right)' \left(F_X(x)\right) f_X(x) dx \geq 0,$$

- We see that we are computing an expectation under a further change of measure that weights large losses when $F_X(x)$ is near zero and discounts large gains when $F_X(x)$ is near unity.

- The entire class $\mathcal{M}$ of supporting pricing measures Q defining a convex cone of acceptable risks is described in detail in Cherny and Madan (2009).

Closed Form formula for Required Capital

- The equation for determining capital C is obtained by merely insisting on the acceptability of the unoptioned cash flow

$$Y = (X + Ce^r) .$$

- We then require for level γ that

$$\int_{-\infty}^{\infty} y d\Psi^\gamma (F_Y(y)) \geq 0.$$

- It follows that our capital requirement is

$$C = -e^{-r} \int_{-\infty}^{\infty} x d\Psi^\gamma (F_X(x)). \qquad (1)$$

- It is useful to have an analytical risk sensitive capital requirement as provided by equation (1) for one may then analyze interactions between risk components and its effects on both capital and equity.

- For the computation of the capital requirement, one may follow the procedure outlined in Cherny and Madan (2009). From a simulation of outcomes from the distribution of cash flows one may sort the outcomes in increasing order as $x_i, i = 1, \cdots, M$, and then evaluate the required capital as

$$C \approx -e^{-r} \sum_{i=1}^{M} x_i \left(\mathbf{\Psi}^{\gamma} \left(\frac{i}{M} \right) - \mathbf{\Psi}^{\gamma} \left(\frac{i-1}{M} \right) \right).$$

Remarks on the Stress Level

- Consider the following two gambles
  - A. Heads you gain a unit tails you lose a unit.
  - B. Heads no outcome, tails you play A.
- One may verify that every distortion prefers B to A as it has a lower ask price and a higher bid price
- We now offer a premium on heads in A of λ that equates the ask prices of the two gambles.
- There is a map from the stress level γ to the premium λ.

Private Incentives Post Capital

- We now enquire if setting capital requirements by such a rule followed by letting firms maximize their equity value results in a system capable of functioning without undue risk exposure.

- More specifically, given a vector of risk choices θ characterizing the distribution F_X, we compute a capital requirement defined by $c(\theta)$, and with this capital in place we then determine equity value as

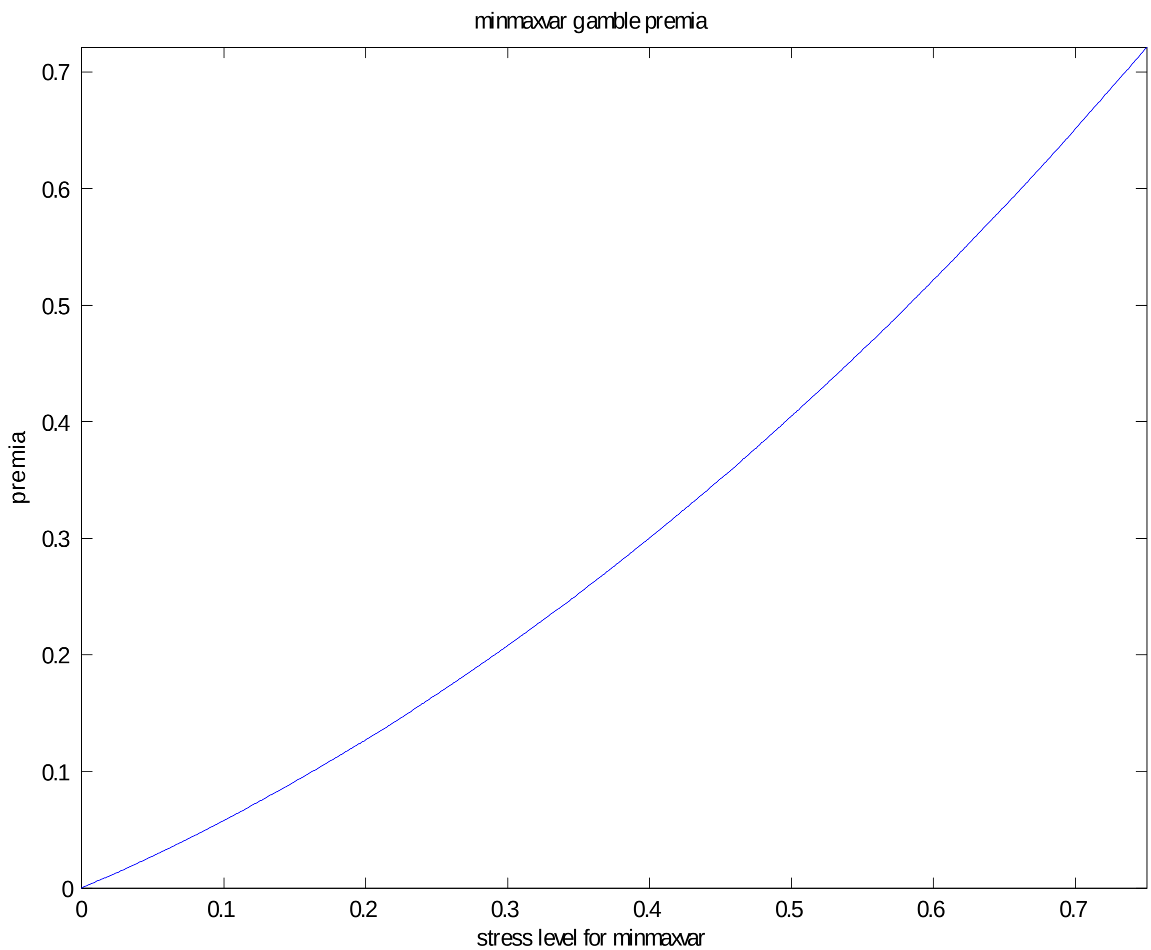
$$w(\theta) = e^{-rT} E\left[(X + c(\theta)e^{r})^{+}\right].$$

- Once we require that actions satisfy

$$c_\theta d\theta \leq 0,$$

so we are enforcing the acceptability of risk taking actions.

Figure 2: Graph of Stress Levels and Premia



- We ask if within this set it should be socially beneficial for companies to pursue the maximization of equity value.

- Our interest is in the resulting risk choices in the capital constrained private sector.

The Risk Model for a typical balance sheet

- We allow our hedge fund to be imbalanced with a long position in dollars of A_L and a short position of A_S dollars. The end of period values are

$$X = A_L e^{\alpha_L + X_L} - A_S e^{\alpha_S + X_S},$$

where X_L, X_S are unit time real valued random variables with unit exponential expectation.

- The asset growth rates are therefore respectively α_L, α_S for the long and short side.

- In order to capture issues of volatility, skewness and kurtosis along with correlation we shall suppose a conditional law for X_L given X_S and a marginal law for X_S that we describe shortly. The correlation will be parametrically built into the conditional law.

Post Capital Equity Value

- The fund has limited liability and posts capital K in future dollars and hence has an equity value of

$$w = e^{-r} E\left[\left(A_L e^{\alpha_L + X_L} - A_S e^{\alpha_S + X_S} + K\right)^+\right], \quad (2)$$

under a suitably chosen market determined expectation operator.

The Risk Model

- We wish to allow the risk distribution used to be fairly flexible and capable of capturing skewness and kurtosis in addition to volatility.

- We employ for this purpose the Variance Gamma family of distributions introduced by Madan and Seneta (1990), and Madan, Carr and Chang (1998).

- The distribution is that of an infinitely divisible random variable obtained from time changing Brownian motion with drift θ and volatility σ by a gamma process with unit mean rate and variance rate ν.

- The joint characteristic function of X_L and X_S is easily derived. The process for X_L may be written as

$$X_L = (\alpha_L + \omega_X)\, t + \beta x(t) + y(t),$$

where $y(t)$ is an independent VG process with parameters $\sigma_y, \nu_y, \theta_y$, and similarly

$$X_S = (\alpha_S + \omega_x)\, t + x(t),$$

with $x(t)$ a VG process with parameters $\sigma_x, \nu_x, \theta_x$.

- The convexity corrections for the exponential are

$$\omega_x = \frac{1}{\nu_x} \ln \left(1 - \theta_x \nu_x - \frac{\sigma_x^2 \nu_x}{2}\right)$$

and

$$\begin{aligned} \omega_X &= \frac{1}{\nu_x} \ln \left(1 - \beta\theta_x \nu_x - \frac{\sigma_x^2 \nu_x \beta^2}{2}\right) \\ &\quad + \frac{1}{\nu_y} \ln \left(1 - \theta_y \nu_y - \frac{\sigma_y^2 \nu_y}{2}\right). \end{aligned}$$

Characteristic Function

- The joint characteristic function of X_L, X_S is

$$\widetilde{\phi}(u,v) = \mathsf{exp}\left(iu\left(\alpha_L + \omega_X\right)t\right)\right) \times$$
$$\mathsf{exp}\left(iv(\alpha_S + \omega_x)t\right) \times$$
$$\left(\frac{1}{1 - i(u\beta + v)\theta_x\nu_x + \frac{1}{2}\sigma_x^2\nu_x\left(u\beta + v\right)^2}\right)^{\frac{t}{\nu_x}} \times$$
$$\left(\frac{1}{1 - iu\theta_y\nu_y + \frac{1}{2}\sigma_y^2\nu_y u^2}\right)^{\frac{t}{\nu_y}}.$$

- This is a nine parameter model describing the risk and return choices facing a typical hedge fund.

Spread Option Algorithm

- In order to price the equity value given by equation (2) we need to price a spread option in a Lévy framework.

- Recently Hurd and Zhou (2009), following Carr and Madan (1999) and Dempster and Hong (2000) develop an efficient two dimensional Fourier inversion methodology for pricing such options for a positive strike and we need it here for a negative strike.

- One may employ put call parity and adapt their algorithm to this case.

The Algorithm

- We wish to price the payoff

$$(A_L \exp(X_L) - A_S \exp(X_S) - K)^+$$

where we have the joint characteristic function

$$E[\exp(iuX_L + ivX_S)] = \phi_X(u, v)$$

for Lévy processes starting at zero.

- Following Hurd and Zhou (2009) we write the price as

$$w = e^{-rT} e^k \times \int dx_L dx_S \left(\begin{array}{c} \left(e^{a_L - k + x_L} - e^{a_S - k + x_S} - 1\right)^+ \times \\ f_X(x_L, x_S) \end{array} \right)$$

- Defining

$$\begin{aligned} y_L &= -x_L, \text{ and } y_S = -x_S, \\ E[\exp(iuY_L + ivY_S)] &= \phi_X(-u, -v) \\ &= \phi_Y(u, v) \end{aligned}$$

we may also write

$$
\begin{aligned}
& w = e^{-rT} e^{k} \times \\
& \int dy_L dy_S \left(\begin{array}{c} \left(e^{a_L - k - y_L} - e^{a_S - k - y_S} - 1\right)^{+} \times \\ f_Y(y_L, y_S) \end{array} \right) \\
= \; & e^{-rT+k} h(a_L - k, a_S - k),
\end{aligned}
$$

where

$$
\begin{aligned}
& h(a,b) = \\
& \int dy_L dy_S \left(\left(e^{a - y_L} - e^{b - y_S} - 1\right)^{+} f_Y(y_L, y_S) \right) \\
= \; & \int dy_L dy_S \times \\
& \left(\begin{array}{c} \left(e^{a - y_L} - e^{b - y_S} - 1\right)^{+} \times \\ e^{a_1(a - y_L) + a_2(b - y_S)} e^{-a_1(a - y_L) - a_2(b - y_S)} \times \\ f_Y(y_L, y_S) \end{array} \right) \\
= \; & e^{-a_1 a - a_2 b} \int dy_L dy_S \times \\
& \left(\begin{array}{c} \left(e^{a - y_L} - e^{b - y_S} - 1\right)^{+} \times \\ e^{a_1(a - y_L) + a_2(b - y_S)} e^{a_1 y_L + a_2 y_S} \times \\ f_Y(y_L, y_S) \end{array} \right).
\end{aligned}
$$

- Hence

$$\begin{aligned} & e^{a_1 a + a_2 b} h(a,b) \\ = & \int dy_L dy_S \begin{pmatrix} \left(e^{a-y_L} - e^{b-y_S} - 1\right)^+ \times \\ e^{a_1(a-y_L)+a_2(b-y_S)} e^{a_1 y_L + a_2 y_S} \times \\ f_Y(y_L, y_S) \end{pmatrix} \\ = & [(e^{x_1} - e^{x_2} - 1)^+ e^{a_1 x_1 + a_2 x_2} * \\ & e^{a_1 y_L + a_2 y_S} f_Y(y_L, y_S)](a,b). \end{aligned}$$

- The Fourier transform of the first function on the right hand side for suitable choices of a_1, a_2 is given by

$$P(u,v) = \frac{\Gamma(-a_1 - a_2 - 1 - iu - iv)\Gamma(a_2 + iv)}{\Gamma(1 - a_1 - iu)}.$$

- The transform of the second function of the above convolution is

$$\phi_Y\left(u - ia_1, v - ia_2\right) = \phi_X(ia_1 - u, ia_2 - v).$$

- Hence the transform of the left hand side is

$$\frac{\Gamma(-a_1 - a_2 - 1 - iu - iv)\Gamma(a_2 + iv)}{\Gamma(1 - a_1 - iu)} \times$$
$$\phi_X(ia_1 - u, ia_2 - v).$$

Capital and Equity

- Hence in this model we may compute both
  - the capital requirement from our closed form formula
  - and the equity value as a call option on a spread with a negative strike.

- We now study the interactions between capital requirements, risk choices and the private sector objectives of shareholder value maximization.

Some Preliminary Comparisons

- We take for a start two independent VG variables for the long and short positions.

- Hence we set β to zero.

- Consider also the case of zero skewness and hence we set $\theta_x = \theta_y = 0$.

- We start with low volatilities of $\sigma_x = \sigma_y = 0.15$ and the Gaussian case with $\nu_x = \nu_y = 0.001$.

- We take the fund to be balanced with $A_L = A_S = 100$ and consider the case $\alpha_L = \alpha_S = r = 0.05$.

- The maturity could be set at a year.

- We use the Cherny and Madan (2009) recommended stress function of $minmaxvar$ at three levels of $0.25, 0.75$, and 1.25 that are progressively higher levels of acceptability.

- The capital required for such a Gaussian hedge fund and the value of equity is as follows.

stress	0.25	0.75	1.25
capital	9.37	24.99	38.12
equity	13.42	25.13	36.66

- The higher the capital, the lower is the negative strike of the call option and the higher the equity value.

- We see some high leverage ratios of 10 to one and 4 to one.

- We now introduce some uniform kurtosis and set $\nu_x = \nu_y = 0.5$.

- The resulting capital and equity values are as follows.

stress	0.25	0.75	1.25
capital	9.58	26.71	43.33
equity	13.40	26.66	41.86

- We see that at a low stress level of 0.25 the capital and equity are not responsive to kurtosis while at 1.25 the response is quite substantial, with a moderate response at 0.75.

- Next we investigate the effect of correlation by changing β to 0.5 for a positively correlated balance sheet and to -0.5 for a negatively correlated one. We report the results in two successive tables.

stress	0.25	0.75	1.25
capital	7.52	14.17	32.65
equity	10.48	15.29	31.33

stress	0.25	0.75	1.25
capital	12.08	33.63	54.72
equity	16.91	33.56	52.49

- We observe that at all levels there is a substantial response to correlation, with positive correlation reducing capital requirements and equity values and negative correlation having the expected opposite effect.

- Finally we consider the effects of skewness. We report the results of $\beta = 0.5$, and $\theta_x = 0.1, \theta_y = -0.1$ and also $\theta_y = 0.1$ in two successive tables.

stress	0.25	0.75	1.25
capital	13.92	41.52	70.66
equity	19.59	41.48	67.81

stress	0.25	0.75	1.25
capital	14.39	41.99	71.20
equity	19.84	41.83	68.29

- We conclude that all dimensions of the distribution of outcomes, volatility, skewness, kurtosis and correlation, have important implications for capital requirements and equity values.

Private Market Risk Incentives in the Presence of Capital Constraints

- We now consider the set of incentives for risk taking in the private market that maximizes equity value once a capital constraint is in place.

- For such a computation we consider two cases, first supposing that market participants effectively adjust correlations to meet capital requirements and we consider the effect on equity value after such a correlation adjustment.

- Second we require continuous capital adjustments in line with risk exposures.

- In the first case, in the context of our model the correlation adjustment is determined for an increase in σ_x, for example by

$$\frac{\partial c}{\partial \sigma_x} d\sigma_x + \frac{\partial c}{\partial \beta} d\beta = 0$$

or

$$\frac{d\beta}{d\sigma_x} = -\frac{\partial c / \partial \sigma_x}{\partial c / \partial \beta}.$$

The effect on equity value post hedge is given by

$$\frac{\partial w}{\partial \sigma_x} + \frac{\partial w}{\partial \beta} \frac{d\beta}{d\sigma_x}.$$

Correlation Adjusted Results

- We perform these calculations for $\sigma_x = 0.15, \nu_x = 0.5, \theta_x = 0.1, \sigma_y = 0.15, \nu_y = 0.5, \theta_y = 0.1, \beta = 0.5$

- and two stress levels of 0.25 and 0.75.

- We report first on the incentives without capital constraints as given by the partial derivatives of equity value with respect to the seven risk parameters.

Equity Value Partials

		Stress
Risk	0.25	0.75
σ_x	23.23	77.71
ν_x	1.06	11.52
θ_x	11.14	43.32
σ_y	46.75	72.88
ν_y	2.41	9.57
θ_y	15.09	20.15
β	-6.59	-17.75

- Observe that in the absence of capital controls leaving participants free to act in all directions the private market incentives are quite perverse, with incentives for increasing volatility and kurtosis, raising the skewness of assets and liabilities, and decorrelating assets from liabilities.

Capital Constrained Partials

- The capital constrained partials of equity value with hedges being adjusted to meet capital constraints are as follows.

Capital Constrained Equity Value Partials

	Stress	
Risk	0.25	0.75
σ_x	-1.67	-1.14
ν_x	0.0053	-0.85
θ_x	-1.88	-0.95
σ_y	-0.89	-0.34
ν_y	-2.06	-1.38
θ_y	-2.17	-1.25

- We see that most of the risk incentives are reversed once the capital constraint is imposed. The stress level of 0.25 is too low and still allows for an incentive to raise residual kurtosis ν_x marginally.

- At the stress level of 0.75 all the incentives are in the right direction leaving the firm to concentrate on generating acceptable cash flows that simultaneously reduce capital requirements.

Capital Adjusted Results

- For the second approach requiring capital adjustment in line with risk exposures we evaluate the derivative of private sector profits with respect to risk characteristics.

- These are given with respect to σ_x, for example by
$$\frac{\partial w}{\partial \sigma_x} - \frac{\partial c}{\partial \sigma_x}.$$

- We report the capital adjusted profit partial derivatives for the two stress levels as follows.

	Capital Adjusted Profit Partials	
	Stress	
Risk	0.25	0.75
σ_x	5.18	-1.45
ν_x	0.29	-0.91
θ_x	1.70	-1.12
σ_y	12.24	-0.63
ν_y	-0.83	-1.43
θ_y	2.58	-1.34
β	-1.18	0.07

- We observe once again that at the stress level of 0.25 there is still a desire to raise volatilities, skewness and kurtosis and decorrelate assets and liabilities.

- This is reversed and all the perverse incentives are corrected at the stress level of 0.75.

- One may therefore correct risk incentives by continuously monitored capital adjustments, provided the stress level has been appropriately calibrated.

The effects of Debt on Required Capital

- We have seen that the capital C required for a balance sheet with access to a random cash flow X that makes $X + C$ acceptable is

$$C = -\int_{-\infty}^{\infty} x d\Psi\left(F_X(x)\right),$$

  excluding discounting considerations.

- This capital is the cash reserve needed to operate the balance sheet. The equity value is then given by

$$E\left[(X + C)^+\right]$$

  and this is our call option on the spread X with a negative strike of $-C$.

- This is the firm value V given by

$$V = (X + C)^+$$

Now if there is in addition some debt D with debt holders receiving

$$V \wedge D$$

and equity holders receiving

$$(V - D)^+,$$

then we may write equity value as

$$\begin{aligned} w &= E\left[\left((X+C)^+ - D\right)^+\right] \\ &= E\left[(X - (D - C))^+\right] \end{aligned}$$

- Now we have a call option on the spread with the higher strike of $-(C - D)$. If the debt proportion is

$$D = \delta C$$

- We now wish to evaluate the risk partials

$$\frac{\partial w}{\partial \sigma_x} - (1-\delta)\frac{\partial c}{\partial \sigma_x}.$$

- We consider capital adjusted profit partials with a debt of 50%.

- Without debt the stress level is 0.75 and the capital for a 100 dollar long short position is 24.31 and the Leverage is 4.1135.

- With a debt fraction of 50% the equity strike is further out of the money and the risk sensitivities may be reduced but so is the capital contribution.

- The net effect requires a raise in stress levels and an associated increase in capital requirements and reduced leverage.

- The stress level now moves to 2.0 to combat the adverse risk incentives. The Capital adjusted profit partials are

Capital adjusted Profit	
50% Debt Case	
σ_x	-10.4059
ν_x	-4.5581
θ_x	-5.6238
σ_y	-0.3463
ν_y	-5.1756
θ_y	-0.8197
β	1.3909

- The capital required at this stress level is 66.6022 and the leverage is substantially reduced to 1.5. Hence the presence of debt reduces the allowed leverage.

Conclusion 1

- We have shown that limited liability distorts private sector risk preferences towards higher volatility and kurtosis, increased skewness for liabilities, reduced skewness for assets, coupled with an incentive to decorrelate assets from liabilities leading to economically poor risk choices.

- In such a context we introduce the concept of acceptable risks operationalized by a positive expectation after distortion of the distribution function.

- This leads to a definition of capital requirements that merely make the risks undertaken acceptable.

- We then show that enforcing these capital requirements can mitigate the perverse risk incentives introduced by limited liability provided that the set of acceptable risks is suitably conservatively defined.

- A careful, critical and external assessment of capital requirements is therefore essential for the efficient and proper functioning of the private sector.

Practical Considerations

- Practical implementation of the procedures introduced here would not require detailed information about specific positions but only aggregate data on the distribution of returns broken down by the two sides of the balance sheet along with the degree of correlation between the balance sheet's two sides.

- The stress level associated with the level of acceptability could be periodically calibrated to evolving market conditions to ensure that private sector incentives are not being perverted by the limited liability features in place.

Using the Equity Option Surface to Determine Required Cash Reserves

- We will use $minmaxvar$ at stress level $\gamma = 0.75$.

- All the computations may be accomplished and the reserve capital required may be identified once we have described the probability law $F(x)$ of the terminal cash flow that we take to be level of risky assets less risky liabilities one year out.

- Our focus in this paper is on the large surviving US banks and we consider the six banks, BAC, C, GS, JPM, MS, and WFC.

Equity as a Spread Option

- We extend the pioneering work of Merton (1973, 1974, 1977) and account for the access to derivative markets that enables transformations of risk exposures and permit positions in a whole range of contingent liabilities.

- There is a component of assets that we shall call cash or money and denote by M, with total assets being $A + M$ where A is the random component of assets that may fluctuate in value.

- On the liability side we also have a relatively fixed or bounded component like risky debt.

- In addition we allow for risky liabilities that are random and may rise in value, in principle without bound.

- Hence we have in place of the Mertonian equation with random assets equalling equity plus risky debt, we write

$$
\begin{aligned}
& Cash \ + \ Risky\ Assets \\
= \ & Equity \ + \ Risky\ Debt \ + \ Risky\ Liabilities \\
& M(t) + A(t) \\
= \ & J(t) + D(t) + L(t)
\end{aligned}
$$

- The limited liability for equity requires us to recognize that at debt maturity T with face value F, we have that

$$J(T) = (M(T) + A(T) - L(T) - F)^{+}.$$

- Debt holders receive

$$D(T) = (M(T) + A(T) - L(T))^{+} \wedge F.$$

- We recognize the relative nonrandomness of money by setting $M(T) = M(t) = M$ and we write

$$J(T) = (M + A(T) - L(T) - F)^{+}$$

- Hence the equity and debt value initially is

$$\begin{aligned} J &= E_0^Q\left[(A(T) - L(T) - (F - M))^+\right] \\ D &= E_0^Q\left[(M + A(T) - L(T))^+ \wedge F\right] \end{aligned}$$

- Apart from the limited liability of equity, the firm also has limited liability and the firm value at maturity is

$$(M + A(T) - L(T))^+$$

- When $L(T)$ is zero and $A(T) \geq 0$ we may as well ignore M as we always have a positive value and there are no capital requirements to be imposed by the general economy.

- Under such assumptions the option to put the firm to the general economy is worthless by construction as the worst outcome of zero assets wipes out debt and equity but does not effect anyone in the rest of the economy.

- As a consequence the external world has no interest in managing the value of this put via the imposition reserve capital requirements.

- With the presence of random liabilities this is no longer the case as we now have state contingent and potentially unbounded liabilities.

- The value of the firm is now the value of a call on the spread of A over L struck at $-M$.

$$\begin{aligned} & J(0) + D(0) \\ = \; & E_0^Q \left[(M + A(T) - L(T))^+ \right] \\ \geq \; & M + A(0) - L(0) \end{aligned}$$

- with the excess being the value of the put option P on $A(T) - L(T)$ at strike $-M$, or

$$P = E_0^Q \left[(-M - (A(T) - L(T))^+ \right].$$

- The external economy must monitor this put value by insisting that M be sufficiently large, reducing $-M$ and the value of the free put option.

- Otherwise market participants will be picking up these free put options all over the place.

- External regulators must set capital requirements at levels such that the required capital M^* is

$$M^* = -de\,(A(T) - L(T), \gamma, 'minmaxvar').$$

  We may compare this level with what was held by banks in terms of cash or cash equivalent reserves once we have the law of the risk held which is $A(T) - L(T)$.

- The risk neutral law of $A(T) - L(T)$ may be modeled as the difference of two exponential Lévy processes that we may simulate forward in time.

- On this path space we may evaluate the path space of equity prices computed as a spread option with payoff

$$J(t) = E_t\left[(A(T) - L(T) - (F - M))^+\right],$$

- We use these paths to construct equity option prices for strike K and maturity t as

$$e^{-rt}E\left[(J(t) - K)^+\right].$$

- We determine the parameters of the joint and correlated risky asset and liability value process to best fit the surface of the equity option surface as seen on the option markets.

- We then compute the value M^* at a level γ that mitigates adverse risk incentives for equity holders and compare this required reserve capital with the level of M obtained from balance sheets to determine which banks were undercapitalized or overcapitalized from the perspective of risk exposure for the external economy.

- We also compute the value of the limited liability put, P, held by the firm.

Net Asset Value Process

- Let us take the risk neutral risky asset and the risky liability as exponential Lévy processes with

$$\begin{aligned} A(t) &= A(0)\exp\left(X(t) + (r + \omega_X)t\right) \\ L(t) &= L(0)\exp\left(Y(t) + (r + \omega_Y)t\right) \end{aligned}$$

  where we now allow for a rich dependence in these processes.

- If we take a linear mixture of just two independent Lévy processes we get jumps occurring on two rays from the origin.

- If the independent processes are variance gamma VG processes for example then we have a VG process running in log space on a particular ray from the origin with the asymmetry parameter on this ray being the skewness parameter of the VG. The VG uses three parameters for each ray which is two sided.

- Given that we operate in a two sided way for each independent Lévy process, we need to cover 180 degrees of possible directions of motion.

- We take 4 VG processes with 12 parameters placed at the degrees 30, 60, 120, and 150.

- This gives us two rays with a positive relation between assets and liability movements and two rays with a negative dependence.

- We shall let the calibration determine the relative variance placed on each of the four rays. For the four angles η_j, $j = 1, \cdots 4$ we have the jumps in assets and liabilities as

$$
\begin{aligned}
x_j &= u_j \cos(\eta_j) \\
y_i &= u_j \sin(\eta_j)
\end{aligned}
$$

  where u_i is the jump in the j^{th} VG process with parameters $\sigma_j, \nu_j, \theta_j$.

- We then have that

$$\begin{bmatrix} X(t) \\ Y(t) \end{bmatrix} = \begin{bmatrix} \cos(\eta_1) & \cos(\eta_2) & \cos(\eta_3) & \cos(\eta_4) \\ \sin(\eta_1) & \sin(\eta_2) & \sin(\eta_3) & \sin(\eta_4) \end{bmatrix} \begin{bmatrix} U_1(t) \\ U_2(t) \\ U_3(t) \\ U_4(t) \end{bmatrix}$$

and our joint law is the linear mixture of 4 independent VG Lévy processes with a prespecified mixing matrix. The joint characteristic function is

$$\begin{aligned} & E\left[\exp\left(iuX(t) + ivY(t)\right)\right] \\ = & \prod_{j=1}^{4} \begin{pmatrix} 1 - i(u\cos(\eta_j) + v\sin(\eta_j))\theta_j\nu_j \\ +\frac{\sigma_j^2\nu_j}{2}(u\cos(\eta_j) + v\sin(\eta_j))^2 \end{pmatrix}^{-\frac{t}{\nu_j}} \\ = & \phi(u, v) \end{aligned}$$

- The value of

$$\begin{aligned} \omega_X &= \sum_{j=1}^{4} \frac{1}{\nu_j} \ln\left(1 - \cos(\eta_j)\theta_j\nu_j - \frac{\sigma_j^2\nu_j\cos^2(\eta_j)}{2}\right) \\ \omega_Y &= \sum_{j=1}^{4} \frac{1}{\nu_j} \ln\left(1 - \sin(\eta_j)\theta_j\nu_j - \frac{\sigma_j^2\nu_j\sin^2(\eta_j)}{2}\right) \end{aligned}$$

and the characteristic function of the logarithm of assets and liabilities is

$$
\begin{aligned}
& E\left[e^{iu\,\mathsf{ln}(A(t))+iv\,\mathsf{ln}(L(t))}\right] \\
= \;& \phi(u,v)\times \\
& \mathsf{exp}(iu\,\mathsf{ln}(A(0))+iv\,\mathsf{ln}(L(0)) \\
& +iu(r+\omega_X)t+iv(r+\omega_Y)t)
\end{aligned}
$$

- Our equity value at any date t given a simulation of $A(t), L(t)$ is the price of a spread option with some strike and maturity using this joint characteristic function with initial values $A(t), L(t)$ and time to maturity $T-t$.

- For the initial value of risky assets and risky liabilities excluding debt, we take these magnitudes from the balance sheet but permit some option market adjustment factor to match the stock price.

- The adjustment factor is calibrated by equating the value of equity computed as a spread option at the strike of debt less initial cash equivalent reserves with the initial stock price at market close on the calibration date.

Balance Sheet and Option Data

- For the balance sheet we access compustat data from WRDS. For each of the six banks we obtained data for the year end 2008.

- We first take data on cash plus short term investments, the variable CHE in compustat and we shall use this value for our initial cash equivalent reserve or the variable M in our calibration procedures.

- For risky assets, A, we take total assets, AT in compustat less CHE.

- For risky liabilities, L, we take all liabilities less long term debt ($DLTT$) plus debt in current liabilities (DLC).

- For the level of debt, D, we take $DLTT$ plus DLC.

- In addition we need the number of shares outstanding, N, and the stock price, S.

- The data is presented in Table 1.

TABLE 1
Balance Sheet on 6 Banks at end of 2008
in billions of dollars

	M	A	L	D	N	S
JPM	368	1807	1009	633	3732	31.59
MS	211	448	181	392	1047	15.16
GS	244	640	299	498	443	82.24
BAC	125	1693	883	633	5017	13.93
WFC	72	1238	781	375	4228	29.86
C	326	1613	769	720	5450	6.88

- The other data we shall bring to bear on the study of capital required is the option surface at year end. Here we have over a hundred options trading at any time.

- We present along with the calibration results later the graphs of the market option prices used in the calibration along with the fitted prices from the compound spread option model. We take option maturities below 1.5 years.

Calibration Details

- For each of the six banks we take data on equity option prices at the date of the balance sheet statement and we describe details for JPM.

- The level of risky assets was 1806903 and risky liabilities were at 1009277. The number of shares was 3732.

- We define $A(0)$, $L(0)$ to be risky assets and liabilities on a per share basis at 484.1647 and 270.4386 respectively.

- The total debt was at 633474 and the value of M was 368149 and this gives us a strike on a per share basis of $(633474 - 368149)/3732 = 71.0945$.

- Technically the strike should be future valued to the maturity but given the low rates and relatively short maturities involved we ignored this adjustment to the strike.

- The stock price was 31.59.

- We take as parameters the maturity of equity as a spread option on the underlying spread of assets over liabilities and the 12 VG parameters on the four rays on which we run our mixture of VG processes.

- The first step is to solve for ξ such that the value of the spread option starting at asset level $A(0) * (1-\xi)$, and liability level $L(0) * (1+\xi)$ equals the observed market stock price of 31.59.

- This is done for the chosen set of VG parameters and ξ is the option market adjustment factor.

- The next step is to generate paths of assets and liabilities daily for 1.5 years and we generated 10000 such paths.

- Then we use the spread option pricing algorithm to compute a grid of prices of equity as a spread option at all the maturities for which we have equity option data.

- This grid is used to interpolate equity values for each of the maturities and all the 10000 paths.

- Given the interpolated equity values we compute the prices of equity options at all the traded strikes and maturities for which we have option data. We then form the mean square error between observed market option prices and the model computed option prices.

- This procedure gives a single value for the objective function that is minimized by an optimizer over the 13 dimensions of the 12 VG parameters and the maturity of the equity as a spread option. The entire calibration for a single name on a single day takes a few hours.

Calibration Results

- We report the results in the order JPM, MS, GS, BAC, WFC, and C.

- The estimated maturities for equity as spread option were close to 5 years and are explicitly 4.4726, 4.9890, 5.0036, 5.0025, 4.9893, and 4.9991.

- We may in future calibrations just set this at 5 years and calibrate the other parameters. We report the VG parameters for the four angles in four separate tables.

$TABLE$ 2
VG 30^0

	σ	ν	θ
JPM	0.0955	0.1558	-0.0178
MS	0.0476	0.1491	-0.0593
GS	0.0018	0.1509	-0.0434
BAC	0.0289	0.1490	-0.0474
WFC	0.0385	0.1594	-0.0476
C	0.0553	0.1501	-0.0505

$TABLE$ 3
VG 60^0

	σ	ν	θ
JPM	0.4018	0.0810	−0.8448
MS	0.1422	0.0843	−0.1927
GS	0.1605	0.0937	−0.1935
BAC	0.0958	0.0744	−0.1792
WFC	0.0735	0.0875	−0.2037
C	0.1990	0.1007	−0.2001

$TABLE$ 4
VG 120^0

	σ	ν	θ
JPM	0.0968	0.1778	0.2967
MS	0.1699	0.2693	0.3217
GS	0.0761	0.2133	0.2092
BAC	0.0016	0.2331	0.2757
WFC	0.1088	0.2564	0.3439
C	0.1098	0.1992	0.2016

•

$TABLE$ 5
VG 150^0

	σ	ν	θ
JPM	0.0116	0.3524	0.0175
MS	0.0240	0.2003	0.0522
GS	0.0117	0.2002	0.0072
BAC	0.0671	0.2175	0.0737
WFC	0.0105	0.2023	0.0614
C	0.0598	0.1999	0.0209

- We observe that skewness is negative on the positive angles and positive on the negative angles. Hence down jumps are more likely in directions where they move together, while up jumps are more likely when they move in opposite directions.

- We present in Figures (3), (4), (5), (6), (7), and (8) the observed and fitted option prices.

Required Reserve Capital Computations

- We present in Table 6 the computed externally required reserve capital at the stress level of 0.75 that was recommended in Madan (2009) for the distortion $minmaxvar$.

- Also presented are the level of cash equivalent capital held, the value of the limited put held by the firm, the ratio of required reserve capital to cash equivalent capital held and the option adjustment factor.

TABLE 6

In Billions of Dollars

	Req. Res.	Res. Held	LL Put	Req. Act. Ratio	Adj. Factor
JPM	698	368	293.96	1.8961	31.54
MS	116	211	29.75	0.5523	41.13
GS	−84	244	3.37	−0.3430	17.96
BAC	246	125	158.17	1.9700	28.40
WFC	367	72	220.14	5.0884	21.07
C	435	326	156.21	1.3344	39.84

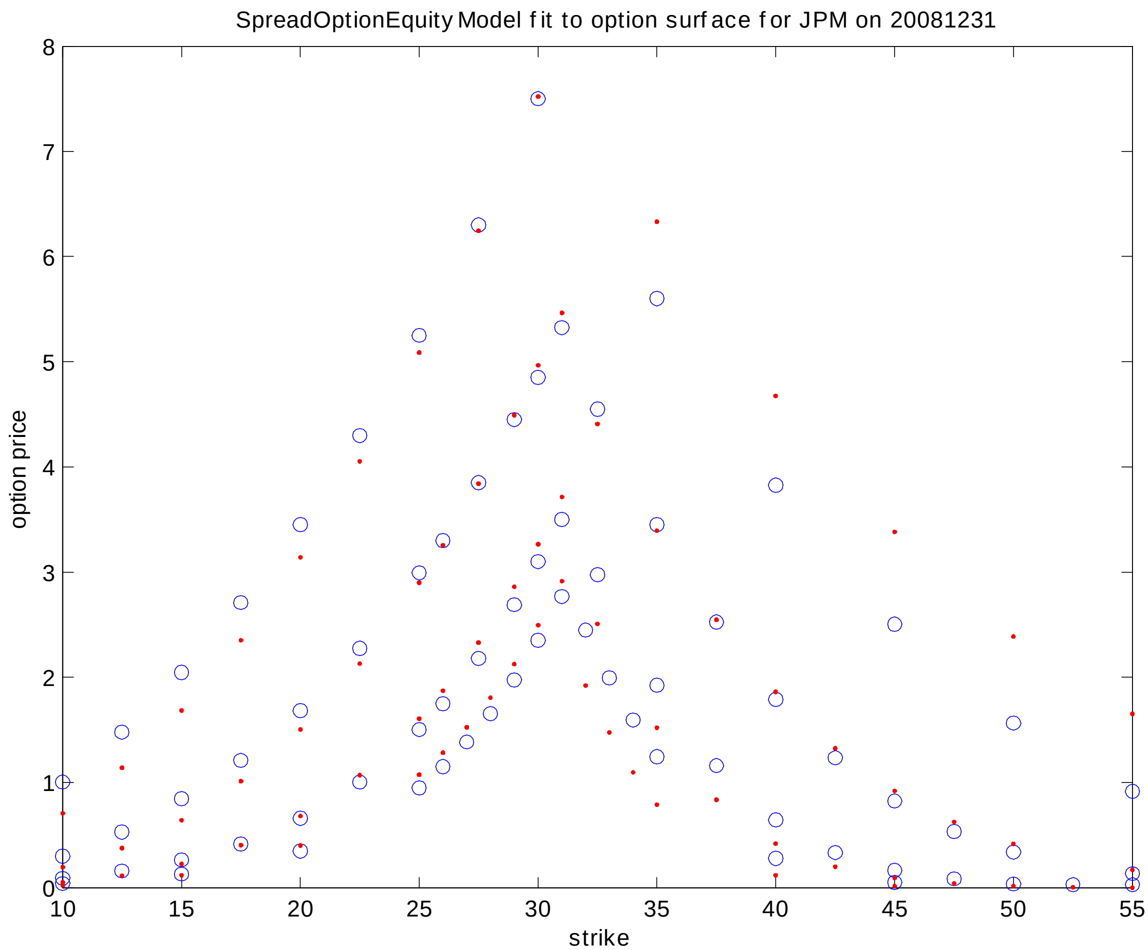


Figure 3: Market Prices and Model Prices for JPM.

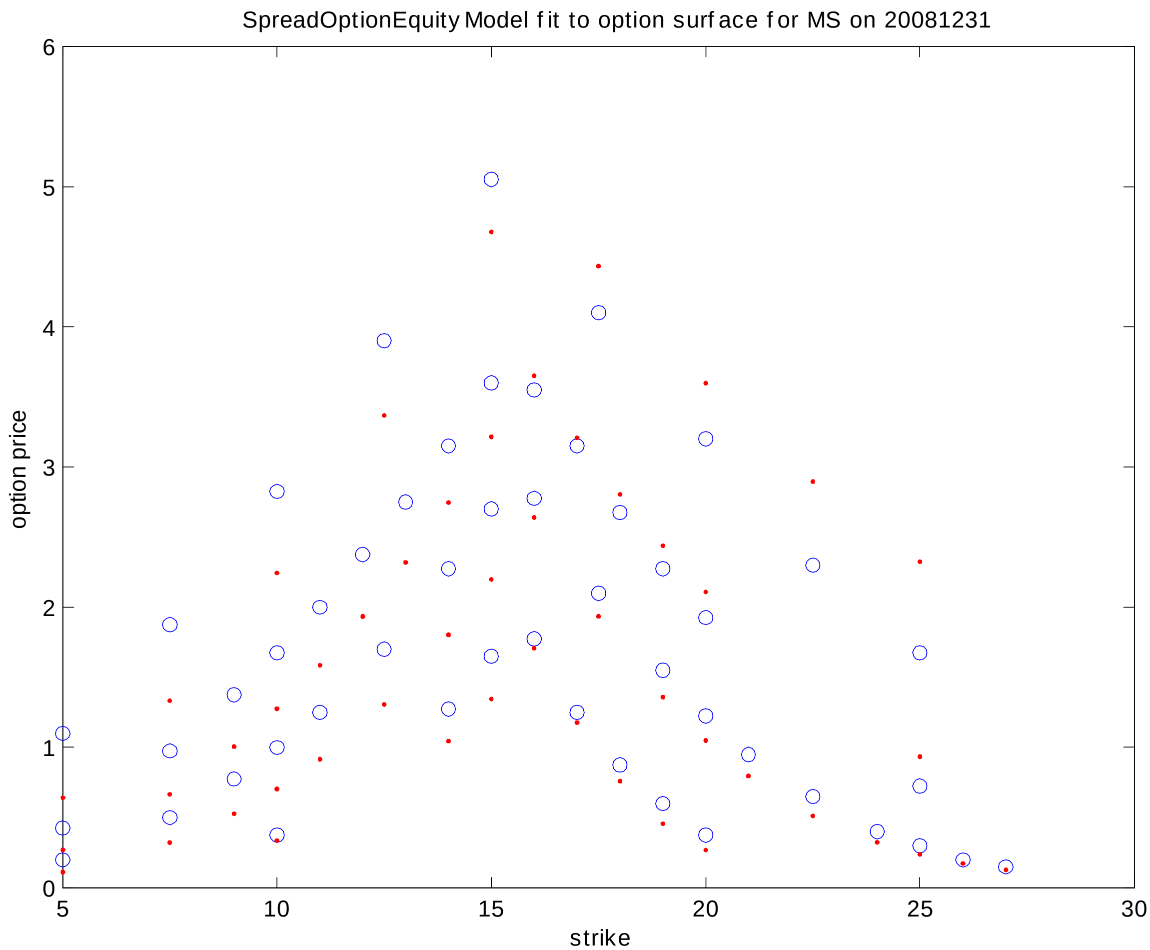


Figure 4: Market Prices and Model Prices for MS.

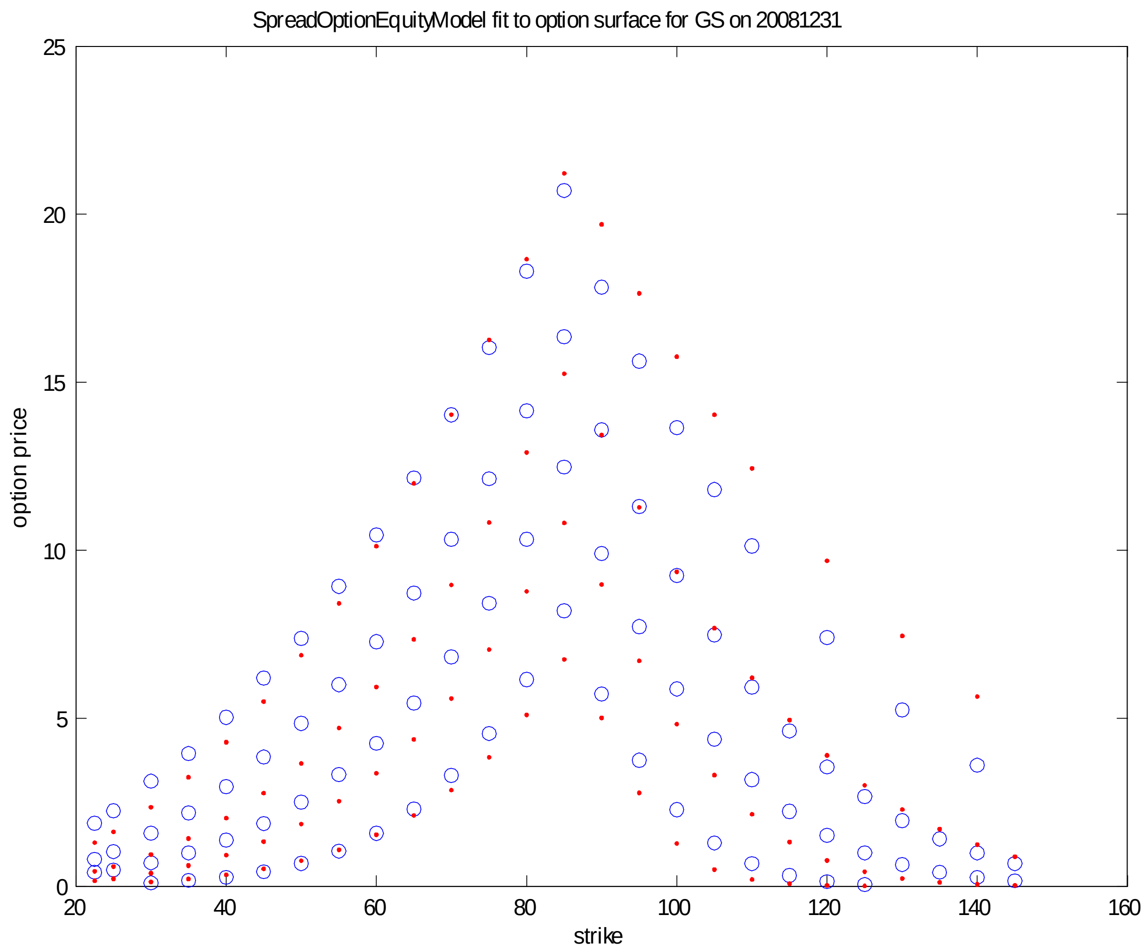


Figure 5: Market Prices and Model Prices for GS.

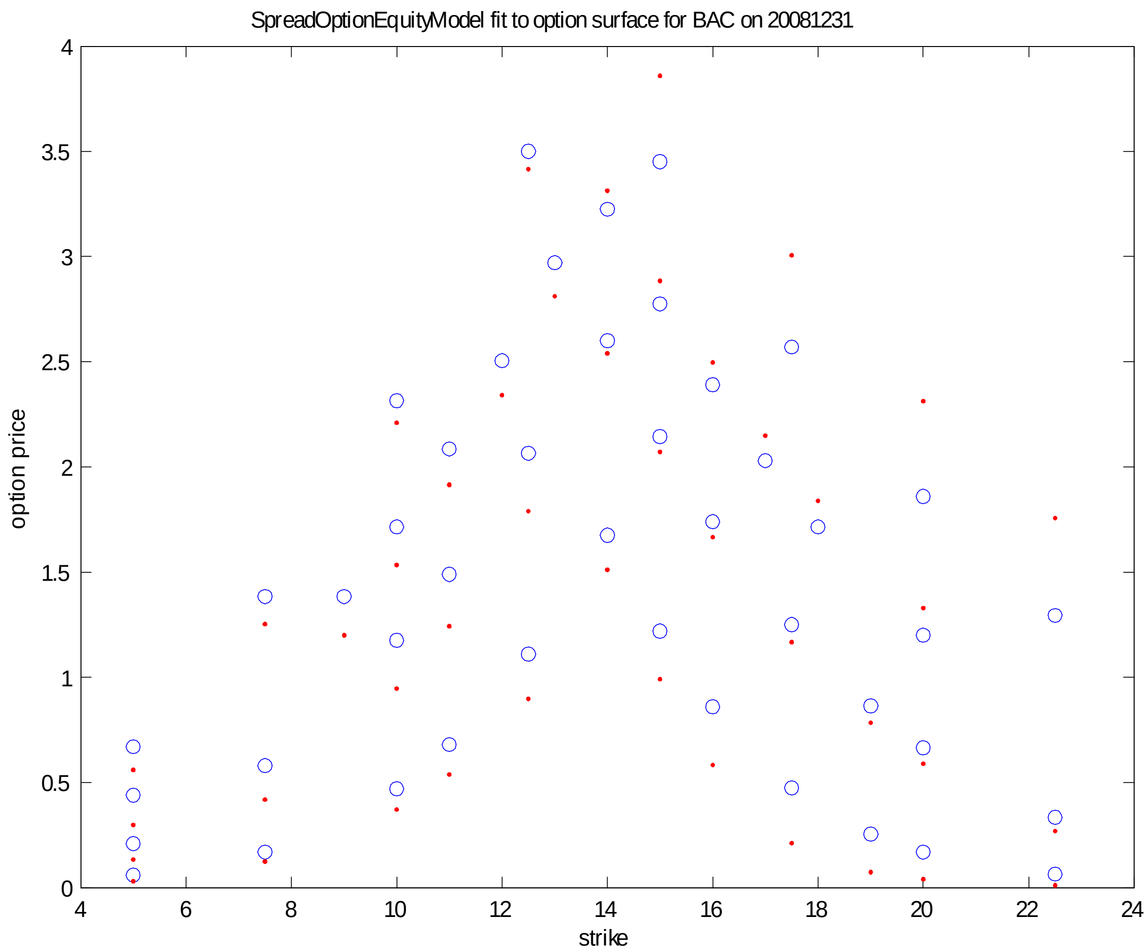


Figure 6: Market Prices and Model Prices for BAC.

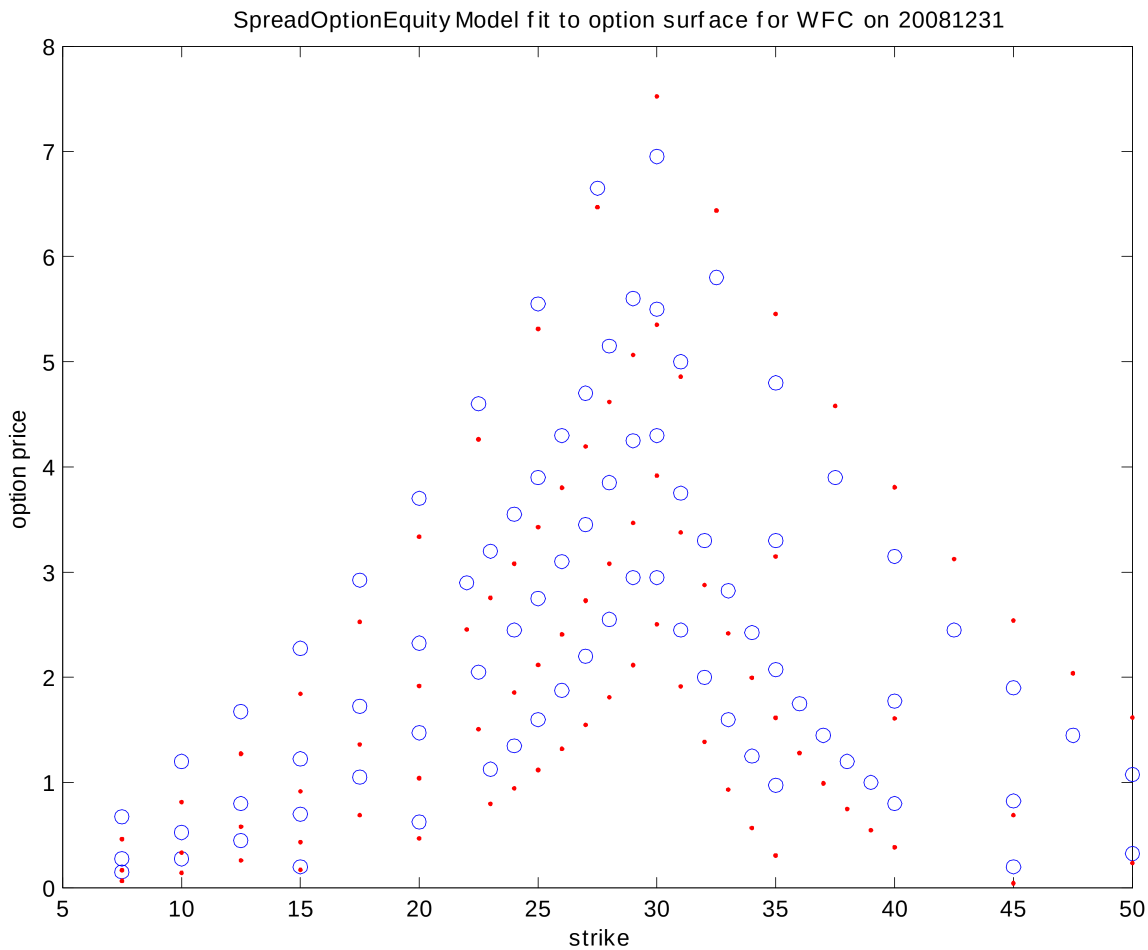


Figure 7: Market Prices and Model Prices for WFC.

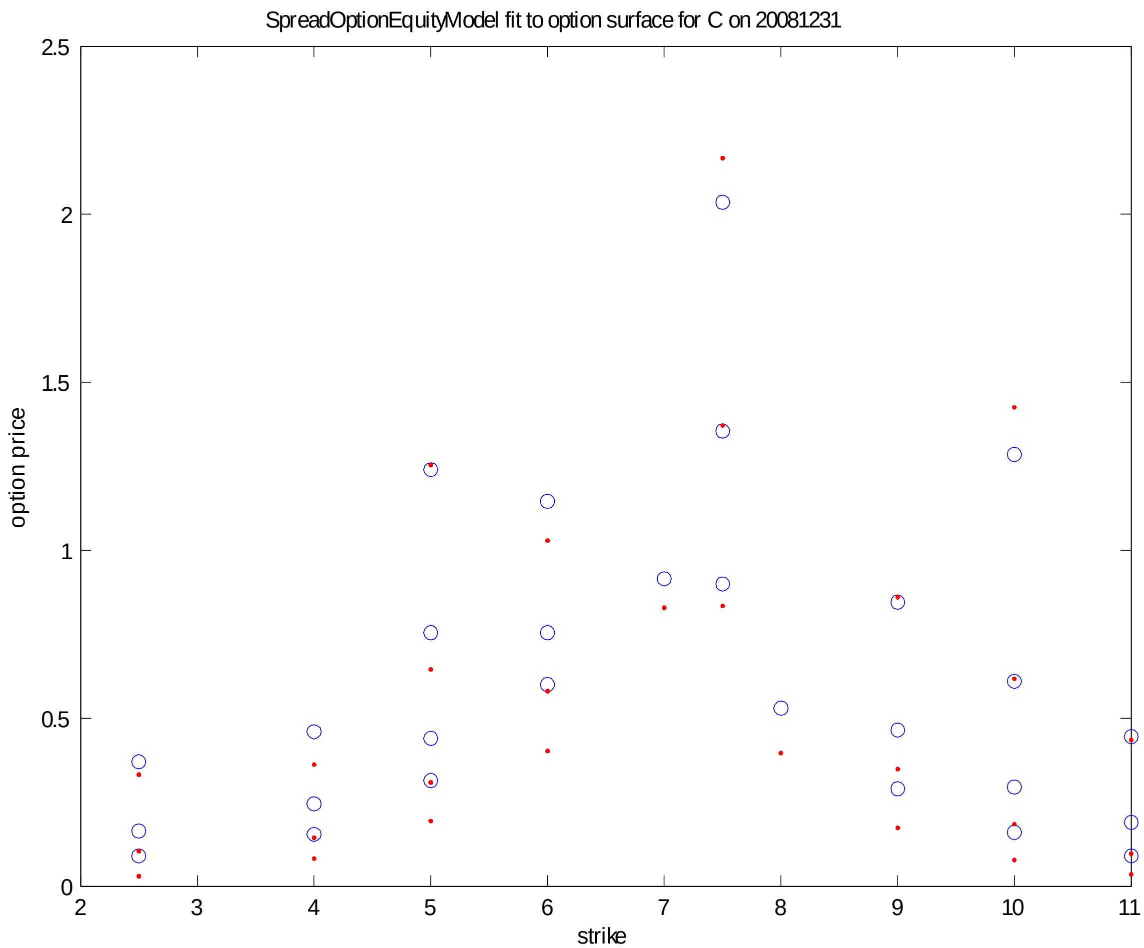


Figure 8: Market Prices and Model Prices for C.

- We observe that GS could remove all cash plus some more as its required reserve capital is negative.

- The value of the limited liability Put is high for WFC and JPM, followed by BAC and C with low values for MS and GS.

- All the others should hold cash equivalent reserves but MS could reduce its cash equivalent holdings by around 50% while the others need to add capital with WFC facing the biggest shortfall.

- The shortfall in the case of WFC may just be a consequence of having recently taken over Wachovia.

Conclusion 2

- We model assets excluding cash plus short term investments and liabilities excluding debt as two positive random processes.

- They are taken as exponentials of two Lévy processes that are modeled as linear mixtures of four independent Lévy processes that may be viewed as factors.

- Two of these factors drive assets and liabilities with positive correlations while the other two of them induce negative correlations.

- As a consequence equity is a call option on the spread of risky assets over risky liabilities.

- We employ recently developed methods by Hurd and Zhou (2009) to value these spread options using a two dimensional Fourier inversion.

- The capital required is computed using the distortion $minmaxvar$ and the stress level 0.75.

- For the calibration we use the equity option surface, simulating first the paths of assets and liabilities, then transforming them to paths of equity values using the model of equity viewed as spread option on the underlying joint asset liability process.

- The resulting equity value path space permits the computation of equity option prices for the traded strikes and maturities that are then compared to market prices to calibrate the joint asset liability process.

- The calibrated process is then used to determine externally required reserve capital which is compared with the cash equivalent capital held.

- We find GS and MS to be sufficiently capitalized while the other four banks are undercapitalized with the greatest shortfall occurring for WFC.