Indices of Acceptability as Performance Measures

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Outline

- Operationally defining Acceptability of Random Cash Flows
- New Examples of Acceptability Indices
  - MINVAR, MAXVAR, MINMAXVAR, MAXMINVAR
- Applications in the Options Domain
- Construction of Required Sharpe Ratios for Hedge Funds
- Pricing Contracts written on Daily Returns
- Pricing Gap Risk
- Pricing CFO contracts

Making Acceptability Operational

- The idea of Pricing and Hedging to acceptability was proposed in Carr, Geman and Madan (JFE 2001).
- The first part of that paper explored the analogy with efficient markets redefined as the absence of no acceptable opportunities and related this to the existence of a pricing operator in the convex hull of the measures defining acceptability.
- The second part considered incomplete markets more fully and explored the generalization of super and sub replication to determining ask and bid prices by hedging to acceptability.
- The primary personal dissatifaction with this work was the absence of an operational definition of acceptability cones capable of producing real prices and hedges for real contracts.

Operational Acceptability

- Consider the task of evaluating trading opportunities and or strategies.
- We agree that an arbitrage is an excellent trade but one that is not likely to be found.
  - There is an extensive literature testing market efficiency understood to be the absence of arbitrage.
  - As this is broadly equivalent to the existence of a pricing kernel or a linear pricing rule, the tests often focus on actually testing for the validity of a particular asset pricing model.

- Another interesting result of Jacod and Shiryaev (1998) describes no arbitrage as zero lying in the interior of the set of possible price moves.
  - Hence no arbitrage is just the acceptance of a positive probability of a loss.

- We accept the absence of arbitrage in this sense though the pricing kernel may be hard to find.

Measuring Efficiency

- As we accept market efficiency in the sense of no arbitrage, we ask what is the level of market efficiency?
- From this perspective, we view the absence of arbitrage as a zero level of efficiency.
- In a really efficient economy one should not be able to come anywhere near an arbitrage.
- This led us to axiomatize the degree of trade acceptability with a positive expectation being a zero level of acceptability while an arbitrage was an infinitely acceptable trade.

Axiomatizing Acceptability Levels

- We work on $L^\infty(\Omega, \mathcal{F}, P)$ a general probability space. We define an acceptability index α to be a map from L^∞ to the extended positive reals $[0, \infty]$, with $\alpha(X)$ being the level of acceptability of the random variable $X \in L^\infty$.
- The set of trades with cash flows X acceptable at level x is given by
$$\mathcal{A}_x = \{X | \alpha(X) \geq x\}.$$

Convexity

- We require that $\mathcal{A}_x$ be a convex set for all x and hence that

$$\begin{aligned} \alpha(X) &\geq x, \alpha(Y) \geq x \\ \text{then for } \lambda, 0 &\leq \lambda \leq 1 \\ \alpha\left(\lambda X + (1-\lambda)Y\right) &\geq x. \end{aligned}$$

- This property was already adopted for acceptability in Artzner, Delbaen, Eber and Heath (1999) and Carr, Geman, Madan (2001) and we merely adopt it for all the acceptability levels x.

Monotonicity

- Domination increases acceptability and we require

  if $X \leq Y$, then $\alpha(X) \leq \alpha(Y)$

- This property is shared with classical utility theory and preference orderings in economic theory generally.

Scale Invariance

- Our interest is in determining the direction of trades and not their scale.
- The scale may be determined by other considerations like market impact, liquidity or depth.
  - Classical theory in our view mistakenly assumes that one may trade at an arbitrary level with no market impact and then uses concavity to determine the optimal trade level.
  - We link our acceptability indices to more traditional performance measures like the Sharpe ratio or the Gain-Loss ratio that are scale invariant.
- Hence we impose that

$$\alpha(\lambda X) = \alpha(X), \text{ for } \lambda > 0.$$

Fatou Property

- This is a continuity or closure property and is a relatively technical condition.
- For X_n with $|X_n| \leq 1$, and $\alpha(X_n) \geq x$ with X_n converging to X in probability we require the $\alpha(X) \geq x$.

Coherent Acceptability Indices

- The four properties on Convexity, Monotonicity, Scale Invariance and the Fatou property define the class of coherent acceptability indices.
- We introduce some other properties that help in formulating operational models of acceptability.
- The first additional property enables us to access the results of Kusuoka (2001) who characterized all law invariant risk measures and hence indirectly law invariant cones of acceptability.

Law Invariance

- We require that the index of acceptability depend on just the probability law of the random variable.
- Hence we require that

  $$\text{if } X \stackrel{law}{=} Y \text{ then } \alpha(X) = \alpha(Y).$$

- This property is shared with traditional performance measures like the Sharpe ratio or the Gain Loss ratio.
- Its main purpose is as a vehicle to concrete examples of acceptability cones.

Three other properties

- Three other properties we note are
  - Second Order Monotonicity, whereby we require that if Y second order stochastically dominates X then $\alpha(X) \leq \alpha(Y)$.

– Arbitrage Consistency, or $X \geq 0$ if and only if $\alpha(X) = \infty$.
  * Acceptability indices depart from traditional preference orderings here as we are effectively converting the entire positive orthant to a bliss point at infinity and we do not rank two positive cash flows from an acceptability perspective.
  * As noted at the start, we do not expect to find zero cost cash flows in the positive orthant so why bother ranking them.

– Expectation Consistency, if $E[X] < 0$, then $\alpha(X) = 0$; and $E[X] > 0$ then $\alpha(X) > 0$.

Characterization Result

- A map $\alpha : L^\infty \to [0, \infty]$ is a coherent acceptability index if and only if there exists a collection $(\mathcal{D}_x)_{x \in \mathbb{R}_+}$ of subsets of probability measures absolutely continuous with respect to P such that $\mathcal{D}_x \subseteq \mathcal{D}_y$ for $x \leq y$ and
$$\alpha(X) = \inf \left\{ x \in \mathbb{R}_+ | \inf_{Q \in \mathcal{D}_x} E^Q[X] < 0 \right\}$$
where $\inf \emptyset = \infty$.
- Alternatively $\alpha(X)$ is the largest x such that $X \in \mathcal{A}_x$.

- The determining sets associated with an index of acceptability are the maximal collections supporting an index of acceptability and these are uniquely identified.
- Arbitrage consistency requires that the $\cup_x \mathcal{D}_x$ coincides with the set of all measures absolutely continuous with respect to P.
- For expectation consistency $\mathcal{D}_0 = \{P\}$.

Performance Measures : Early Examples

- The Sharpe Ratio defined by

$$SR = \frac{E[X]}{\sigma(X)}, \text{ if } E[X] \geq 0$$
$$0 \text{ otherwise}$$

  is scale invariant, law invariant, and expectation consistent and has the Fatou property.

- It is well known not to satisfy the monotonicity property and is not arbitrage consistent.

- The Gain-Loss ratio defined as
$$GLR(X) = \frac{E[X^+]}{E[X^-]} - 1, \text{ if } E[X] > 0$$
$$0 \; otherwise$$
- This measure is law and scale invariant, arbitrage and expectation consistent, it is monotonic and has the Fatou property. It is a coherent acceptability index.
- It also satisfies convexity and second order monotonicity.

- A dissatisfaction with this measure is the absence of a unique identification of the extreme measures attaining the infimum of $E^Q[X]$ for $Q \in \mathcal{D}_x$.
- The density of the extreme measure for $\mathcal{D}_x$ has the form
$$c\left(1 + x\mathbf{1}_{X \leq b}\right)$$
- Another disadvantage is that the extreme measure does not exaggerate large losses relative to small ones and treats them uniformly.

- The Tilt Coefficient is defined as the largest negative exponential tilt such that the tilted expectation is positive or

$$TC(X) = \inf\left\{\lambda \in \mathbb{R}_+ | E[Xe^{-\lambda X}] < 0\right\}.$$

- This measure is monotone, law invariant and has the Fatou property.
- It is not convex or scale invariant and is therefore not a coherent acceptability index.
- It also fails second order monotonicity.

The Law Invariant Measures

- An important building block of law invariant cones of acceptability is $TVAR$ the measure for tail value at risk. It is defined as $-u^{\lambda}(X)$ where
$$u^{\lambda}(X) = \inf_{Q \in \mathcal{D}_{\lambda}} E^{Q}[X]$$
$$\mathcal{D}_{\lambda} = \left\{ Q << P | \frac{dQ}{dP} \leq \frac{1}{\lambda} \right\}.$$
- For X with a continuous distribution the infimum is attained at
$$\frac{dQ^{*}}{dP} = \frac{1}{\lambda} \mathbf{1}_{X \leq q^{\lambda}(X)}$$
where $q^{\lambda}(X)$ is the $\lambda - quantile$ of X.

- An acceptability index based on tail value at risk is defined by

$$AIT(X) = \frac{1}{\inf\{\lambda \in (0,1] | u^{\lambda}(X) \geq 0\}} - 1$$

- We have here a coherent acceptability index that is arbitrage and expectation consistent and the extreme measures are identified.
- We do ignore gains completely and treat all losses uniformly.

Weighted Value at Risk

- Tail Value at Risk leads us to weighted value at risk by integrating with respect to a measure on the quantile levels. Let μ be a measure on $(0,1]$ and define
$$u^{\mu}(X) = \int_{(0,1]} u^{\lambda}(X)\mu(d\lambda)$$
- To define an acceptability index we are going to have to find a sequence of measures μ_x indexed by the level of acceptability and it is not clear how to go about doing this.
- There is however an alternative equivalent characterization of weighted value at risk defined by relating the measure μ to a concave distribution function on the unit interval Ψ^{μ} defined by
$$\Psi^{\mu}(y) = \int_0^y dz \int_z^1 \frac{1}{\lambda}\mu(d\lambda)$$

- We may then establish that
$$u^\mu(X) = \int_{\mathbb{R}} y d\Psi^\mu(F_X(y))$$
or the expectation of X under the concave distortion of the distribution function of X from F_X to $\Psi^\mu(F_X)$.
- It is hard to visualize a sequence of measures μ^x but relatively easy to formulate a sequence of concave distortions Ψ^x from which the measures may be extracted by
$$\mu(dy) = -y\Psi''(dy).$$

- We may define the index of acceptability as the largest level x such that expectations under concave distortions Ψ^x are positive or

$$AIW(X) = \inf \left\{ x \in \mathbb{R}_+ | \int_{\mathbb{R}} y d\Psi^x(F_X(y)) < 0 \right\}$$

where $\inf \emptyset = \infty$.

- The extreme measures are identified as

$$\frac{dQ^*}{dP} = \Psi'(F_X(x))$$

- We also have a characterization of the the sets $\mathcal{D}_x$ in terms of the conjugate dual of Ψ.

The Determining System

- Define by $\Phi_x(y)$ the conjugate dual of $\Psi_x^+(z) = \lim_{\varepsilon \downarrow 0} \Psi_{x+\varepsilon}(z)$ by
$$\Phi_x(y) = \sup_{z \in [0,1]} \left(\Psi_x^+(z) - yz \right)$$
- The determing system of densities $\mathcal{D}_x$ are given by all densities Z for which
$$E\left[(Z-y)^+ \right] \leq \Phi_x(y), \; y \in \mathbb{R}_+.$$

Characterization for Law Invariant Indices

- For a coherent acceptability index α, the following conditions are equivalent
  - α is law invariant.
  - α is monotone and second order monotone.
  - α is dilatation monotone, i.e. for any $X \in L^\infty$, and $\mathcal{G} \subseteq \mathcal{F}$,
  $$\alpha(E\left[X|\mathcal{G}\right]) \geq \alpha(X).$$
  - the determining system of α is law invariant, i.e. for any $x \in \mathbb{R}_+$, and any density $Z \overset{law}{=} Z'$ then if $Z \in \mathcal{D}_x$, it is also the case that $Z' \in \mathcal{D}_x$.
  - there exists a family $(\alpha^\gamma)_{\gamma \in \Gamma}$ of AIW indices such that
  $$\alpha(X) = \inf_{\gamma \in \Gamma} \alpha^\gamma(X)$$

Remark on Law Invariant Result

- This result relies on the characterization by Kusuoka (2001) of all law invariant monetary utilities (negative risk measures) as the infimum of a collection of weighted value at risk measures, specifically there exists a set of measures $\mathcal{M}$ on $[0,1]$ with elements μ such that
$$u(X) = \inf_{\mu \in \mathcal{M}} u^{\mu}(X).$$
- This result was an important step forward in the operationalization of acceptability cones.

The New Acceptability Cones: MINVAR

- The first family of concave distortions we considered was
$$\Psi^x(y) = 1 - (1-y)^x$$
- It is simple to observe that X is acceptable under this distortion just if the expectation of the minimum of x independent draws from the distribution of X is still just positive.
- Hence we refer to this measure as $MINVAR$ as it is based on the expectation of minima.

- The extreme measure in this case is
$$\frac{dQ^*}{dP} = (x+1)\,(1 - F_X(X))^x\,,\; x \in \mathbb{R}_+$$
- A potential drawback is that large losses have a maximum weight of $(x+1)$ in the densities of the determining system $\mathcal{D}_x$.
- Asymptotically large gains receive a weight of zero.

MAXVAR

- The next concave distortion is based on the maxima of independent draws and is defined by
$$\Psi^x(y) = y^{\frac{1}{1+x}}$$
- Here we take expectations from a distribution G such that the law of the maxima of x independent draws from this distribution matches the distribution of X.
- The extreme measure now is
$$\frac{dQ^*}{dP} = \frac{1}{1+x}\left(F_X(X)\right)^{-\frac{x}{x+1}},\ x \in \mathbb{R}_+$$
- Large losses now receive unbounded large weights in the determining system, but large gains have a minimum weight of $(x+1)^{-1}$.

MAXMINVAR and MINMAXVAR

- We combine the two distortions in two ways to define MAXMINVAR by
$$\Psi^x(y) = \left(1 - (1-y)^{x+1}\right)^{\frac{1}{x+1}}$$
- and MINMAXVAR by
$$\Psi^x(y) = 1 - \left(1 - y^{\frac{1}{x+1}}\right)^{x+1}$$
- The densities in the determining system now have weights tending to infinity for large losses and zero for large gains.

Application to Unhedged Option Writes

- We ask what are levels of acceptability attained on these measures by strategies of writing options and holding the position to maturity and paying out the requisite cash flow.
- We took data on all out of the money options on the SPX and the FTSE from December 2000 to December 2005 and bucketed these by 7 ranges for moneyness and 4 ranges for maturity.

- We thereby obtained 28 cash flow distributions for writing options bucketed by moneyness and maturity.
- We then computed the level of each of 8 performance measure for each of the 2 indices.
- We report the results in eight 7 by 4 tables for each index.

Remarks on Option Writes

- The TC is the smallest measure suggesting that exponential tilting is quite a severe risk adjustment.
- $MINMAXVAR$ and $MAXMINVAR$ are next and comparable with each other.
- This followed by $MAXVAR$ indicative of the large weighting of losses in this measure.
- The highest measure is that for $MINVAR$ among the new coherent indices.
- The values are generally below unity with the exception of $MINVAR$ for short maturity out of the money calls.

SPX Options

Strike Ranges	SR Maturity Range 0-.25	.25-.5	.5-.75	.75-1.0	TC Maturity Range 0-.25	.25-.5	.5-.75	.75-1.0
.85-.9	0.3166	0.1790	0.2136	0.4585	0.0520	0.0247	0.0345	0.1346
.9-.95	0.1970	0.1234	0.1748	0.2802	0.0266	0.0128	0.0246	0.0600
.95-1.0	0.1281	0.0741	0.0857	0.1043	0.0132	0.0050	0.0067	0.0100
1.0-1.05	0.0697	0.0203	0.0127	0.0566	0.0044	0.0004	0.0002	0.0031
1.05-1.1	0.1761	0.2732	0.2879	0.2347	0.0284	0.0683	0.0763	0.0520
1.1-1.15	0.3544	0.3423	0.4538	0.3667	0.0846	0.0939	0.1536	0.1084
1.15-1.2	0.5336	0.4849	0.4950	0.4409	0.1262	0.1393	0.1574	0.1366

	GLR				RAROCX10			
.85-.9	2.9789	0.8292	0.8997	2.0282	0.3328	0.1073	0.1244	0.2954
.9-.95	1.2354	0.4962	0.6636	0.9792	0.1437	0.0682	0.0961	0.1596
.95-1.0	0.6234	0.2528	0.2768	0.2921	0.0773	0.0380	0.0439	0.0551
1.0-1.05	0.2498	0.0578	0.0345	0.1444	0.0361	0.0100	0.0063	0.0302
1.05-1.1	0.5665	0.9487	1.0264	0.7847	0.1070	0.1861	0.2036	0.1612
1.1-1.15	2.1914	1.5448	2.1740	1.4472	0.2734	0.2344	0.3170	0.2389
1.15-1.2	6.8759	3.8486	2.7580	2.0353	0.7889	0.4555	0.3674	0.2935

	AIMIN				AIMAX			
.85-.9	0.9115	0.3461	0.4710	1.1245	0.3397	0.1667	0.2059	0.4718
.9-.95	0.4708	0.2762	0.2889	0.4581	0.2003	0.1306	0.1341	0.2304
.95-1.0	0.2508	0.0763	0.1169	0.1884	0.1169	0.0398	0.0593	0.1075
1.0-1.05	0.0644	0.0077	0.0216	0.0245	0.0348	0.0044	0.0123	0.0152
1.05-1.1	0.2461	0.3635	0.4193	0.2764	0.1823	0.2791	0.3238	0.2254
1.1-1.15	0.6849	0.4636	0.8665	0.6080	0.3699	0.3050	0.4894	0.3805
1.15-1.2	1.2659	1.1474	0.9899	0.8882	0.5485	0.5218	0.4718	0.4546

	AIMAXMIN				AIMINMAX			
.85-.9	0.2249	0.1076	0.1359	0.2994	0.2145	0.1049	0.1315	0.2780
.9-.95	0.1330	0.0856	0.0885	0.1458	0.1291	0.0838	0.0866	0.1404
.95-1.0	0.0772	0.0259	0.0388	0.0669	0.0759	0.0257	0.0384	0.0657
1.0-1.05	0.0224	0.0028	0.0078	0.0093	0.0223	0.0028	0.0078	0.0093
1.05-1.1	0.1010	0.1500	0.1727	0.1193	0.0983	0.1440	0.1649	0.1155
1.1-1.15	0.2207	0.1721	0.2823	0.2170	0.2095	0.1648	0.2629	0.2050
1.15-1.2	0.3362	0.3166	0.2871	0.2726	0.3127	0.2944	0.2678	0.2546

FTSE Options

Strike Ranges	SR Maturity Range 0-.25	.25-.5	.5-.75	.75-1.0	TC Maturity Range 0-.25	.25-.5	.5-.75	.75-1.0
.85-.9	0.2505	0.0650	-0.0475	0.0988	0.0396	0.0038	0.0000	0.0090
.9-.95	0.1103	0.0064	-0.1085	-0.0858	0.0100	0.0000	0.0000	0.0000
.95-1.0	0.0462	-0.0214	-0.1615	-0.2076	0.0020	0.0000	0.0000	0.0000
1.0-1.05	-0.0149	-0.0541	-0.2304	-0.2873	0.0000	0.0000	0.0000	0.0000
1.05-1.1	0.5915	0.4471	0.4948	0.4469	0.2599	0.1775	0.2244	0.1822
1.1-1.15	0.6761	0.6869	0.7332	0.5958	0.2326	0.3341	0.4149	0.2851
1.15-1.2	0.5219	0.7185	1.0434	0.8054	0.1167	0.2588	0.5540	0.4188

	GLR				RAROCX10			
.85-.9	1.8315	0.2645	0.0000	0.2802	0.2089	0.0356	0.0000	0.0528
.9-.95	0.5638	0.0203	0.0000	0.0000	0.0686	0.0033	0.0000	0.0000
.95-1.0	0.1802	0.0000	0.0000	0.0000	0.0245	0.0000	0.0000	0.0000
1.0-1.05	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1.05-1.1	3.2350	1.9633	2.0273	1.8022	0.4959	0.3532	0.4291	0.3623
1.1-1.15	9.1868	5.7715	4.2449	2.7973	1.0758	0.7685	0.6863	0.4782
1.15-1.2	17.4153	23.4815	16.0821	5.1221	2.0014	2.8906	2.3423	0.7614

	AIMIN				AIMAX			
.85-.9	0.4579	0.0269	0.0000	0.1385	0.2314	0.0157	0.0000	0.0849
.9-.95	0.1657	0.0000	0.0000	0.0000	0.0867	0.0000	0.0000	0.0000
.95-1.0	0.0526	0.0000	0.0000	0.0000	0.0289	0.0000	0.0000	0.0000
1.0-1.05	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1.05-1.1	1.1565	0.6869	0.8379	0.7031	0.6760	0.5104	0.6533	0.5422
1.1-1.15	1.7939	1.3843	1.5108	1.0274	0.7746	0.7789	0.9108	0.6721
1.15-1.2	2.1375	2.8520	3.2708	1.8586	0.9083	1.1088	1.2811	0.9130

	AIMAXMIN				AIMINMAX			
.85-.9	0.1447	0.0099	0.0000	0.0517	0.1401	0.0098	0.0000	0.0509
.9-.95	0.0556	0.0000	0.0000	0.0000	0.0549	0.0000	0.0000	0.0000
.95-1.0	0.0185	0.0000	0.0000	0.0000	0.0184	0.0000	0.0000	0.0000
1.0-1.05	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1.05-1.1	0.3770	0.2675	0.3313	0.2805	0.3429	0.2492	0.3028	0.2599
1.1-1.15	0.4557	0.4271	0.4886	0.3619	0.4120	0.3852	0.4314	0.3288
1.15-1.2	0.5162	0.6262	0.7293	0.5220	0.4655	0.5498	0.6187	0.4591

Acceptability Levels For Hedge Funds

- For data on hedge fund returns we had 60 monthly returns on 527 hedge funds and we centered and scaled this data to zero mean and unit variance.
- To get a better assessment of the distribution function we fitted a probability model, and in particular the centered and scaled $CGMY$ model for $Y = .5$. We therefore estimated just two parameters, G, M.
- We then simulated cash flows from these distributions and evaluated the new coherent indices on this data as well as similar data for 27 stocks and 10 as a benchmark.
- The results are reported in a table.

TABLE 2

Acceptability Levels of Funds From Standardized CGMY

Based on		quantiles .05	.5	.95
	S	0	.1177	.3243
MN	I	0	.1265	.2492
	F	.1640	.7175	2.0668
	S	0	.0883	.2779
MX	I	0	.0957	.1964
	F	.1374	.4966	1.2346
	S	0	.0494	.1426
MXMN	I	0	.0535	.1062
	F	.0738	.2679	.6591
	S	0	.0488	.1372
MNMX	I	0	.0527	.1032
	F	.0726	.2495	.5645

Remarks on Hedge Fund Levels

- Hedge Funds do access higher levels of acceptability than one may obtain from stocks or indices.
- The highest levels are again for $MINVAR$, followed by $MAXVAR$ and the composite measures.
- The measures are generally below unity indicating a fairly efficient economy.

From Required Returns to Required Sharpe Ratios

- Investment in a return distribution X may be standardized on defining Z such that

$$X = \mu + \sigma Z$$

- Let F_Z be the distribution function of the standardized variate. We show that for each coherent acceptability index, X is acceptable at level x just if

$$\begin{aligned} SR &= \frac{\mu}{\sigma} \geq c_Z(x) \\ c_Z(x) &= -\int_{-\infty}^{\infty} z d\left(\Psi^x(F_Z(z))\right). \end{aligned}$$

- Hence hedge funds may affect distributions by accessing different skews and kurtosis with a view to raising Sharpe ratios but required Sharpe ratios for acceptability may rise by even more.
- A regression of required Sharpe ratios on skewness and kurtosis indicates a preference for skewness.
- Kurtosis is confused as higher kurtosis appears to lower required Sharpe ratios.
- We split Kurtosis into tailweightedness and peakedness and find the latter is preferred while the former is shunned.

TABLE 3

Regression Coefficients of Required Sharpe Ratios on Skewness and Kurtosis

	Constant	Skewness	Kurtosis	R^2
MN	0.5536	0.00076	-0.0055	83.45
	t-stat	(0.98)	(-7.07)	
MX	0.7572	-0.0529	-0.0037	64.08
	t-stat	(-29.10)	(-2.06)	
MXMN	0.9954	-0.0566	-0.0062	76.54
	t-stat	(-35.36)	(-3.90)	
MNMX	1.1674	-0.0763	-0.0071	74.88
	t-stat	(-35.65)	(-3.30)	

TABLE 4

Regression Coefficients of Required Sharpe Ratios
Skewness, Peakedness and Tailweightedness

	Const	Skew	Peak	Tail	R^2
MN	1.0738	0.00832	-0.8662	2.0137	96.70
	t-stat	(23.21)	(-115.69)	(30.58)	
MX	0.8121	-0.0536	-0.3494	3.889	59.01
	t-stat	(-26.73)	(-8.34)	(10.56)	
MXMN	1.2864	-0.0539	-0.7447	4.8321	64.26
	t-stat	(-26.41)	(-17.45)	(12.87)	
MNMX	1.4479	-0.0741	-0.7966	5.7761	64.57
	t-stat	(-28.20)	(-14.52)	(11.97)	