Sato Processes in Finance

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January 16-27 2012, Mumbai, India

OUTLINE

1. The impossibility of Lévy processes for the surface of option prices

2. The information content of option prices

   (a) Dynamic vs Static Arbitrage

   (b) Markov Martingales

3. Sato Processes

The impossibility of calibrating Homogeneous Lévy Processes across maturities.

- The log characteristic function of homogeneous Lévy processes is linear in time to maturity.

- This property has the easily computed consequence that

  i) the t period annualized volatility of
  log returns is constant,
  ii) the absolute skewness of t period
  log returns is proportional to $t^{-0.5}$,
  iii) excess kurtosis or kurtosis-3
  is proportional to t^{-1}.

- The following figures show the quarterly average moments (annualized volatility, absolute skewness and excess kurtosis) for the risk neutral density as functions of time to expiration for S&P500 Index in 1999.

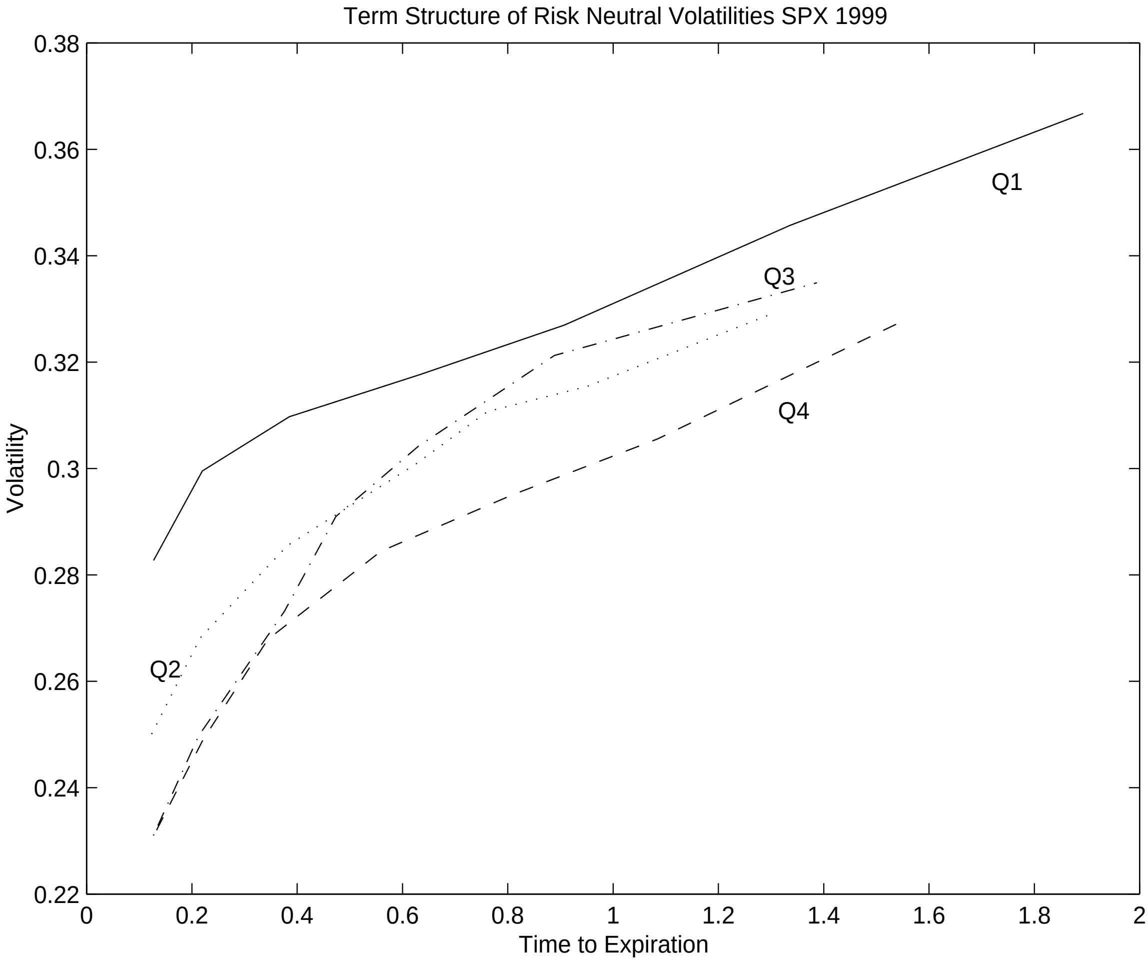

Figure 1: SPX Volatility 1999

- We can easily see from these graphs that the respective moments are increasing in time to maturity.

- These observations are inconsistent with the assumption that log returns follow a homogeneous Lévy process.

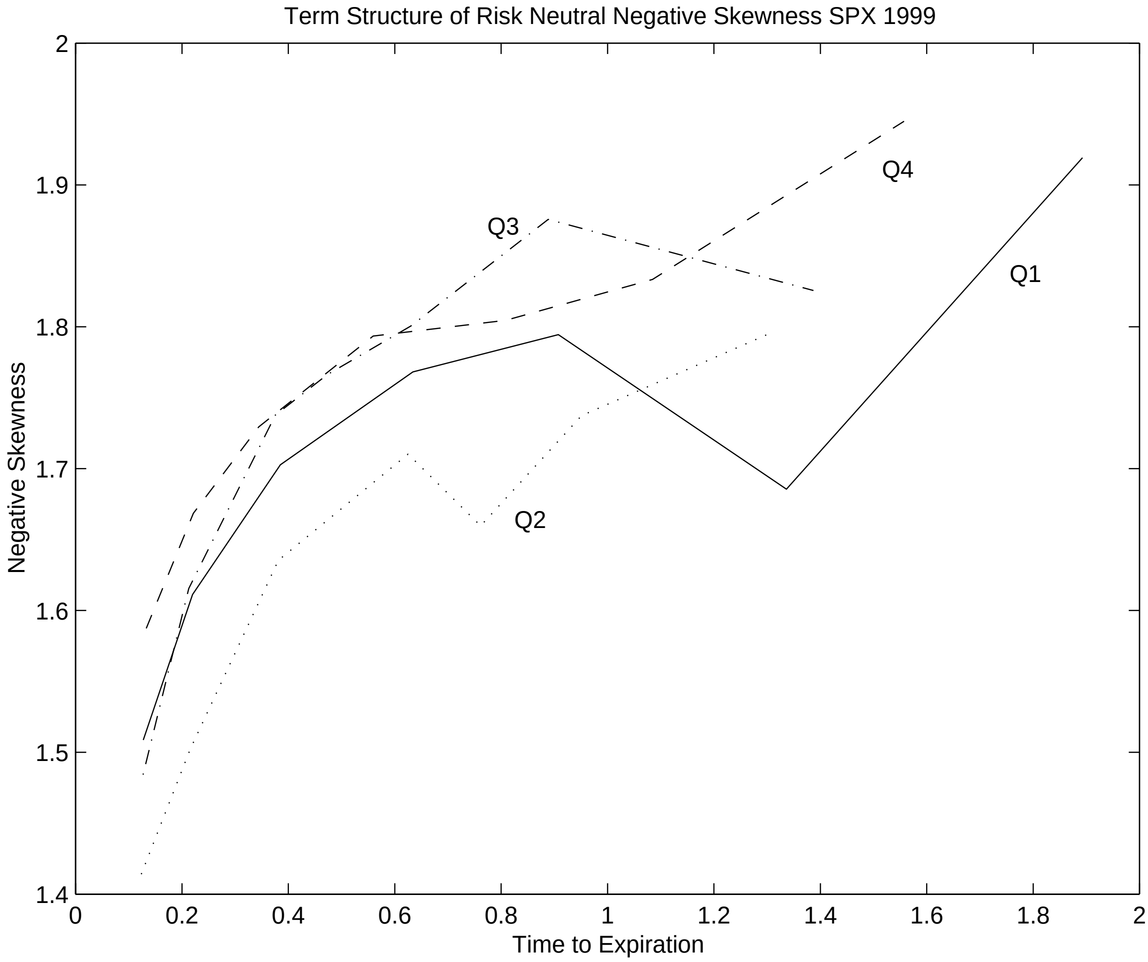

Figure 2: SPX Skewness 1999

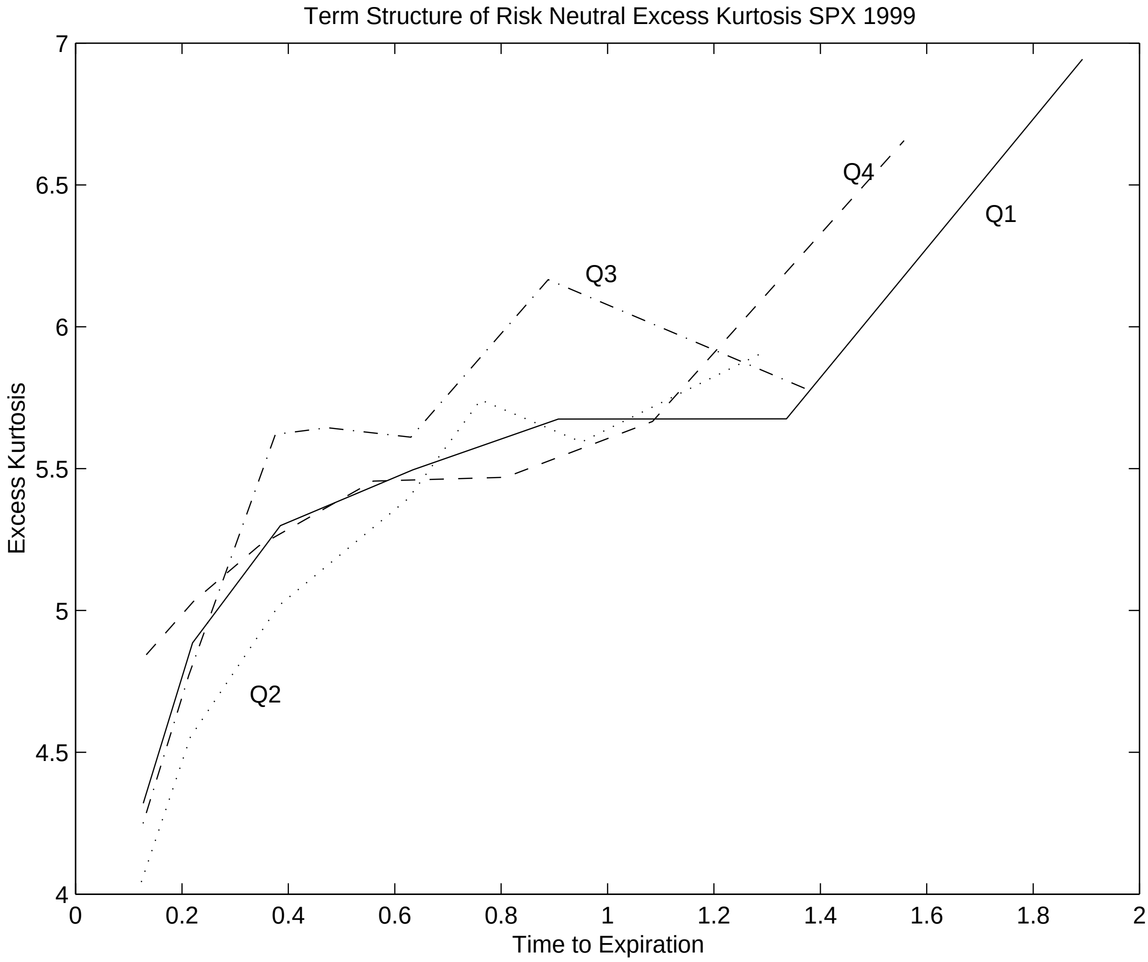

Figure 3: SPX Excess Kurtosis 1999

The Information Content of Option Prices

- Breeden and Litzenberger showed that one may recover from option prices the risk neutral density of the stock price at each maturity.

- We have

$$C(K,T) = e^{-rT} \int_K^\infty (S - K) f(S) dS$$

- and it follows that

$$f(K) = e^{rT} C_{KK}.$$

- We further asked when a screen of option prices was free of static arbitrage as opposed to when dynamic paths of asset prices were free of dynamic arbitrage.

- For the absence of dynamic arbitrage the necessary and sufficient condition is the presence of an equivalent martingale measure (EMM) where the martingale property is attained with respect to the original filtration associated with the asset price paths.

- For the absence of just static arbitrage we introduce as static securities (with zero interest rates and divs) the zero cost securities paying at time $t_2 > t_1$ the cash flow

$$\mathbf{1}_{S_{t_1}>K}\left(S_{t_2} - S_{t_1}\right).$$

Calendar Spread Inequality

- In the absence of arbitrage opportunities the non-negative cash flow,

$$(S(t_2) - K)^+ - (S(t_1) - K)^+ \\ -\mathbf{1}_{S(t_1)>K} \left(S\left(t_2\right) - S\left(t_1\right)\right)$$

  must have a non-negative initial price.

- This implies that call prices for strike K and maturities t_1, t_2 satisfy

$$C(K, t_2) > C(K, t_1).$$

- This now implies that the densities extracted by Breeden and Litzenberger are now increasing in the convex order. That is for every convex function $\phi(S)$

$$\int \phi(S) f(S, t_1) dS \leq \int \phi(S) f(S, t_2) dS.$$

- Kellerer (1972) then showed (See also Follmer and Schied Stochastic Finance (2004)) that there must exist a Markov Martingale in some filtration to be constructed with the property that the marginal densities of this martingale match those implied by the options.

- Hence the absence of Static arbitrage is equivalent to the existence of (MMM) Markov martingale marginals in some filtration.

Explicit Constructions

- For marginals that scale Madan and Yor (2002, Bernoulli) provide three methods for the construction of such Markov martingales.

- In a discrete time context Davis and Hobson (2007, Mathematical Finance) describe the equations to be solved for such a process construction.

For m_{ij} as the candidate joint density for the stock to be at level a_i at time t and level a_j at the next time $s > t$ we require that

$$\sum_j m_{ij} = q_i$$
$$\sum_i m_{ij} = q_j$$

where q_i, q_j are the relevant marginal densities for the levels a_i, a_j respectively.

In addition we require for the martingale condition that

$$\sum_j \left(a_j - a_i\right) m_{ij} = 0$$

These linear restrictions may be imposed in selecting the values m_{ij} using linear programming for any criterion linear in probabilities. There are many such operational choices.

Sample Criteria

- Minimum variance

$$\sum_i \sum_j \left(a_j - a_i\right)^2 m_{ij}$$

- Matching initial moments

$$\sum_{ik} | \sum_j \left(a_j - a_i\right)^k m_{ij} - c_k |$$

Sato Processes

- Limit Laws and stock price motion

- Summary of The Self Decomposable Laws attained at unit time.

  – The VGSSD model.

  – The NIGSSD model.

  – The MXNRSSD process.

  – The Hyperbolic Processes VC,VS,VT

- Data and Summary of Results.

- Conclusions.

The Use of Limit Laws for the unit time distribution

- A classical motivation for using the Gaussian model is that it is a limit law and over any substantial period there are many independent effects on the stock price. This is often appealed to in elementary classes presenting the Gaussian model for the first time.

- Limit laws have been studied as far back as Lévy (1937) and Khintchine (1938) and the class of such laws, once one allows for arbitrary scaling and centering coefficients, are the self decomposable laws.

- A probability law of a random variable X is said to self decomposable just if for every constant c, $0 < c < 1$ there exists an independent random variable $X^{(c)}$ such that

$$X \stackrel{law}{=} cX + X^{(c)}.$$

- From a financial perspective this an important class of random variable models for the unit time distribution as independent effects on the return may need to be scaled to be brought to comparable orders of magnitude before scaling by the square root of n becomes relevant. Such considerations motivate arbitrary scaling factors and point to self decomposable laws as candidate models.

- Self decomposable laws are infinitely divisible and may be characterized nicely in terms of the Lévy density that must have the form

$$\frac{h(x)}{|x|}\mathbf{1}_{x<0} + \frac{h(x)}{x}\mathbf{1}_{x>0}$$

where h is increasing for negative x and decreasing for positive x.

- We call the function $h(x)$ the self decomposability characteristic.

- We note in passing that many jump diffusion models in the literature employing either Laplace or Gaussian jump size distributions are not self decomposable laws in their jump component.

Processes associated with self decomposable laws at unit time

- Given a candidate risk neutral self decomposable law at unit time we consider risk neutral laws at other time points defined by the scaling property. Specifically we consider defining a process $Y(t)$ with the property

$$Y(\lambda t) \stackrel{law}{=} a(\lambda) Y(t).$$

- It is easily seen on applying the above property to $\lambda\mu$ in two ways that we must have

$$a(t) = t^{\gamma}.$$

- Sato shows that one may construct additive self similar processes that match at unit time a pre-specified unit time self decomposable law. The Lévy system for the additive process may be explicitly identified in terms of the self decomposability characteristic and is given by $g(y,t)$ where

$$g(y,t) = \begin{cases} -\frac{\gamma h'\left(\frac{y}{t^\gamma}\right)}{t^{1+\gamma}} & y > 0 \\ \frac{\gamma h'\left(\frac{y}{t^\gamma}\right)}{t^{1+\gamma}} & y < 0 \end{cases}$$

- Note on making the change of variable

$$u = \frac{y}{t^\gamma}$$

and writing

$$g(y,t)dydt = \begin{cases} -h'(u)\, du\ dlogt & y > 0 \\ \gamma h'(u)\, du\ dlogt & y < 0 \end{cases}$$

that we may expect to see the process $Y(t)$ as a scaled homogeneous process evaluated in log time.

- Jeanblanc, Pitman and Yor show that this is indeed the case and we may write for example that

$$Y(t) = Y(1) + \int_1^{\log(t)} e^{\gamma s} dU(s)$$

for a Lévy process $U(t)$ that one constructs from additive process $Y(t)$.

- We may regard $U(t)$ as the underlying uncertainty in the economy that has been time changed by the logarithm and scaled by the exponential.

- The process $U(t)$ is in fact an underlying $BDLP$ in the sense defined by Barndorff-Nielsen and Shepard. Specifically one may construct a stationary solution to the OU equation

$$dZ = -\gamma Z dt + dU$$

and relate $Y(t)$ to this stationary process as shown by Lamperti in the form

$$Y(t) = t^{\gamma} Z_{\log(t)}.$$

The Stock Price Models

- Our Stock price models are formulated in terms of our additive processes as discounted exponential martingales in the form

$$S(t) = S(0)\frac{\exp((r-q)t + Y(t))}{E\left[\exp\left(Y(t)\right)\right]}$$

  where the normalizing expectation may be explicitly obtained from the characteristic function of the additive process.

- We investigate 6 scaled selfdecomposable processes, termed SSD.

- Each of these has just four parameters and to our pleasant surprise they do a remarkable job of calibrating the vanilla options surface consistently across time and a wide range of assets.

Summary of The Self Decomposable Laws

1. $NIGSSD$

- Define by T_t^ν the time it takes Brownian motion with drift ν to reach the level t. It is well known that

$$\begin{aligned} & E\left[\exp\left(-\lambda T_t^\nu\right)\right] \\ = \; & \exp\left(-t\left((2\lambda+\nu^2)^{1/2}-\nu\right)\right) \end{aligned}$$

- Now evaluate another independent Brownian motion with drift θ and volatility σ at T_t^ν to get the NIG process

$$X_{NIG}(t;\sigma,\nu,\theta) = \theta T_t^\nu + \sigma W(T_t^\nu)$$

- The characteristic function is

$$
\begin{aligned}
& \phi_{NIG}(u;\alpha,\beta,t\delta) \\
= \; & \exp\left(-t\delta(A-B)\right) \\
A^2 = \; & \alpha^2 - (\beta - iu)^2 \\
B^2 = \; & \alpha^2 - \beta^2 \\
\beta = \; & \frac{\theta}{\sigma^2} \\
\alpha^2 = \; & \frac{\nu^2}{\sigma^2} + \frac{\theta^2}{\sigma^4} \\
\delta = \; & \sigma
\end{aligned}
$$

- The Lévy density is

$$k_{NIG}(x) = \left(\frac{2}{\pi}\right)^{1/2} \delta\alpha^2 \frac{e^{\beta x} K_1(x)}{|x|}$$

- The NIGSSD log characteristic function is

$$\psi_{NIG}(u, t; \sigma, \nu, \theta) = -\sigma \left(\left(\frac{\nu^2}{\sigma^2} + \frac{\theta^2}{\sigma^4} - \left(\frac{\theta}{\sigma^2} + iut^\gamma \right)^2 \right)^{1/2} - \frac{\nu}{\sigma^2} \right)$$

- The NIG Self Decomposability Characteristic is

$$h_{NIG}(x) = \left(\frac{2}{\pi} \right)^{\frac{1}{2}} \sigma \alpha^2 e^{\frac{\theta}{\sigma^2} x} K_1 (|x|)$$

2. $VGSSD$

- Define by G_t^ν the gamma process with mean rate unity and variance rate ν. It is well known that

$$E\left[\exp\left(-\lambda G_t^\nu\right)\right] = (1+\lambda\nu)^{-t/\nu}$$

- Now evaluate another independent Brownian motion with drift θ and volatility σ at T_t^ν to get the VG process

$$X_{VG}(t;\sigma,\nu,\theta) = \theta G_t^\nu + \sigma W(G_t^\nu)$$

- The characteristic function is

$$\begin{aligned} & \phi_{VG}(u; \sigma, \nu, \theta) \\ = & \left(1 - iu\theta\nu + \sigma^2\nu u^2/2\right)^{-t/\nu} \end{aligned}$$

- The Lévy density illustrates a classic self decomposable law

$$\begin{aligned} k_{VG}(x) &= \begin{cases} \frac{C\exp(Gx)}{|x|} & x < 0 \\ \frac{C\exp(-Mx)}{|x|} & x > 0 \end{cases} \\ C &= \frac{1}{\nu} \\ G &= \left(\left(\frac{\theta^2\nu^2}{4} + \frac{\sigma^2\nu}{2}\right)^{1/2} - \frac{\theta\nu}{2}\right)^{-1} \\ M &= \left(\left(\frac{\theta^2\nu^2}{4} + \frac{\sigma^2\nu}{2}\right)^{1/2} + \frac{\theta\nu}{2}\right)^{-1} \end{aligned}$$

- The $VGSSD$ characteristic function is

$$\phi_{VGSSD}(u,t) = \left(\frac{1}{1 - iu\theta\nu t^{\gamma} + \frac{\sigma^2\nu}{2}u^2 t^{2\gamma}}\right)^{\frac{1}{\nu}}.$$

- The VG self decomposability characteristic is

$$h_{VG}(x) = Ce^{Gx}\mathbf{1}_{x<0} + Ce^{-Mx}\mathbf{1}_{x>0}$$

3. $MXNRSSD$

- The Meixner Process introduced by Grigelionis (1999) and Schoutens (2001) has characteristic function

$$\phi_{MXNR}(u; a, b, d) = \left(\frac{\cos\left(\frac{b}{2}\right)}{\cosh\left(\frac{au-ib}{2}\right)} \right)^{2dt}$$

- The process

$$X_{MXNR}(t; a, b, d) = aX_{MXNR}(dt, 1, b, 1).$$

- The process $X_{MXNR}(t; 1, b, 1)$ is obtained from $X_{MXNR}(t; 1, 0, 1)$ by applying an Esscher transform.

- The process $X_{MXNR}(t; 1, 0, 1)$ is an independent Brownian motion $\beta(t)$ evaluated at

$$\int_0^1 (R_{4t}(s))^2 ds$$

where R_{4t} is the Bessel process of dimension $4t$.

- The density at unit time is obtained on Fourier inversion by

$$f(x;a,b,d) = \frac{2\cos\left(\frac{b}{2}\right)^{2d}}{2a\pi\Gamma(2d)} \exp\left(\frac{b}{a}x\right)\left|\Gamma(d+i\frac{x}{a})\right|^2$$

- The Lévy density is

$$k_{MXNR}(x) = d\frac{\exp\left(\frac{b}{a}x\right)}{x\sinh\left(\frac{\pi x}{a}\right)}$$

- The $MXNRSSD$ characteristic function is

$$\phi_{MXNRSSD}(u,t) = \left(\frac{\cos\left(\frac{b}{2}\right)}{\cosh\left(\frac{aut^{\gamma}-ib}{2}\right)} \right)^{2d}$$

- The $MXNR$ self decomposability characteristic is

$$h_{MXNR}(x) = d \frac{\exp\left(\frac{b}{a}x\right)}{\left|\sinh\left(\frac{\pi x}{a}\right)\right|}.$$

4. Processes associated with the hyperbolic functions.

- Two increasing processes denoted C_t, S_t defined by

$$\begin{aligned} C_t &= \inf \{s \,| |B_s| = t\} \\ S_t &= \inf \{s \,| BES(3, s) = t\} \end{aligned}$$

have Laplace transforms

$$\begin{aligned} E\left[e^{-\lambda C_t}\right] &= \left(\frac{1}{\cosh\left((2\lambda t)^{1/2}\right)}\right) \\ E\left[e^{-\lambda S_t}\right] &= \left(\frac{(2\lambda t)^{1/2}}{\sinh\left((2\lambda t)^{1/2}\right)}\right) \end{aligned}$$

- Alternative characterizations are

$$\begin{aligned} C_t &= \inf \{s \,| M_s - B_s = t\} \\ S_t &= \inf \{s \,| 2M_s - B_s = t\} \end{aligned}$$

where $M_t = \sup_{s \le t} B_s$.

- We allow for drifts and define

$$B_t^{(\nu)} = \nu t + B_t$$

and define

$$\begin{aligned} C_t^{(\nu)} &= \text{inf}\left\{ s \left| M_s^{(\nu)} - B_s^{(\nu)} = t \right. \right\} \\ S_t^{(\nu)} &= \text{inf}\left\{ s \left| 2M_s^{(\nu)} - B_s^{(\nu)} = t \right. \right\} \end{aligned}$$

- We also consider a one dimensional diffusion $Z_t^{(\nu)}$ with infinitesmal generator

$$\frac{1}{2}\frac{\partial^2}{\partial x^2} + \nu \, \text{tanh}(\nu x)\frac{\partial}{\partial x}$$

and define

$$T_t^{(\nu)} = \text{inf}\left\{ s \left| \left| Z_s^{(\nu)} \right| = t \right. \right\}$$

- We note that

$$\left(\left| Z_t^{(\nu)} \right|, t \geq 0 \right) \stackrel{(d)}{=} \left(\left| B_t^{(\nu)} \right|, t \geq 0 \right).$$

- We change measure to accomodate the drift ν and evaluate

$$E\left[e^{-\lambda C_t^{(\nu)}}\right] = \frac{\exp\left(-\nu t\right) K_\lambda}{K_\lambda \cosh\left(tK_\lambda\right) - \nu \sinh\left(tK_\lambda\right)}$$

$$E\left[e^{-\lambda S_t^{(\nu)}}\right] = \frac{\sinh(\nu t)}{\nu} \frac{K_\lambda}{\sinh(tK_\lambda)}$$

$$E\left[e^{-\lambda T_t^{(\nu)}}\right] = \frac{\cosh\left(\nu t\right)}{\cosh\left(tK_\lambda\right)}$$

$$K_\lambda = (\nu^2 + 2\lambda)^{1/2}$$

- The processes VC, VS, VT are constructed by evaluating Brownian motion with volatility σ at these times and then performing a measure change using an Esscher transform with transform parameter θ.

- We also considered evaluating Brownian motion with drift at these times but the resulting models did not perform well.

- The characteristic functions are obtained as

$$E^{(\theta)}\left[e^{iu\sigma B(H_t)}\right] = \frac{E\left[e^{iu\sigma B(H_t)+\theta\sigma B(H_t)}\right]}{E\left[e^{\theta\sigma B(H_t)}\right]} = \frac{E\left[e^{i(u-i\theta)\sigma B(H_t)}\right]}{E\left[e^{i(-i\theta)\sigma B(H_t)}\right]}$$

where $H_t \in \left\{C_t^{(\nu)}, S_t^{(\nu)}, T_t^{(\nu)}\right\}$.

- The characteristic functions prior to the Esscher measure change are

$$E\left[e^{iu\sigma B(C_t^{(\nu)})}\right] = \frac{\exp\left(-\nu t\right) L_u}{L_u \cosh\left(tL_u\right) - \nu \sinh\left(tL_u\right)}$$

$$E\left[e^{iu\sigma B(S_t^{(\nu)})}\right] = \frac{\sinh(\nu t)}{\nu}\frac{L_u}{\sinh\left(tL_u\right)}$$

$$E\left[e^{iu\sigma B(T_t^{(\nu)})}\right] = \frac{\cosh\left(\nu t\right)}{\cosh\left(tL_u\right)}$$

$$L_u = (\nu^2 + \sigma^2 u^2)^{1/2}$$

Data and Results

- We obtained data on option prices for 21 names for 14 mid week days. In all we had 11988 option prices for out-of-the-money options.

- We present first the average percentage errors by model across the entire set.

TABLE 1

Average Percentage Errors Across Names and Days

Model	Proportion Below .05	Mean	Std. Dev.
VG	0.9320	0.02488	0.008937
NIG	0.9456	0.02492	0.009001
MXNR	0.9456	0.02487	0.008966
VC	0.9149	0.02539	0.008741
VS	0.9014	0.02600	0.008728
VT	0.9150	0.02548	0.008853

- For all days and names we ranked the models on the basis of APE and the average ranks are as follows.

TABLE 2

Average Rank of Model

VG	NIG	MXNR	VC	VS	VT
1.97	3.25	3.68	4.21	4.29	3.58

- Graphs of the densities of pricing errors displays the model comparabilities.

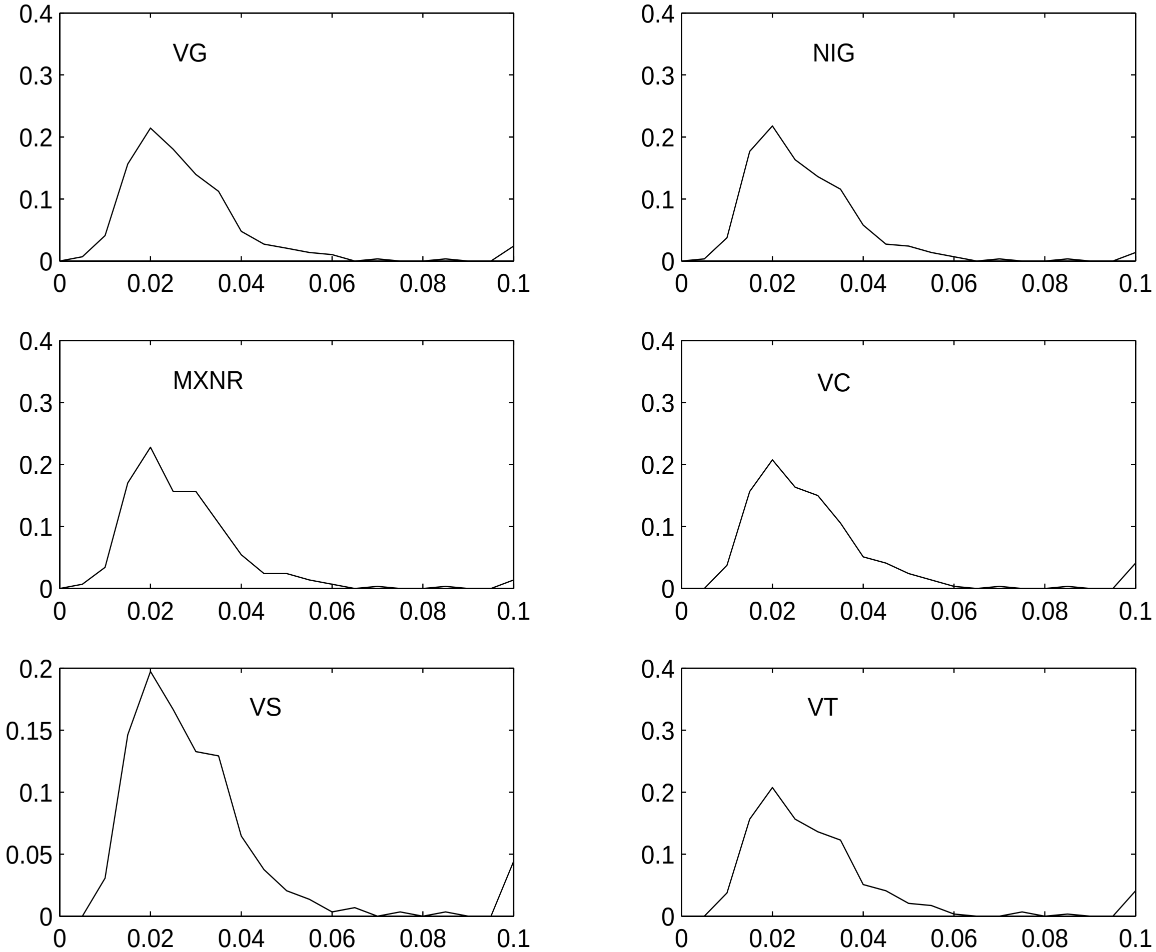

Figure 4: Densities or average percentage errors

- Model Rankings across names and days separately are as follows.

TABLE 5
Average Model Rankings

	Across Names	Across Days
VG	2.0	2.36
NIG	2.62	2.79
MXNR	2.57	1.43
VC	4.76	4.86
VS	4.24	4.64
VT	4.81	4.93

- For each model we present sample parameter values on each name averaged across the 14 estimation days.

- We note the relative stability of the VG and $MXNR$ parameters as judged by the standard deviation estimates.

TABLE 6

Average Parameter Values and (Std. Dev.) for VG

Name	σ	ν	θ	γ
amzn	0.7721	0.7077	-1.1354	0.4465
	(0.1641)	(0.1963)	(0.4983)	(0.0202)
bkx	0.2892	0.6003	-0.1463	0.4713
	(0.0079)	(0.0443)	(0.0022)	(0.0193)
csco	0.5479	0.4244	-0.5871	0.4197
	(0.0366)	(0.0392)	(0.0733)	(0.0153)
ibm	0.3661	0.5064	-0.2897	0.4178
	(0.0123)	(0.0492)	(0.0085)	(0.0166)
intc	0.4774	0.2986	-0.5181	0.4172
	(0.0182)	(0.0268)	(0.0594)	(0.0138)
msft	0.4107	0.6185	-0.2606	0.4339
	(0.0152)	(0.0545)	(0.0089)	(0.0172)
spx	0.1783	0.5848	-0.1914	0.4677
	(0.0033)	(0.0347)	(0.0049)	(0.0201)

TABLE 7

Average Parameter Values and (Std. Dev.) for NIG

Name	σ	ν	θ	γ
amzn	0.8847	2.1919	-4.5988	0.4538
	(0.0194)	(2.4009)	(21.596)	(0.0183)
bkx	0.3463	1.4036	-0.2597	0.4725
	(0.0121)	(0.2763)	(0.0142)	(0.0194)
csco	0.8748	3.3396	-3.4704	0.4214
	(0.1930)	(5.2986)	(12.983)	(0.0154)
ibm	0.5146	2.2696	-1.0283	0.4198
	(0.0377)	(3.6108)	(1.8754)	(0.0166)
intc	0.9912	4.8189	-4.2061	0.4181
	(0.1797)	(12.8605)	(29.669)	(0.0139)
msft	0.4840	1.3941	-0.4511	0.4390
	(0.0202)	(0.3605)	(0.0485)	(0.0163)
spx	0.2299	1.7070	-4.2667	0.4700
	(0.0047)	(0.309)	(0.0239)	(0.0202)

TABLE 8

Average Parameter Values and (Std. Dev.) for MXNR

Name	a	b	d	γ
amzn	1.4021	-1.8664	0.0643	0.4506
	(1.0201)	(0.5696)	(0.1359)	(0.0184)
bkx	0.5341	-0.9343	0.5041	0.4717
	(0.0333)	(0.0761)	(0.0349)	(0.0194)
csco	0.7403	-1.4104	0.9559	0.4205
	(0.0995)	(0.2629)	(0.3136)	(0.0154)
ibm	0.5851	-1.2506	0.7238	0.4189
	(0.0611)	(0.1444)	(0.2101)	(0.0166)
intc	0.5567	-1.1976	1.3588	0.4177
	(0.0372)	(0.0139)	(0.6237)	(0.0138)
msft	0.7497	-1.1732	0.5079	0.4379
	(0.0615)	(0.1647)	(0.0423)	(0.0162)
spx	0.2859	-1.5852	0.5473	0.4689
	(0.0081)	(0.2143)	(0.0286)	(0.0201)

TABLE 9

Average Parameter Values and (Std. Dev.) for VC

Name	σ	ν	θ	γ
amzn	1.0742	-7.3852	-6.2111	0.4381
	(0.1531)	(7.0528)	(4.7875)	(0.0187)
bkx	0.2820	-11.258	-1.6672	0.4718
	(0.0121)	(0.6763)	(0.4167)	(0.0194)
csco	0.9323	-3.6493	-5.0193	0.4136
	(0.1351)	(40.581)	(4.3293)	(0.0157)
ibm	0.4645	-1.4936	-2.3992	0.4180
	(0.0421)	(3.177)	(1.2864)	(0.0179)
intc	1.1205	-7.6666	-3.8770	0.4171
	(0.2467)	(18.874)	(2.2711)	(0.0138)
msft	0.4234	-0.7815	-2.0143	0.4397
	(0.0277)	(2.8996)	(2.3860)	(0.0173)
spx	0.2016	-1.1092	-6.0776	0.4664
	(0.0041)	(0.4466)	(4.6708)	(0.0218)

TABLE 10

Average Parameter Values and (Std. Dev.) for VS

Name	σ	ν	θ	γ
amzn	0.6762	-.0033	-4.7136	0.4295
	(0.1274)	(0.0010)	(8.0115)	(0.0259)
bkx	0.4362	0.0039	-2.6679	0.4698
	(0.0187)	(.0004)	(0.8780)	(0.0192)
csco	0.7537	-0.0077	-4.5348	0.4202
	(0.0891)	(0.00005)	(13.115)	(0.0164)
ibm	0.4878	-0.6834	-7.3176	0.4193
	(0.0415)	(7.9552)	(26.632)	(0.0166)
intc	0.7587	-0.0556	-2.4846	0.4205
	(0.0581)	(0.0343)	(7.1605)	(0.0139)
msft	0.5402	0.0038	-5.0184	0.4362
	(0.0329)	(0.0009)	(23.191)	(0.0161)
spx	0.2454	-0.0063	-6.9053	0.4687
	(0.0054)	(0.00002)	(4.4521)	(0.0202)

TABLE 11

Average Parameter Values and (Std. Dev.) for VT

Name	σ	ν	θ	γ
amzn	0.9807	-0.6425	-5.6473	0.4483
	(0.1113)	(9.3187)	(3.3910)	(0.0226)
bkx	0.3037	-0.6830	-1.7716	0.4716
	(0.0102)	(0.4989)	(0.3735)	(0.0193)
csco	1.0567	-7.1514	-4.7568	0.4194
	(0.2363)	(14.482)	(2.7087)	(0.0152)
ibm	0.4766	-2.0124	-2.4433	0.4185
	(0.0348)	(3.3663)	(1.2919)	(0.0167)
intc	1.0955	-7.7125	-3.7720	0.4171
	(0.2349)	(17.249)	(2.1290)	(0.0138)
msft	0.4234	-0.8295	-1.5959	0.4396
	(0.0185)	(0.7021)	(0.3757)	(0.0173)
spx	0.2034	-1.6843	-6.1219	0.4690
	(0.0038)	(0.5312)	(4.2939)	(0.0202)