ICTS Summer School on Numerical Relativity: Course work

10-21 June 2013
ICTS-TIFR Building, IISc Campus, Bangalore, India.

C1: Numerical relativity: Mathematical formulation (Thomas Baumgarte)

<u>Day 1: A brief Review of GR</u>

- Tensors
  - expansion into bases
  - covariant derivative
  - Christoffel symbols
  - Riemann tensor
- Einstein's Field Equations
- Some Important Solutions
  - weak fields: gravitational radiation
  - Schwarzschild (in different coordinate systems)
  - Tolman-Oppenheimer-Volkoff

<u>Day 2: The 3+1 Decomposition</u>

- Foliations of spacetime
  - normal vector
  - spatial metric
  - projections of tensors
  - 3D covariant derivative
- Extrinsic Curvature
- Interlude: The Lie Derivative
- Projections of the Riemann tensor

<u>Day 3: The 3+1 Decomposition continued</u>

- Gauss, Codazzi and Ricci Equations
- The ADM equations
- Examples: decompositions of Schwarzschild
- The role of the lapse and shift
- Comparison with Maxwell's equations

<u>Day 4: Solving the constraint equations</u>

- Freely specifiable versus constrained variables
- Conformal transformations
- Solving the Hamiltonian constraint
  - time symmetry
  - examples
- Solving the Momentum constraint
  - the CTT decomposition
  - examples
- The puncture method

<u>Day 5: Solving the evolution equations</u>

Comparison with Maxwell's equations
The "generalized harmonic" approach
Reformulations of the ADM equations
- Choosing the lapse and shift: slicing and gauge conditions
- The moving-puncture method

<u>Suggested references</u>

- Thomas Moore, *A General Relativity Workbook*, University Science Books, or any other textbook on general relativity.
- Thomas Baumgarte & Stuart Shapiro, *Numerical Relativity: Solving Einstein's Equations on the Computer*, Cambridge University Press
- Miguel Alcubierre, *Introduction to 3+1 Numerical Relativity,* Oxford University Press
- Eric Gourgoulhon, *3+1 Formalism in General Relativity*, Springer
- Carles Bona, Carlos Palenzuela-Luque & Carles Bona-Casas, *Elements of Numerical Relativity and Relativistic Hydrodynamics*, Springer

C2: Introduction to theory and numerics of partial differential equations (Sascha Husa)

- *Lectures (90 mins x 5):*
- *Tutorials: (90 mins x 5):*

C3: Introduction to numerical hydrodynamics (David Neilsen)

This is a rough outline of these lectures, but it will certainly change as I spend more time putting things together. I am not sure what the schedule will be, but I am thinking of one lecture on fluids and one lecture on methods per day. The tutorials will be used for students to work on some computational examples.

<u>Lecture 1: Relativistic Fluids 1</u>

1. The fluid equations in special relativity
    a. Ideal fluid
    b. Stress-energy tensor
    c. Equations of motion & conservation laws
2. Thermodynamics and equations of state
    a. ideal gas equation of state
    b. degenerate Fermi gas
    c. nuclear equations of state
3. Conserved and primitive variables
    a. Balance laws
    b. Solving for primitive variables

<u>Lecture 2: Numerical Methods 1</u>

This is an introduction to simple numerical techniques that will be explored in the tutorial session.

1. Nonlinear equations
    a. characteristics
    b. weak solutions
    c. numerical issues
2. Conservative schemes
    a. Example of Burger's equation
3. Solving ODEs
    a. Solving the TOV equations

   b. The method of lines

Lecture 3: Relativistic Fluids 2

1. Riemann problem for relativistic fluids
2. Fluid equations in general relativity (Valencia formulation)
3. Ideal MHD and astrophysical applications

Lecture 4: Numerical Methods 2

1. Linear systems of conservation laws
2. High-resolution shock-capturing methods
3. Shock-tube tests

Suggested References

Relativistic Fluids

1. Jose Maria Marti and Ewald Müller, Numerical Hydrodynamics in Special Relativity, *Living Rev. Relativity,* **6**, (2003)
2. Jose A. Font, Numerical Hydrodynamics and Magnetohydrodynamics in General Relativity, *Living Rev. Relativity,* **11** (2008), 7
3. Nils Andersson and Gregory L. Comer, Relativistic Fluid Dynamics: Physics for Many Different Scales, *Living Rev. Relativity,* **10** (2007), 1
4. Eric Gourgoulhon, Introduction to Relativistic Hydrodynamics, arXiv:gr-qc/0603009 (2006).

Numerical Methods

1. Randy J. LeVeque, *Nonlinear Conservation Laws and Finite Volume Methods for Astrophysical Fluid Flow,* 1998.

Python Tutorials

1. How to Think Like a Computer Scientist (A general python tutorial.)
2. Matplotlib Tutorial (A simple introduction to matplotlib, a plotting package for python.)
3. matplotlib.org (The homepage for matplotlib. Includes documentation and examples.)
4. Python tutorial (The python tutorial from python.org)

C4: Advanced course in theory and numerics of partial differential equations (Mark Hannam)

- *Lectures (90 mins x 4):*
- *Tutorials: (90 mins x 4):*

C5: Overview of current research in numerical relativity (Harald Pfeiffer)

- *Lecture (90 mins x 1):*

C6: Overview of current research in GW astronomy (B. S. Sathyaprakash)

- *Lecture (90 mins x 1):*