

Elliptic equations: Exercises 1

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1. Solve the linear one-dimensional elliptic equation

$$u'' = -e^x(-2 + 2x + 5x^2 + x^3),$$

on the domain $x \in [0, 1]$ with $u(0) = u(1) = 0$. Use a cell-centered grid and the tridiagonal method. Verify second-order convergence.

2. Solve the same problem on a staggered grid, with the Neumann boundary condition $u'(0) = 1$. (And the same condition at $x = 1$, $u(1) = 0$.) Is there a significant difference in accuracy between the two methods, for the same numbers of grid points?
3. (Probably Tuesday.) Solve the radial elliptic equation

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \phi}{\partial r} \right) = -\frac{r^2}{1 + r^6},$$

with the boundary conditions $\phi'(0) = 0$ and $\phi(R) = 0$, where R is the outer boundary of the domain. Verify second-order convergence. How does the solution change with the location of R ? How large does the solution change with respect to R , compared to changes with respect to spatial resolution?