The Stock-Flow-Consistent Framework - 2

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Module #4

A simple SFC model

A very very simple model (model SIM)

1. Only household, non-financial firms and the government
2. Only government money as a financial asset
3. No investment
4. All profits (if any) are distributed

The corresponding SAM

	Prod.	Hous.	Non-fin.	Gov.	Cap. Acc.	Total
Production		C		G		Y
Households	W		Π			Yh
Non-fin.firms	Π					Π
Government		T				Yg
Capital account		S		Sg		0
Total	Y	Yh	Π	T	0	

…and the Flow of Fund Matrix

	Hous.	Gov.	Total
Money	$+\Delta H$	$-\Delta H$	0
Total	S	Sg	0

Accounting identities from the SAM

First row

$$Y = C + G$$

First column

$$Y = W + \Pi$$

Second row

$$Yh = W + \Pi$$

Second column

$$Yh = C + T + S$$

Third row and column

$$Yg = T = G + Sg$$

Fourth row and column (implied by the previous identities)

$$S + Sg = 0$$

Accounting identities from the FoF and the balance sheet

$$\Delta H = S$$

$$-\Delta H = Sg \text{ (redundant)}$$

$$H(t) = H(t-1) + \Delta H$$

Accounting model

Starting	Normalized
1. $Y = C + G$	1. $Y = C + G$
2. $Y = W + \Pi$	2. $\Pi = Y - W$
3. $Yh = W + \Pi$	3. $Yh = W + \Pi$
4. $Yh = C + T + S$	4. $S = Yh - C - T$
5. $Yg = T = G + Sg$	5. $Sg = T - G$
6. $[S + Sg = 0]$	6. $H(t) = H(t-1) + S$
7. $\Delta H = S$	**Simplified**
8. $[-\Delta H = Sg]$	1. $Y = C + G$
9. $H(t) = H(t-1) + \Delta H$	2. $S = Y - C - T$
	3. $Sg = T - G$
	4. $H(t) = H(t-1) + S$

Accounting model

Simplified model	Variables
1. Y = C + G 2. S = Y - C - T 3. Sg = T - G 4. H(t) = H(t-1) + S	***Endogenous*** • H; S; Sg; Y ***Pre-determined*** Exogenous • C; G; T Lagged endogenous • H(t-1)

Decision on what to endogenize (and how!)

Model closure

T = t0 + t1*Y

C(t) = c0 + c1*[Y(t)-T(t)] + c2*H(t-1)

Two additional variables:

t0; c0

Three parameters

t1; c1; c2

Full model

1. $Y(t) = C(t) + G(t)$
2. $S(t) = Y(t) - C(t) - T(t)$
3. $Sg(t) = T(t) - G(t)$
4. $H(t) = H(t-1) + S(t)$
5. $T(t) = t0 + t1*Y(t)$
6. $C(t) = c0 + c1*[Y(t)-T(t)] + c2*H(t-1)$

Must imply

$Sg(t) = -S(t)$

A recursive model

1. C(t) = c0 + c1*[Y(t-1)-T(t-1)] + c2*H(t-1)
2. Y(t) = C(t) + G(t)
3. T(t) = t0 + t1*Y(t)
4. S(t) = Y(t) – C(t) – T(t)
5. Sg(t) = T(t) – G(t)
6. H(t) = H(t-1) + S(t)

In each period, consumption is obtained by pre-determined variables only. Other variables can be obtained without any feedback

Rearranging the equations

1. C(t) = c0 + c1*[Y(t-1)-T(-1)] + c2*H(t-1)
2. Y(t) - C(t) = G(t)
3. T(t) - t1*Y(t) = t0
4. S(t) - Y(t) + C(t) + T(t) = 0
5. Sg(t) - T(t) = – G(t)
6. H(t) – S(t) = H(t-1)

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -t1 & 1 & 0 & 0 & 0 \\ 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} C \\ Y \\ T \\ S \\ Sg \\ H \end{bmatrix} = \begin{bmatrix} c1 & -c1 & c2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} Y_{-1} \\ T_{-1} \\ H_{-1} \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} c0 \\ t0 \\ G \end{bmatrix}$$

Back to the full model

1. $Y(t) = C(t) + G(t)$
2. $S(t) = Y(t) - C(t) - T(t)$
3. $Sg(t) = T(t) - G(t)$
4. $H(t) = H(t-1) + S(t)$
5. $T(t) = t0 + t1*Y(t)$
6. $C(t) = c0 + c1*[Y(t)-T(t)] + c2*H(t-1)$

Changes in the values of exogenous variables imply changes in Consumption, and therefore income Y, and therefore taxes, and consumption…

Solving by substitution

Y(t) = c0 + c1*(1-t1)*Y(t) -c1*t0 + c2*H(t-1) + G(t)

so that

$$Y_t = \frac{1}{1 - c_1 \cdot (1 - t_1)} (c_0 - c_1 \cdot t_0 + c_2 \cdot H_{t-1} + G_t)$$

which is a standard Keynesian multiplier result

Let's now adopt the simplifying assumptions of G&L, i.e. that $c_0 = t_0 = 0$

So that

$$Y_t = \frac{G_t + c_2 \cdot H_{t-1}}{1 - c_1 \cdot (1 - t_1)}$$

Solving by substitution #2

Notice that

S(t) = G(t) – T(t)

so that, after manipulation

$$S_t = G_t - t_1 \cdot \left[\frac{G_t + c_2 \cdot H_{t-1}}{1 - c_1 \cdot (1 - t_1)} \right]$$

$$S_t = \frac{[1 - c_1(1 - t_1) - t_1] \cdot G_t}{1 - c_1 \cdot (1 - t_1)} - \frac{c_2 \cdot t_1 \cdot H_{t-1}}{1 - c_1 \cdot (1 - t_1)}$$

and using the equation for the change in the stock of money

$$H_t = H_{t-1} + S_t$$

we get

Solving by substitution #3

$$H_t = \frac{[1 - c_1(1 - t_1) - t_1] \cdot G_t}{1 - c_1 \cdot (1 - t_1)} +$$

$$\frac{1 - c_1 \cdot (1 - t_1) - c_2 \cdot t_1}{1 - c_1 \cdot (1 - t_1)} \cdot H_{t-1}$$

So the dynamics of the stock of wealth depends on

$$1 - \frac{c_2 \cdot t_1}{1 - c_1 \cdot (1 - t_1)}$$

Stability

$$1 - \frac{c_2 \cdot t_1}{1 - c_1 \cdot (1 - t_1)} < 1$$

if

$$\frac{c_2 \cdot t_1}{1 - c_1 \cdot (1 - t_1)} > 0$$

and

$$1 - \frac{c_2 \cdot t_1}{1 - c_1 \cdot (1 - t_1)} > 0$$

if

$$1 - c_1 \cdot (1 - t_1) - c_2 \cdot t_1 > 0$$
$$c_1 \cdot (1 - t_1) + c_2 \cdot t_1 < 1$$

Steady state solutions

In steady state, by definition, stocks are stable, so that

$$H_t = H_{t-1} \rightarrow S_t = 0$$

$$\frac{[1 - c_1(1 - t_1) - t_1] \cdot G_l}{1 - c_1 \cdot (1 - t_1)} - \frac{c_2 \cdot t_1 \cdot H_l}{1 - c_1 \cdot (1 - t_1)} = 0$$

$$H_l = \frac{(1 - c_1)(1 - t_1)}{c_2 \cdot t_1} \cdot G_l$$

and

$$Y_l = \frac{G_l + c_2 \cdot H_l}{1 - c_1 \cdot (1 - t_1)}$$

$$Y_l = \frac{1}{1 - c_1 \cdot (1 - t_1)} \cdot \left[G_l + c_2 \cdot \frac{(1 - c_1)(1 - t_1)}{c_2 \cdot t_1} \cdot G_l \right]$$

Steady state solutions #2

$$Y_l = \frac{1}{1 - c_1 \cdot (1 - t_1)} \cdot \left[\frac{t_1 + 1 - t_1 - c_1 + c_1 t_1}{t_1}\right] \cdot G_l$$

$$Y_l = \frac{1}{t_1} \cdot G_l$$

Implied stock-flow norm

$$\frac{H_l}{Y_l} = \frac{(1 - c_1)(1 - t_1)}{c_2 \cdot t_1} \cdot G_l \Bigg/ \frac{1}{t_1} \cdot G_l = \frac{(1 - c_1)(1 - t_1)}{c_2}$$

An example in Eviews

You can download model SIM in Eviews from

http://gennaro.zezza.it/software/eviews/glch03.php

An Excel version is available at

http://models.sfc-models.net/gl2007/ModelSIM.xls

At

http://models.sfc-models.net/

You also find versions for R and Matlab