School and Workshop on Cocompact Imbeddings, Profile Decompositions, and their Applications to PDE, TIFR-CAM, January 03-12, 2012

Conference abstracts

Hajer Bahouri
Université Paris-Est Creteil

On the lack of compactness of some critical Sobolev embedding

The aim of this talk is to present two recent results on the characterization of the lack of compactness of some critical Sobolev embedding. This topic is very useful in Calculus of Variations for instance, but also in the studyof various types of Partial Differential Equations (semilinear wave and Schrdinger
equations, exponential type nonlinearities, Navier-Stokes equations). In this talk we shall concentrate on two types of critical embeddings. The first one concerns an abstract framework which includes Sobolev, Besov, Triebel-Lizorkin, Lorentz, Hölder and BMO spaces. The second one concerns the lack of
compactness of $H^1(R^2)$ into the Orlicz space. The two results are expressed in the same manner and relate the lack of compactness to a concentration phenomenon via an asymptotic decomposition, but the elements involved in the decomposition are of completely different types.

Claudio Bonanno,
University of Pisa

Solitons and vortices for nonlinear field equations

In this talk I will focus on the existence of soliton solutions, that is orbitally stable solitary waves, for nonlinear field equations. The existence of solitons is obtained by finding positive solutions of an
associated nonlinear elliptic equation which minimizes the energy functional. I will consider also the case of solutions with non-vanishing angular momentum and dwell upon the problem of their stability.

Mónica Clapp

Universidad Nacional Autónoma de México

Multiple solutions to the Bahri-Coron problem in domains with nontrivial topology

We consider the Bahri-Coron problem

$-\Delta u = |u|^{2^*-2}u$ in Ω, $u = 0$ on $\partial\Omega$,

where Ω is a bounded smooth domain in R^N, $N \geq 3$, and $2^* := \frac{2N}{N-2}$ is the critical Sobolev exponent.
Concerning existence, the most remarkable result was obtained by Bahri and Coron in 1988. They showed that, if Ω has nontrivial homology, this problem has a positive solution.
Multiplicity of solutions is still a largely open question. Recently, Pacella and I considered annular domains, with a hole of arbitrary size, and proved existence of multiple solutions in the presence of enough - but still nite - symmetries.
In this talk we present a multiplicity result for other types of domains having nontrivial topology. This is joint work with J. Faya.

Juan Dávila
Universidad de Chile

Solutions with point singularities for a MEMS equation with fringing field

We construct solutions of the equation

$-\Delta u = \lambda(1 + |\nabla u|^2)(1-u)^{-2}$, $0 < u < 1$

in a bounded smooth domain of R^2 with Dirichlet boundary condition, for $\lambda > 0$ small. These solutions approach 1 as $\lambda \to 0$ at one point, and if Ω is not simply connected we find solutions forming singularities at many points. The equation arises in the modeling of a MEMS with fringing field. A surprising connection with plasma problem is found. This is joint work with Juncheng Wei, Chinese University of Hong Kong.

Manuel del Pino
Universidad de Chile

Minimal surfaces and the Allen-Cahn equation

We review some results on construction of solutions to the Allen Cahn equation with its nodal set close to a given, largely dilated minimal surface. In particular we present a counterexample to De Giorgi's conjecture in dimensions 9 or higher.

Olivier Druet,
ENS Lyon

Stability issues for systems of elliptic PDEs

We will study the question of stability of strongly coupled systems of elliptic PDEs with critical Sobolev growth. Some stability as well as instability results will be presented, depending on the coupling potential in the system.

Pierpaolo Esposito
University of Rome 3

Singular mean-field equations on compact Riemann surfaces

In a joint work with Pablo Figueroa, we study the existence of non-compact sequence of solutions for mean-field problems on a compact Riemann surface. The presence in the equation of singular sources in the form of Dirac delta measures makes the problem more degenerate and more difficult to deal with.

Marco Ghimenti
University of Pisa

Existence, stability, concentration properties and dynamics of solitons in Nonlinear Schroedinger Equation in presence of a strong nonlinearity

I present some result about the dynamics of solitons (stable solitary waves) for the Nonlinear Schroedinger equation in presence of an external potential.
We'll focus on the role on the nonlinearity in formation, stability and concentration properties of the soliton.
I will discuss both the case of confining potential and bounded potential. The second result is obtained by a suitable notion of barycenter that seem to be useful to study the dynamics of solitons in more general
frameworks.

Ignacio Guerra
Universidad de Santiago de Chile

Ground states for a semilinear elliptic equation with mixed Sobolev growth.

We consider the problem

$\Delta u = u^p + u^q,\ u > 0$ in R^N, $u(x) \to 0$ as $|x| \to \infty$,
where $N \geq 3$ and $N/(N-2) < p < /N+2)/(N-2) < q$
We prove existence of radial slow decay solutions for p near $N/(N-2)$and q close to the critical exponent $(N+2/(N-2)$, when certain relations between p and q are satisfied. We also find existence of singular radial fast decay solutions for q large and p close to $(N+2)/(N-2)$, when p and q are linked by certain relations. Finally, we discuss on parameter regions where we have existence of bounded slow decay solutions and singular fast decay solutions.

This work is in collaboration with Juan Dávila.

Emmanuel Hebey
Univ. Cergy-Pontoise

Static KGMP equations in closed manifolds

We investigate existence of a solution and existence of a priori bounds for the Klein-Gordon-Maxwell-Proca system. In doing so we discuss phase compensation and prove the existence of resonant states when a priori bounds do not hold true.

Stéphane Jaffard
Univ. Paris XII

Oscillation spaces and beyond: New function spaces defined by wavelet conditions.

We will show how recent interactions between harmonic and functional analysis led to new collections of function spaces which, up to now, have been used for classification purposes, but, hopefully, might find their way through PDEs.
The seminal concepts of multifractal analysis were reformulated in a wavelet setting, thus leading to new collections of function spaces, related with Besov spaces. We will show that, on a more abstract level, they allow to associate new scales of shift invariant function spaces to a given pointwise regularity condition and vice-versa. Examples of explicit such couples are supplied by the weak-scaling exponent of Y. Meyer, which leads to Besov spaces, the usual pointwise Hölder exponent, which leads to oscillation spaces, and the ${T_u}^p$ regularity condition of Calderon and Zygmund, which leads to Seeger spaces.
We will finally show in which sense the concepts of multifractal analysis also yield extensions of the spaces to negative Lebesgue exponents p.

Rowan Killip
UCLA

The cubic Klein-Gordon equation in two dimensions.

We discuss the proof of global well-posedness and scattering for the defocusing cubic nonlinear Klein-Gordon equation in two space dimensions. We will also describe results in the focusing case, which take the form of a dichotomy between scattering and finite-time blow up. This is joint work with Besty Stovall and Monica Visan.

Chang-Shou Lin
National Taiwan University

Bubbling solutions for Abelian Chern-Simons-Higgs model on a torus - known results and open problems.

In this lecture I will talk about the bubbling solutions of the Abelian Chern-Simons -Higgs equation as the coupling constant is tending to zero. Our goal is to find all possible bubbling solutions. To do so, we are aware that there are two different PDE to govern the limiting equations. One is the Chern-Simos-Higgs equation in R^2, the other is the mean field equation on the torus.So, we not only want to find bubbling solutions but also want to specify the limiting equations. In spite of intensive research for the past 10 years, there are still many important questions that remains answered. One of important questions is to locate the blow-up points, where a set of algebraic equations associated Green functions are derived. However, we conjecture that the location is related to the classical Lame solution of Lame equation, in other words, it is exactly the branched points of the hyper-elliptic curve associated the torus.

Gianni Mancini
University of Rome III

Sharp exponential inequalities and extremal functions

Part 1. Exponential inequalities in compact manifolds: a review:
- Moser-Onofri inequality, extremals and applications
- Moser-Trudinger inequalities on compact manifolds, extremals

Part 2. Exponential inequalities in manifolds of infinite volume:
-Scale invariant inequalities and imbeddings in Orlicz spaces
-Sharp Moser-Trudinger inequality in unbounded open subsets of R^2
and on conformal discs of infinite volume
-extremals for sharp M-T on some manifold of infinite volume

Roberta Musina
Universitá di Udine

Weighted biharmonic operators, Rellich potentials and lack of compactness

Motivated by the celebrated Caffarelli-Kohn-Nirenberg first-order inequalities, much attention has been paid in recent years to noncompact elliptic problems involving weights that are powers of the distance from the origin. In the recent papers [1], [2] with Paolo Caldiroli (Torino, Italy), we started to investigate the corresponding second-order inequalities for the weighted L^2- norm of Δu. An example of such inequalities is

$$(1)\quad \int_{R^n} |x|^{\alpha}|\Delta u|^2 dx \geq S_\alpha \left(\int_{R^n} |x|^{\frac{n\alpha}{n-4}}|u|^{**} dx \right)^{2/2^{**}}, \; u \in C_c{}^{\infty}\,(R^n)$$

where $n \geq 5,\ \alpha > 4-n$ and $2^{**} = 2n/(n-4)$ is the critical Sobolev exponent. Notice that (1) reduces to the standard Sobolev inequality if α = 0. In particular, in this talk I will give partial answers to the following questions:

(i) is the best constant S_α positive?
(ii) is S_α achieved on a suitable function space?

(iii) are extermals radially symmetric or symmetry breaking occurs?

The question raised in (ii) is deeply related to the existence of nontrivial solutions to the following Brezis-Nirenberg type problem:

$(2)\quad \Delta^2 u - \lambda \Delta u = |u|^{2^{**}-2} u$ on B, $u = |\nabla u| = 0$on ∂B,

where B is the unit ball in R^n and λ is a given parameter.
Problem (2) has an independent interest. Some partial existence and nonexistence results can be found in [2], where in particular we prove that n = 5 is the only ”critical dimension” for problem (2).

[1] P. Caldiroli, R.M. Rellich inequalities with weights, Calc. Var. Partial Differential Equations, to appear
[2] P. Caldiroli, R.M. Caffarelli-Kohn-Nirenberg type inequalities for the weighted biharmonic operator in cones, Milan J. Math, to appear.

Monica Musso
Pontificia Universidad Católica de Chile

Critical problems in R^2

We study the following boundary value problem

(1) $\Delta u + \lambda u^{p-1} e^{u^p} = 0,\ u > 0$ in Ω,
$u = 0$ on Ω,
where Ω is a bounded domain in R^2 with smooth boundary, $\lambda > 0$ is a small parameter and $1 \le p \le 2$.
We construct the bubbling solutions to problem (1) by Lyapunov-Schmidt reduction procedure.

Frank Pacard
Ecole Polytechnique

Sign changing solutions for the nonlinear Schrödinger equation

I will present some recent joint work with W. Ao, M. Musso and J. Wei on the existence of finite energy standing waves for the nonlinear Schrödinger equation in n-dimensional Euclidean space. This result leads in particular to solutions which do not have any symmetry. The construction is inspired by a similar construction in the framework of constant mean curvature surfaces by N. Kapouleas or M. Jleli and myself. I shall also describe a similar construction by F. Ting and J. Wei for the Ginzburg-Landau equation. All constructions exploit the non compactness of the space of solutions of the equation.

Filomela Pacella
University of Rome I

Solutions of Lane Emden problems in tubular neghborhoods of manifolds

I present some results about the existence of new positive solutions for the classical Lane-Emden equation in bounded domains which are tubular neighborhoods of compact manifolds, with Dirichlet boundary conditions. The exponent of the nonlinearity can be either subcritical or critical , or supercritical.

The domain will depend on a small positive parameter, but the solutions will not concentrate in points when the parameter goes to zero.
These results are contained in a joint paper with F.Pacard and B.Sciunzi.

Angela Pistoia
University of Rome I

On the stability of the Paneitz-Branson equation

In this talk, I will describe a joint work (with G. Vaira) about the existence of blowing-up solutions for linear perturbations of the Paneitz-Branson equation.

Frederic Robert
University of Nancy

Sign-changing blow-up for scalar curvature type equations

Given *(M,g)* a compact Riemmanian manifold of dimension *n>2*, we are interested in the existence of blowing-up sign-changing families $(u_\epsilon)_{\epsilon>0} \subset C^{2,\theta}(M)$, $\theta \in (0,1)$, of solutions to

$\Delta_g u_\epsilon + hu_\epsilon = |u_\epsilon|^{\frac{4}{n-2}-\epsilon} u_\epsilon$ in *M*,

where $\Delta_g := div_g \nabla$ and $h \in C^{0,\theta}(M)$ is a potential. We prove that such families exist in two main cases: in small dimension $n \in \{3,4,5,6\}$ for any potential *h* or in any dimension when $h \equiv \frac{n-2}{4(n-1)} K_g$ and *(M,g)* is locally conformally flat. These examples complete previous existence and nonexistence results on blowing-up solutions and allow to have a complete panorama of the stability/instability of critical elliptic equations of scalar curvature type on compact manifolds. The changing of the sign is necessary due to the compactness results of Druet and Schoen.

P. N. Srikanth
TIFR-CAM

Semilinearly perturbed elliptic equations with solutions concentrating on a 1-dimensional orbit

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Kyril Tintarev
Uppsala University

Weak continuity properties of the Trudinger-Moser functional.

We show that the functional $\int_\Omega (e^{4\pi u^2} - 1)$ on the set $\{||\nabla u||_2 \leq 1\}$

lacks weak continuity only (up to a remainder vanishing in the gradient norm) on sequences of Moser functions concentating at arbitrary points. The proof uses a profile decompostion based on logarithmic dilations. We also show that Palais-Smale sequences associated with the Turidnger-Moser functional have asymptotic profiles shaped like a toy pyramid. Most of this work is in collaboration with Adimurthi.

Jérome Vétois
University of Nice

Bubble tree decompositions for critical anisotropic equations

We describe the asymptotic behavior in energy space of Palais-Smale sequences for an anisotropic problem on a domain in the Euclidian space. This description is well-known in the isotropic case (Struwe '84). In the general case, we emphasize the crucial role played by the geometry of the domain.

Monica Visan
UCLA

The energy-supercritical nonlinear wave equation

We present the proof of global well-posedness and scattering for the energy-supercritical nonlinear wave equation in three space dimensions for solutions with bounded critical norm. This is joint work with Rowan Killip.