ICTS Summer School on Numerical Relativity 2013 – Mathematical formulation

Tutorial # 5

5.1 (a) Use the results of Problem 3.5 to show that $\partial_t K \geq 0$ for *geodesic slicing* (as long as $\rho + S \geq 0$.)
(b) Now consider a vacuum spacetime (say, a gravitational wave), split K_{ij} into its trace and trace-free parts, and neglect the trace-free part to find a differential equation for K alone.
(c) Solve the differential equation of part (b), assuming that $K = K_0 > 0$ at some time $t = t_0$. Argue that this solution represents a lower limit for K for all $t \geq t_0$ even when the trace-free part of K_{ij} is nonzero. Find an upper limit for the time at which a coordinate singularity will develop, as $K \to \infty$. Express your answer in terms of K_0 and t_0.

5.2 (a) Consider *harmonic slicing* with

$$\partial_t \alpha = -\alpha^2 K \qquad\qquad \beta^i = 0. \tag{1}$$

Use the results of Problem 3.5 to show that

$$\alpha = C(x^i)\,\gamma^{1/2}, \tag{2}$$

where $C(x^i)$ is a constant of integration that depends on the spatial coordinates x^i only.
(b) Now consider "1+log" slicing with

$$\partial_t \alpha = -2\alpha K \tag{3}$$

to show that $\alpha = 1 + \ln\gamma$, where a constant of integration has been chosen appropriately. This explains the name of this slicing condition.

5.3 Consider the reformulation of Maxwell's equations that we discussed in class,

$$\begin{aligned}
\partial_t A_i &= -E_i - D_i\Phi \\
\partial_t E_i &= -D_j D^j A_i + D_i\Gamma - 4\pi j_i \\
\partial_t \Gamma &= (a^2 - 1)D^i E_i - D_i D^i \Phi - 4\pi a^2 \rho.
\end{aligned} \tag{4}$$

Show that constraint violations $\mathcal{C} \equiv D_i E^i - 4\pi\rho$ satisfy the wave equation

$$(-\partial_t^2 + a^2 D_i D^i)\mathcal{C} = 0. \tag{5}$$