A Survey of Kleinian Groups: Lattices

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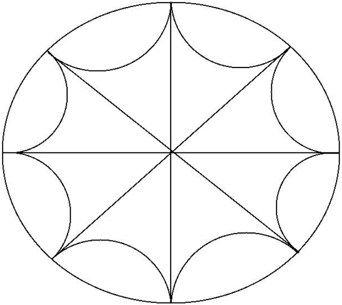

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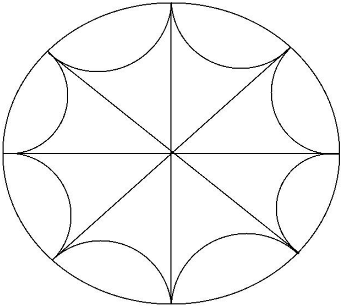

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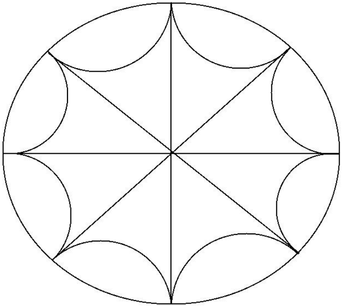

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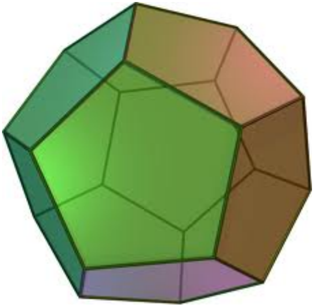
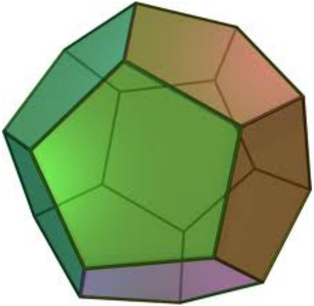


Example: The Regular Dodecahedron
Glue via 3/5ths twist.
Example of a cocompact lattice.



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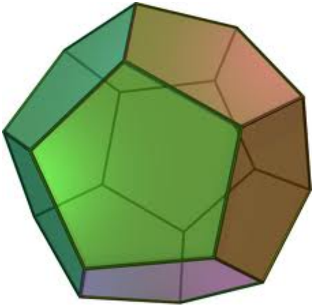

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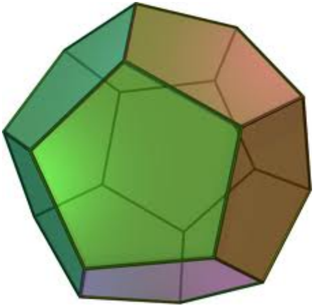

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18. Does every hyperbolic 3-manifold have a finite-sheeted cover which fibers over the circle? This dubious-sounding question seems to have a definite chance for a positive answer.

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(1) Surface Subgroup Conjecture: – Kahn and Markovic
Every closed hyperbolic 3-manifold has immersed incompressible surfaces.

Broad Scheme:

a) Find (almost) flat pairs of hyperbolic pants ($\pi_1 = F_2$) all whose cuffs are of length R.

b) Glue pairs along common geodesic boundary by small bending and twist by (about) 1.

c) Use mixing properties to show that perfect pairing is posssible

d) Small bending ensures that resulting surface is "almost geodesic".

e) Get very large collection of surfaces.

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(2) Group Theoretic Criterion for Virtual Fibering: – Agol

Definition

A group G is residually finite $\mathbb{Q}$-solvable or RFRS if there is a sequence of subgroups $G = G_0 > G_1 > G_2 > \cdots$ such that G_i's are normal, $\cap_i G_i = \{1\}$, $[G : G_i] < \infty$ and G_{i+1} contains the i-th term of the rational derived series.

Theorem

Let M be a connected orientable aspherical 3-manifold such that $\pi_1(M)$ is RFRS. Then M is virtually fibered.

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A group G is residually finite $\mathbb{Q}$-solvable or RFRS if there is a sequence of subgroups $G = G_0 > G_1 > G_2 >$ such that G_i's are normal, $\cap_i G_i = \{1\}$, $[G : G_i] < \infty$ and G_{i+1} contains the i-th term of the rational derived series.

Theorem

Let M be a connected orientable aspherical 3-manifold such that $\pi_1(M)$ is RFRS. Then M is virtually fibered.

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(3) CAT(0) Cubulated Groups: – Sageev, Haglund-Wise

CAT(0) = singular non-positive curvature.

Cubulated = built of Euclidean cubes.

G – acts cocompactly, properly discontinuously on a proper CAT(0) cubulated cell-complex.

Theorem

(Haglund-Wise) Let M be a cubulated hyperbolic 3-manifold all whose finitely generated subgroups are separable. Then M is virtually special.

Virtually Special = Embedded hyperplanes in M such that proper transversality conditions hold in the cubulated category.

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